POLITONG - SHANGHAI

BASIC AUTOMATIC CONTROL

September, 2011 Academic Year 2010/11

Exam grade

NAME (pinyin/italian)..

MATRICULATION NUMBER...

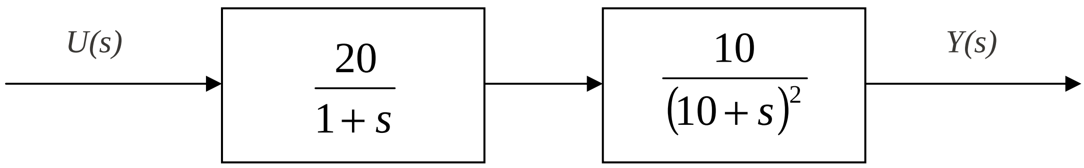
SIGNATURE……………………………………………

Exercise grades

- Use only these pages (including the back) for answers.
- Do not use additional sheets.
- Use of any book, note, or other didactic material is not allowed.
- Write clearly and be explicit and concise in your answers.
- [N] in the text must be substituted with the number of letters of your given name.
- <u>In case of doubts on the text</u>, write “I assume…” and continue coherently.

EXERCISE 1

Given the following linear system



with *u*= [N]step(*t*), determine:

- all the qualitative characteristics of the output
- the analytical expression of the output.

Assume, for instance, [N]=5.

$u(t) = 5\,\text{step}(t) \qquad U(s) = \frac{5}{s}$

$G(s) = \frac{20}{1+s} \cdot \frac{10}{(10+s)^2} = \frac{\cancel{200}\ 2}{\cancel{100}(1+s)(1+0,1s)^2} = \frac{2}{(1+s)(1+0,1s)^2}$

$Y(s) = G(s)\,U(s) = \frac{5}{s} \cdot \frac{2}{(1+s)(1+0,1s)^2} = \frac{10}{s(1+s)(1+0,1s)^2}$

$\mu = 2$

All poles are real $\rightarrow$ no oscillation

4 poles $\rightarrow$ $s=0$, $s=1$, $s=10$ mult 2

THE G(S) IS AS. STABLE BECAUSE IT IS A SERIES OF TWO AS. STABLE COMPONENT.

$\lim_{t\to 0} y(t) = \lim_{s\to\infty} sY(s) = 0 \qquad \lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = 10 \qquad \lim_{t\to 0} \dot{y}(t) = \lim_{s\to\infty} s^2 Y(s) = C$

$$\lim_{t\to 0} \ddot{y}(t) = \lim_{s\to\infty} s^3 Y(s) = 0$$

To determine the analytical expression we have to use Heaviside decomposition

$$\frac{10}{s(1+s)(1+0.1s)^2} = \frac{A}{s} + \frac{B}{1+s} + \frac{C}{1+0.1s} + \frac{D}{(1+0.1s)^2}$$

Which gives the following system of linear equations:

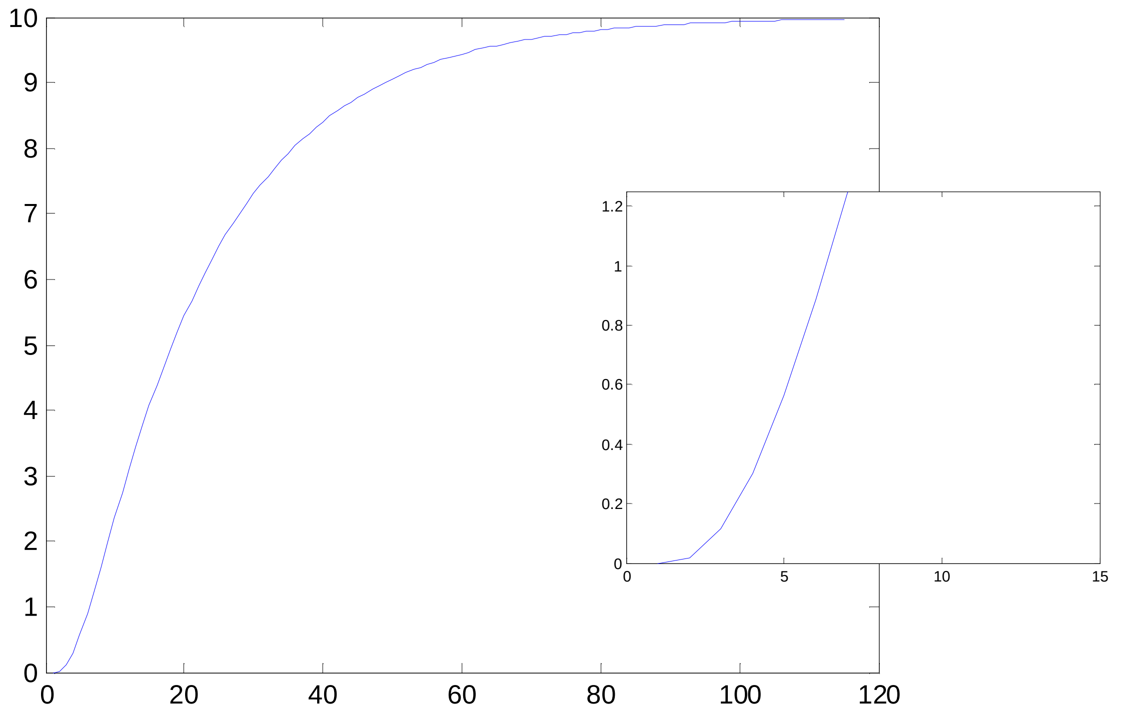
$$\begin{cases} A+B+C=0 \\ 21A+20B+11C+D=0 \\ 120A+100B+10C+D=0 \\ A=10 \end{cases}$$

