POLITONG - SHANGHAI

BASIC AUTOMATIC CONTROL

Exam grade

March 22, 2011 Academic Year 2010/11

NAME (pinyin/italian)..
MATRICULATION NUMBER...
SIGNATURE……………………………………………

- Use only these pages (including the back) for answers.
- Do not use additional sheets.
- Use of any book, note, or other didactic material is not allowed.
- Write clearly and be explicit and concise in your answers
- [N] in the text must be substituted with the number of letters of your given name.

EXERCISE 1

Determine the stability of the equilibrium states of the following nonlinear system and how they vary when the parameter p goes from -∞ to +∞.

$$\begin{aligned}\dot{x}_1 &= x_1 - p x_1^3\\ \dot{x}_2 &= [N]x_1 - x_2\end{aligned}$$

Assume [N] = 7 (n. of letters in Giorgio). Then, to compute the equilibrium states we have:

$$\begin{matrix} 0 = x_1 - p x_1^3 \\ 0 = 7x_1 - x_2 \end{matrix} \quad \text{that is} \quad \begin{matrix} x_1(1 - p x_1^2) = 0 \\ x_2 = 7x_1 \end{matrix} \quad \text{which means} \quad \begin{matrix} x_1 = 0 \\ x_2 = 0 \end{matrix} \quad \text{or} \quad \begin{matrix} x_1 = \pm\sqrt{1/p} \\ x_2 = \pm 7\sqrt{1/p} \end{matrix}.$$

The last 2 equilibria are defined only for $p>0$ and thus we have only 1 equilibrium (the origin) for $p \le 0$ and 3 equilibria for positive p.

To analyse their stability, we linearize the system

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 1 - 3p x_1^2 & 0 \\ 7 & -1 \end{bmatrix}. \quad \text{If we evaluate this matrix in the origin, we obtain } \begin{bmatrix} 1 & 0 \\ 7 & -1 \end{bmatrix}.$$

The matrix is triangular and thus the eigenvalues are on the diagonal. They are 1 and -1, which means the origin in a saddle (unstable) whatever the value of p.

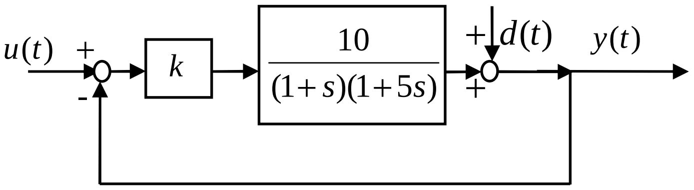
In the other two equilibria, we substitute the value of x_1 and obtain:

$$A = \begin{bmatrix} 1 - 3p\frac{1}{p} & 0 \\ 7 & -1 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 7 & -1 \end{bmatrix},$$ which is again triangular and independent from p.

The eigenvalues are thus -2 and -1 and the equilibria are nodal sinks (stable).

EXERCISE 2

Given the control system in the figure



a) Fix a value for the gain *k* such that the effect on the output of a disturbance *d*(*t*)= [N]step(t-3) is attenuated at least 10 times for *t*→∞.

b) For the value of *k* defined at the previous point, say what the approximate duration of the transient will be.

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a) The transfer function from *d* to *y* is $\frac{1}{1+\frac{10k}{(1+s)(1+5s)}}=\frac{(1+s)(1+5s)}{(1+s)(1+5s)+10k}$.

The Laplace transform of the input is $\frac{7e^{-3s}}{s}$. However, the initial delay of 3 is clearly irrelevant for our problem (we have to look at the effect of *d* for *t*→∞) and thus can be disregarded.

We can simply use the final value theorem and write: $\lim_{t\to\infty} y(t)=\lim_{s\to 0} s\frac{7}{s}\frac{(1+s)(1+5s)}{(1+s)(1+5s)+10k}\leq\frac{7}{10}$

that is $\frac{7}{1+10k}\leq\frac{7}{10}$ and $k\geq 0.9$. For instance, $k = 1$.

