POLITONG - SHANGHAI

BASIC AUTOMATIC CONTROL

July, 2011 Academic Year 2010/11

Exam grade

NAME (pinyin/italian)...

MATRICULATION NUMBER..

Exercise grades

SIGNATURE……………………………………………

- Use only these pages (including the back) for answers.
- Do not use additional sheets.
- Use of any book, note, or other didactic material is not allowed.
- Write clearly and be explicit and concise in your answers.
- [N] in the text must be substituted with the number of letters of your given name.
- In case of doubts on the text, write "I assume…" and continue coherently.

EXERCISE 1

Determine the stability and the type of the equilibrium states of the following nonlinear system

$$\dot{x}_1 = -2x_1 - x_2 + 5$$
$$\dot{x}_2 = 3x_1 x_2 + [N]$$

Draw the approximate behaviour of the trajectories around the equilibria.

We first compute the equilibria (say [N]=6)

$$\begin{matrix} -2x_1 - x_2 + 5 = 0 \\ 3x_1 x_2 + 6 = 0 \end{matrix} \quad \text{from which} \quad \begin{matrix} x_2 = -2x_1 + 5 \\ 3x_1(-2x+5)_1 + 6 = 0 \end{matrix} \quad \text{hence } x' = \begin{bmatrix} 2.85 \\ -0.7 \end{bmatrix} \quad \text{and } x'' = \begin{bmatrix} -0.35 \\ 5.7 \end{bmatrix}$$

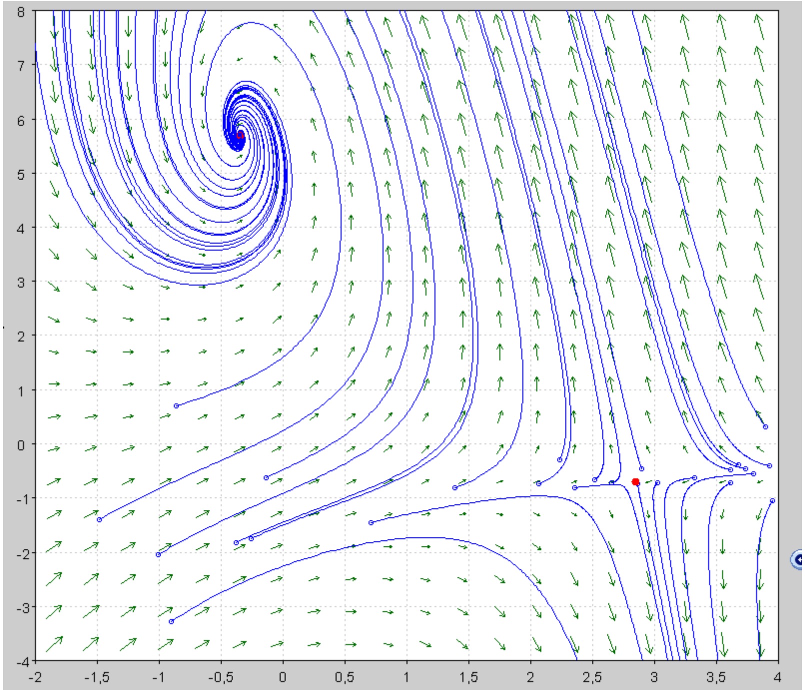
To determine the type and stability of equilibria, we linearize

$\frac{\partial f}{\partial x} = \begin{bmatrix} -2 & -1 \\ 3x_2 & 3x_1 \end{bmatrix}$ which we evaluate in correspondence to equilibria

$A' = \begin{bmatrix} -2 & -1 \\ -2.1 & 8.5 \end{bmatrix}$ whose eigenvalues are λ_1= 8.7, λ_2 = -2.2 i.e. the equilibrium is a saddle (unstable)