Which means $A=10, \quad B=-12.35, \quad C=2.35, \quad D=11.11$, i.e.

$$y(t)=10-12.35e^{-t}+2.35e^{-0.1t}+11.11te^{-0.1t}$$

This is coherent with the values in 0 and ∞ with the qualitative analysis above.

The result is shown in the figure below, with a zoom of the initial values. For a unit step, the final value if [N] times smaller.



EXERCISE 2

Say, clearly justifying the answer, which of the three linear continuous-time systems characterized by the following state transition matrices has the shortest settling time and what is its value.

$$A_1 = \begin{bmatrix} -1 & 0 & 1/3 & 0 \\ 2 & -2 & -1 & 5 \\ 0 & 0 & -[N] & 0 \\ 0 & 0 & 3 & -4/[N] \end{bmatrix} \quad A_2 = \begin{bmatrix} -2 & 1 \\ -3 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} 2 & 0 & 1/2 \\ 1 & -1 & 2 \\ -1 & 0 & 1/[N] \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -2 & 1 \\ -3 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} s+2 & 1 \\ -3 & s-1 \end{bmatrix} \qquad s^2+s+1=0 \qquad s_{1,2} = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$$

$$\begin{cases} \operatorname{tr} A_2 < 0 \\ \det A_2 > 0 \end{cases} \quad \text{As. STABLE} \qquad t'' = 10$$

$$A_1 = \left[\begin{array}{cc|cc} -1 & 0 & 1/3 & 0 \\ 2 & -2 & -1 & 5 \\ \hline 0 & 0 & -5 & 0 \\ 0 & 0 & 3 & -4/5 \end{array}\right] \xrightarrow[R_4 = R_4 + \frac{3}{5}R_3]{R_2 = R_2 + 2R_1} \begin{bmatrix} -1 & 0 & 1/3 & 0 \\ 0 & -2 & -1/3 & 5 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -4/5 \end{bmatrix} \qquad t' = 5 \cdot \frac{5}{4} = \frac{25}{4} = 6{,}25$$

The matrix is block-triangular it can also be analyzed by just looking at the 2 x 2 matrices on the diagonal.

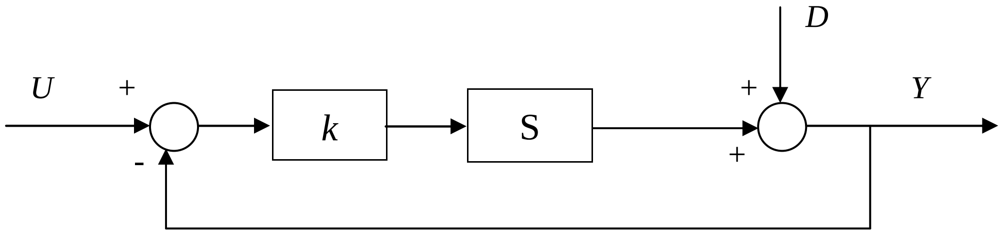
$$A_3 = \begin{bmatrix} 2 & 0 & 1/2 \\ 1 & -1 & 2 \\ -1 & 0 & 1/5 \end{bmatrix} \qquad \operatorname{tr} A = 2 - 1 + 1/5 > 0$$