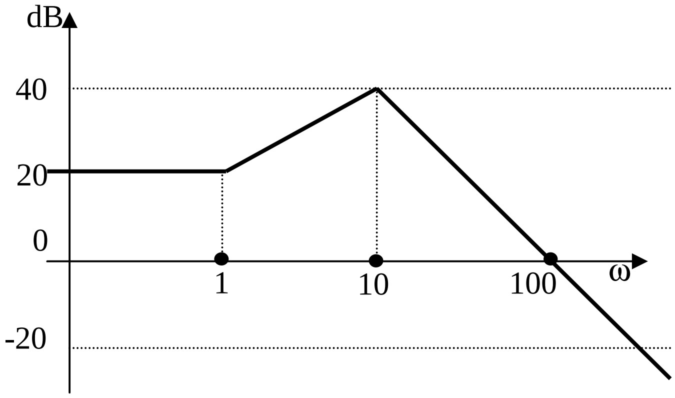
Note that the text asks for an attenuation of 10 times, which means that the component of the output due to the disturbance, must be 10 times smaller than the disturbance itself. If the disturbance is 7, its asymptotic effect on the output must be smaller than 0.7.

b) Assuming $k = 1$, the denominator of the feedback transfer function becomes $5s^2+6s+11$, that can be solved for the poles (it's second order), thus $s_{1,2}=\frac{-6\pm\sqrt{36-220}}{10}$. The real part of the poles is negative, which means the system is asymptotically stable and thus the transient lasts for about 5 times the dominant time constant, which is the inverse of the opposite of the real part of the poles. Thus about $5\cdot 10/6= 8.33$ time units.

Other solution methods are possible.

EXERCISE 3

A minimum phase system has the phase Bode plot in the figure.



a) Draw the corresponding phase plot.
b) Write a possible transfer function for the system.
c) Determine the system response to an impulsive input.
d) Determine the asymptotic output when the input is u(t)=10sin(t)+sin(10t).
e) Explain clearly if a direct feedback with such a system is asymptotically stable and why.

Please note that the answer to questions c-d above may be qualitative or quantitative.

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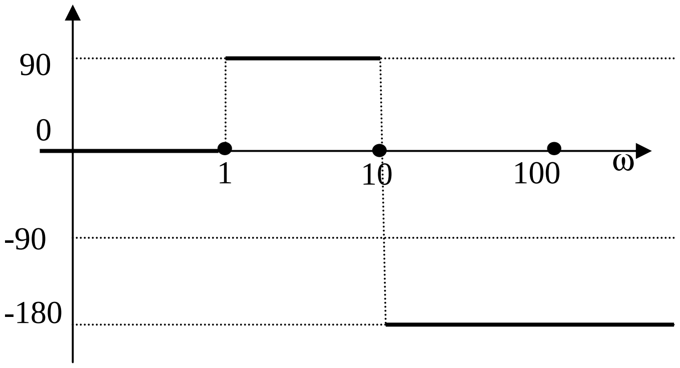
a) In minimum phase systems, the phase plot can be derived from the magnitude one. Taking into account the slope of the three segments in the magnitude plot, we can draw →



b) Thus $G(s)=\dfrac{10(1+s)}{(1+0.1s)^3}$

c) The Laplace transform of an impulsive input is 1 and thus Y(S)=G(s). For a qualitative response, it is sufficient to determine the initial and final limits.

$$\lim_{t\to 0} y(t) = \lim_{s\to\infty} sG(s) = 0$$

$$\lim_{t\to 0} \dot{y}(t) = \lim_{s\to\infty} s^2G(s) = 100$$

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sG(s) = 0$$

Thus the output starts from 0, with a positive high slope and returns asymptotically to 0. Alternatively, one can antitransform the transfer function.

d) The asymptotic behavior is the sum of the output of the two sinusoidal inputs, i.e.