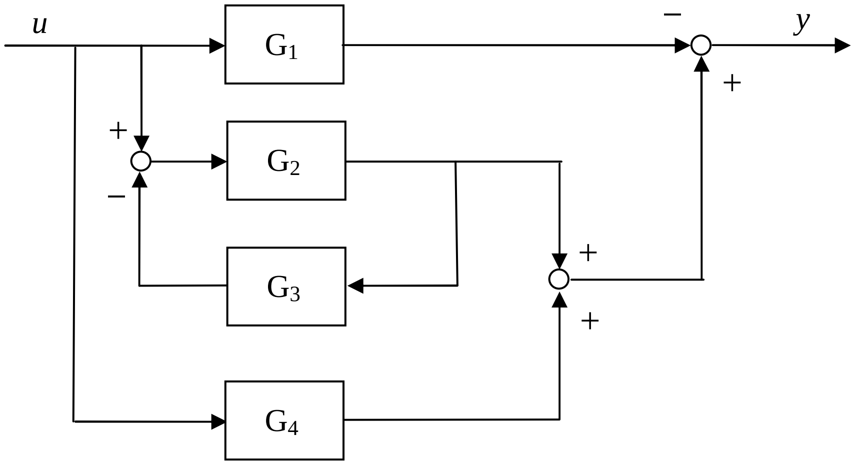
$A'' = \begin{bmatrix} -2 & -1 \\ 17.1 & -1 \end{bmatrix}$ whose eigenvalues are λ_1= -1.5+j4.1, λ_2 = -1.5-j4.1i.e. the equilibrium is a spiral sink (stable)

The trajectories are shown in the picture.



EXERCISE 2

Given the system in the figure



where

$$G_1(s) = \frac{5}{s^2+2s+1} \qquad G_2(s) = \frac{1}{s+2} \qquad G_3(s) = \frac{1}{s} \qquad G_4(s) = \frac{1}{s+1}$$

a) determine if the system is asymptotically stable;
b) determine the dominant time constant of the system;
c) compute the approximate response *y* when the input is *u*=2step (*t*-4)

The system is the parallel of G_1, G_4 and the feedback between G_2 and G_3.

A parallel connection does not modify the eigenvalues (i.e. the stability) and G_1 and G_4 are obviously stable. Hence the only problem is the stability of the feedback between G_2 and G_3.

Its transfer function G = $G_2/(1+G_2G_3)$ = $\dfrac{\frac{1}{s+2}}{1+\frac{1}{s+2}\frac{1}{s}} = \dfrac{s}{s^2+2s+1} = \dfrac{s}{(s+1)^2}$.

Thus also the feedback is stable and the overall system is stable as well.

The overall transfer function G_{tot} is thus

$$G_{tot} = \frac{5}{(s+1)^2} + \frac{s}{(s+1)^2} + \frac{1}{s+1} = \frac{5+2s+1}{(s+1)^2}$$

and there is only one time constant =1, which is obviously also the dominant one.

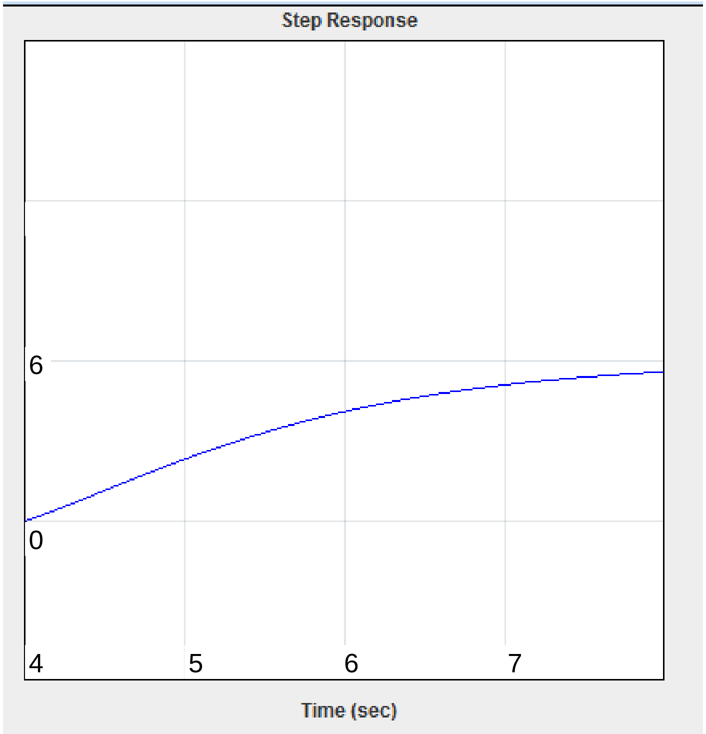
To determine the approximate response, we first notice that it will start at *t*=4 (nothing happens before) and we use the limit theorems.