Not stable,
No settling time

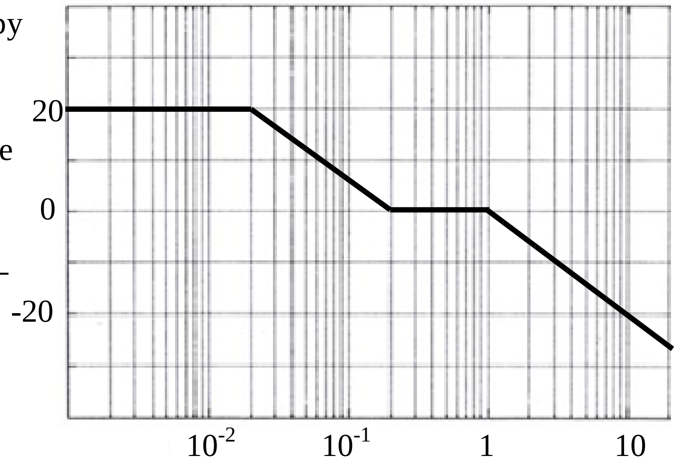
A_1 has the shortest settling time and the value is <u>6.25</u>

EXERCISE 3

Consider the control system shown below



where S is a minimum phase system defined by the magnitude Bode plot in the figure.



Determine a value of *k* such that the effect on the output of a constant disturbance is reduced asymptotically at least [N] times.

$$S(s) = \frac{10(1+5s)}{(1+50s)(1+s)}$$

$$J(s) = \frac{Y(s)}{D(s)} = \frac{1}{1+\frac{10k(1+5s)}{(1+50s)(1+s)}} = \frac{(1+50s)(1+s)}{(1+50s)(1+s)+10k(1+5s)}$$

$$= \frac{s^2+51s+1}{s^2+(51+50k)s+1+10k}$$

If we want to reduce the constant disturbance we need to move the bode plot down of 14 dB (Feedback J(s) plot)

$$\lim_{s \to 0} \cancel{s \frac{1}{s}} J(s) = \frac{1}{1+10k} < \frac{1}{5}$$

$$1+10k > 5$$

$$10k > 4$$

$$k > 0{,}4$$

EXERCISE 4
Answer the following questions, using only the available space.

a) Given the linear continuous-time system below, is it possible to find a control law $u=kx+v$ that stabilizes the system? Why?

$$A=\begin{bmatrix} 2 & 0 \\ [N] & -2 \end{bmatrix} \quad b=\begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}$$

$$R=\begin{bmatrix} 1 & 2 & 2 & 4 \\ -1 & 0 & 7 & 10 \end{bmatrix} \qquad \text{rank}(R)=2$$

The system is completely reachable and thus it is possible to find the required control law.

b) What is the “phase margin” of a control system?

THE PHASE MARGIN IS A VALUE USED TO RAPPRESENT THE ROBUSTNESS OF THE SYSTEM, WITH THE GAIN MARGIN. IT'S CALCULATE WITH THIS FORMULA $\varphi_m = 180 - \varphi_c$, WHERE φ_c IS THE CRITICAL PHASE. IT'S USED ON BODE CRITERION TO CONTROL THE STABILITY.

c) When the Nyquist stability criterion is useful?

THE NYQUIST STABILITY CRITERION IS USEFUL WHEN WE NEED TO FIND THE STABILITY OF THE CLOSED LOOP SYSTEM WHEN WE HAVE THE OPEN LOOP TF AND WE CAN EASILY RAPPRESENT IT.

d) A continuous-time linear system is used for a number of experiments, having each time a different sinusoidal input. The asymptotic output is measured for each case and is reported in the table.

Input u(t)	**Output y(t)**
10 sin(0.1t)	0.2 sin(0.1t+φ_1)
sin(t)	2 sin(t+φ_2)
0.1 sin([N]t)	0.5 sin([N]t+φ_3)
5 sin(100t)	0.1 sin(100t+φ_4)

What can we say about the filtering properties of the system?

THE SYSTEM IS A PASS BAND FILTER, IN FACT, THE VALUES ON FREQUENCY ω=0.1 AND ω=100 ARE ATTENUATED. THE VALUES ON ω= 1 AND ω=5 PASS TO THE OUTPUT AND THEY ARE AMPLIFIED