$$y(t)= 10|G(j1)|\sin(t+\angle G(j1))+ |G(j10)|\sin(10t+\angle G(j10))$$

The magnitudes and the phases can be computed analytically or one can just see the two plots. From the first, the two magnitudes are about 10 and about 100, the two phases about 30° and more negative in $\omega=10$ (the effect of the 3 poles must be stronger than the effect of the single zero). Indeed the precise values are:

$|G(j1)| =13.9 \quad \angle G(j1)=0.48=28° \quad |G(j10)|=35.5 \quad \angle G(j10)= -0.88=-50°$

e) To check the stability of the direct feedback connection, one can use either the Bode or the Nyquist criteria. One can say for instance that the phase for ω_c (where the magnitude in db is 0) is less than 180° and thus the phase margin is larger than 0.

EXERCISE 4
Answer the following questions, using only the available space.

a) Given the linear continuous-time system below, is it possible to find a control law $u=kx+v$ such that both the eigenvalues become equal to -3? Why?

$$A=\begin{bmatrix}[N] & 0\\ 1 & -2\end{bmatrix}\qquad b=\begin{bmatrix}1\\ 0\end{bmatrix}\; c=\begin{bmatrix}0 & 1\end{bmatrix}$$

Again using $[N]=7$, we have $R=|b\ Ab|=\begin{bmatrix}1 & 7\\ 0 & 1\end{bmatrix}$, whose rank is 2 ($\det(R)\neq 0$) and thus the eigenvalues can be fixed freely with a suitable control law of the type $u=kx+v$. In particular, they can be fixed both to -3.

b) The eigenvalues of a linear DISCRETE TIME system are $s_1=1/[N]$, $s_2=-0.2$, $s_3=1$. Is the system asymptotically stable? Why?

Th condition for asymptotic stability in discrete time systems is $|s_i|<1$ for every i. Thus the system in the text cannot be asymptotically stable. It is indeed only stable, since one eigenvalue is equal to 1 and the others are smaller (in magnitude).

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c) Given the Bode plots of an open-loop system in the figure, what are (approximately) the gain and phase margins when connected in a direct feed-back loop?

The phase margin can be read on the lower diagram, when the first intercepts the horizontal axis. In this case, the critical phase is about 110° and thus the phase margin is 180-110=70°.

The gain margin is read on the magnitude plot when the phase is equal to 180°. In this case, the phase never reaches such value and thus the gain margin is infinite.

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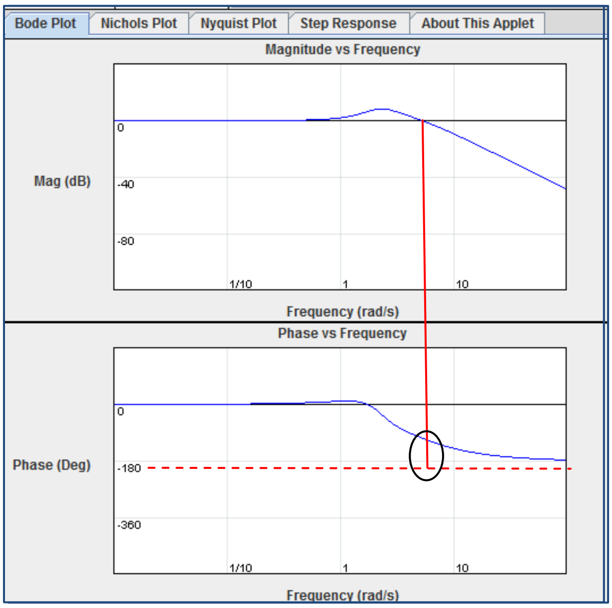

d) A system with transfer function $G(s)=\dfrac{(1+2s)}{(1+s)(1+[N]s)^2}$ is connected with a direct feedback loop. Is the closed loop asymptotically stable? Why?

The system has a unit gain and thus its Nyquist plot rotates always inside the unit circle, starting from the real value 1. It cannot ever reach -1 and thus the Nyquist theorem guarantees its asymptotic stability.

Note that many other solutions are possible (e.g. Bode criterion).