$$\lim_{t\to 0} y(t) = \lim_{s\to\infty} s\frac{2}{s}G_{tot} = 0$$

$$\lim_{t\to 0} \dot{y}(t) = \lim_{s\to\infty} s^2\frac{2}{s}G_{tot} = 2$$

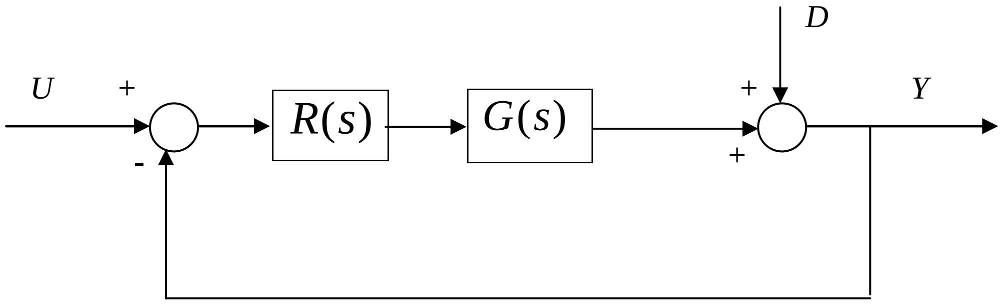
$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} s\frac{2}{s}G_{tot} = 6$$

So the output starts from the 0 at time 4, with a positive (2) derivative and ends in 6 without oscillations (real poles).



EXERCISE 3

Consider the following control system



where:

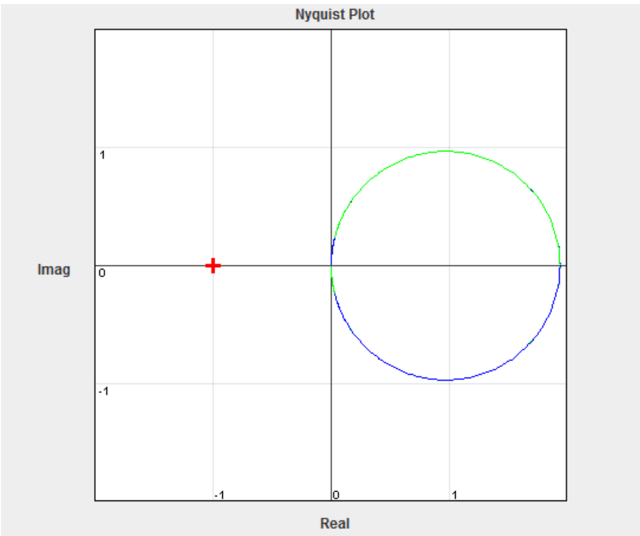
$$G(s) = \frac{10[N]s}{(1+10\,s)\ (1+0.1s)}$$

a) Verify the stability of the system without the regulator (i.e. $R(s)$=1) and determine the phase margin.

b) Determine whether it is possible to reduce hundred times the amplitude of a sinusoidal disturbance D having a period T=100, using a purely proportional regulator , i.e. $R(s) = k$.

Assume for instance [N]=7.

If R(s) = 1, the open loop transfer function is again G(s), it is second order with stable poles, thus its Nyquist plot cannot turn around -1 and, for the Nyquist criterion, we are sure of the stability of the feedback system (the figure is not in the correct scale, for ω=0, μ=70).



For the second question, we compute the feedback transfer function F(s) between D and Y

$$F(s) = \frac{1}{1+\dfrac{70ks}{(1+10s)(1+0.1s)}} = \frac{(1+10s)(1+0.1s)}{s^2+(10.1+70k)s+1}$$

Now we substitute $s=j\omega$ and what we want to obtain is a value of k such that, for $\omega=2\pi/100=0.063$ Re(F(j0.063)) ≤ 0.01 with some approximations this means $\frac{56k+30}{426k^2+123k+30} \leq 0.01$ i.e. $k \geq 15$.

Note that the final result obviously depends on [N] and can also be obtained by checking some numerical values for k instead of solving the inequality (as somebody did).

Alternatively, one may obtaining a similar result by moving the Bode plot in the required position: below -40dB for $\omega=0.063$.

EXERCISE 4
Answer the following questions, using only the available space.

a) Given the linear continuous-time system below, is it possible to find a control law $u=kx+v$ that stabilizes the system? Why?

$$A=\begin{bmatrix}2 & 0\\ 1 & -2\end{bmatrix} \quad b=\begin{bmatrix}[N]\\ 0\end{bmatrix} \quad c=\begin{bmatrix}0 & 3\end{bmatrix}$$

Compute R = [b Ab] = $\begin{bmatrix}[N] & 2[N]\\ 0 & [N]\end{bmatrix}$ *since its rank is 2, eigenvalues can be arbitrarily fixed with a state feedback control law. So the system can also be stabilized.*

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b) The eigenvalues of a linear continuous time system are $s_1=1/[N]$, $s_2=-0.2$, $s_3=1$. What is the approximate value of the settling time? Why?

There is no settling time for this system, because there are positive eigenvalues and thus the system is unstable.

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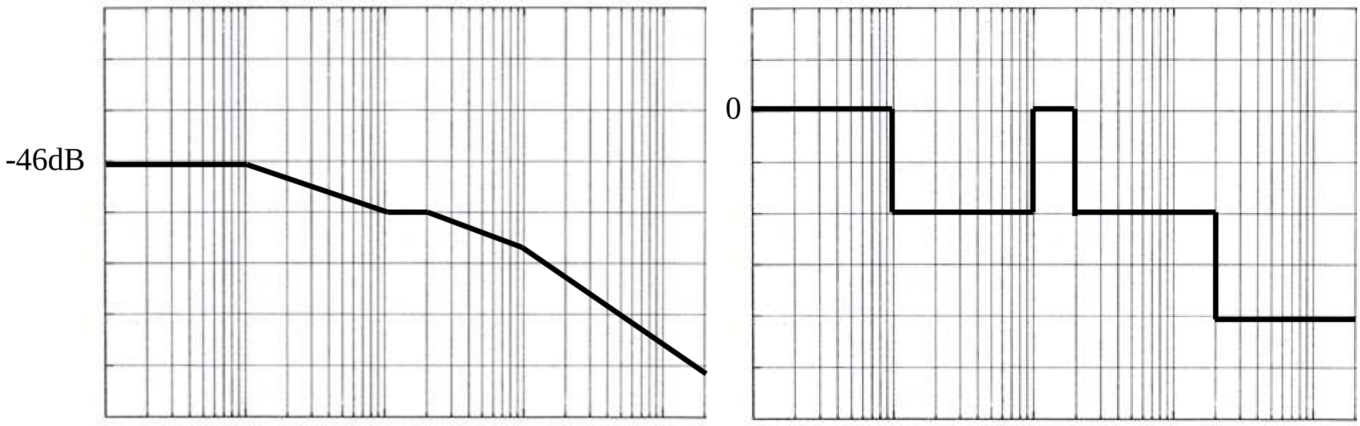
c) Draw the approximate Bode plots of the system $G(s)=\dfrac{s+10}{(s+1)(s+20)(s+100)}$



d) The step response of a linear continuous time system has the characteristics in the table. Draw its approximate behavior.

Steady-state value	10
Settling time	25
Peak time	5
Maximum overshoot	4
Damping factor	2=A/B