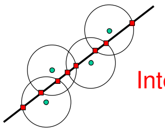
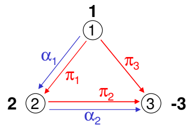


Integer and Linear Programs involving Consecutive-1 Matrices

Horst W. Hamacher *



Copy of these transparencies at:
http://optimierung.mathematik.uni-kl.de/~hamacher/2007_03_12_Milano.pdf

* partially supported by the Politecnico di Milano

In case you get these transparencies from the internet:

Please note that these transparencies are designed to support (and not to replace) an oral presentation. Parts of the material may not be understandable without.

Contents

C1 Matrix

$$Y = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} [2,4[\\ [1,3[\\ [4,6[\\ [3,6[\end{pmatrix}$$

1 2 3 4 5 6

C1-Decomposition of Integer Matrices

Given: Non-negative integer MxN matrix $A = (a_{mn})$

Find: „Good“ decomposition of A into C1 matrices

$$A = \sum_{t \in T} \alpha_t Y_t$$

with $\alpha_t \geq 0$ for all t.

T ... index set of all C1P matrices
or
T... subset of this index set

Decomposition Example

$$A = \begin{pmatrix} 2 & 5 & 3 \\ 3 & 5 & 2 \end{pmatrix}$$

$$= 2\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + 1\begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix} + 2\begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Decomposition Time

minimize $\quad DT(\alpha) := \sum_{t \in T'} \alpha_t$

such that

$$\sum_{t \in T'} \alpha_t Y_t = A$$
$$\alpha_t \geq 0$$
$$Y_t \ \text{C1-matrix}$$
$$T' \subseteq T$$

$$A = 2\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + 1\begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix} + 2\begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix} \qquad DT(\alpha)=5$$

Decomposition Cardinality

minimize $\quad DC(\alpha) := \left|\{\alpha_t : \alpha_t > 0\}\right|$

such that

$$\sum_{t \in T'} \alpha_t Y_t = A$$
$$\alpha_t \geq 0$$
$$Y_t \ \text{C1-matrix}$$
$$T' \subseteq T$$

$$A = 2\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + 1\begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix} + 2\begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix} \qquad DC(\alpha)=3$$

Decomposition Sequence

minimize $\quad DS(\alpha) := \sum_{t=1}^{T} \alpha_t + \sum_{t=1}^{T-1} s_{\sigma(t)\sigma(t+1)}$

such that

$$\sum_{t=1,\dots,T} \alpha_t Y_t = A$$
$$\alpha_t \geq 0$$
$$Y_t \ \text{C1-matrix}$$

e.g. $s_{\sigma(t)\sigma(t+1)} = \text{dist}(Y_t, Y_{t+1})$ (e.g. = maximal "movement" of ones)

$$A = 2\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + 1\begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix} + 2\begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix} \qquad DS(\alpha)=5+(1+1)$$

Objectives are contradictory

(Marc Nussbaum, 2006)

$$A = \begin{bmatrix} 8 & 5 & 6 \\ 5 & 3 & 6 \end{bmatrix}$$

	DT	DC	DS
$= 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} + 1\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} + 3\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} + 2\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$	9	4	15
$= 5\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} + 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} + 6\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$	14	3	17
$= 2\begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} + 3\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} + 1\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} + 4\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$	10	4	14

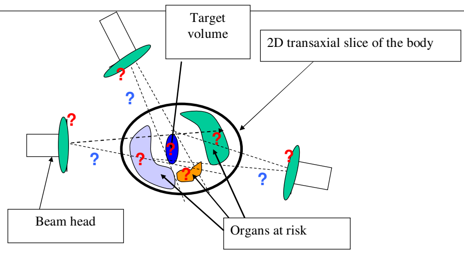
Application of C1-Dec: MLC

Modulate uniform radiation field using
Multileaf Collimator (MLC)

Intensity Modulated Radiation Therapy (IMRT)

(Project with German Cancer Association, BMBF, Univ. of Auckland, ...)

Three OR Problems in IMRT



Geometry Problem: Where does the gantry stop?

Intensity Problem: How much radiation is sent off ?

Realization Problem: How is the radiation modulated?

Survey Papers

- D.M. Shepard, M.C. Ferris, G.H. Olivera, and T.R. Mackie, Optimizing the delivery of radiation therapy to cancer patients, SIAM Rev 41 (1999), 721–744.
- A. Holder. Radiotherapy treatment design and linear programming. Technical Report 70, Trinity University, San Antonio, TX, 2002.
- OR & Oncology web site at www.trinity.edu/aholder/HealthApp/oncology

Intensity Modulated Radiation Therapy

- Geometry Problem
- Intensity Problem
- Realization Problem

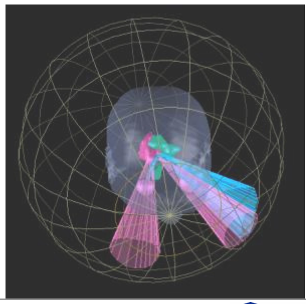
OR Problem: Location on the Sphere

Where does the gantry stop?

Ahmad S.A. Sultan, Ph.D. Thesis Kaiserslautern, ITWM, (2006)

M. Ehrgott and A. Holder, Beam Selection in Radiotherapy Design (2007)

Mangalika Jayasundara, Spherical location problem (Ph.D. thesis, 2005)

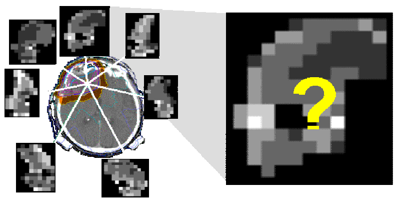


Intensity Modulated Radiation Therapy

- Geometry Problem
- Intensity Problem
- Realization Problem

Intensity Problem

Which intensity profile gives the best conformal picture of tumor?



Intensity Problem: Optimization Technique - Multi Criteria LP

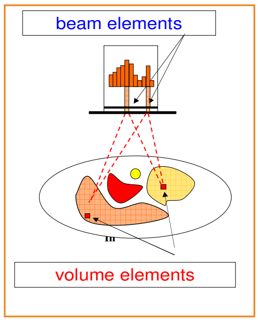
Conflicting criteria:

- high radiation in target volume (cancer cells should be destroyed)
- low radiation in organs at risk (organs should stay functional)

Calculation of Dose Distribution

Discretize

- radiation beams into beam elements (bixels)
- body parts into volume elements (voxels)



currently:
equidistant discretization

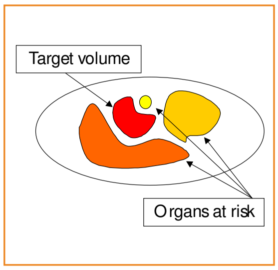
Calculation of Dose Distribution

- P(i,j) = dose in voxel i irradiated from bixel j under unit intensity
- dose volume D = P x
  (x_j = (unknown!) radiation intensity in bixel j)
- partitioned into
  target volume ($k=1$) $D_1 = P_1 x$
  and
  organs at risk ($k=2,...,K$): $D_k = P_k x$

Objective

Find intensity profile x such that dosage D=Px controls cancer without destroying healthy organs !

- **Target volume :**
  50 Gray (lower bound)
- **Organs at risk**
  20 Gray (upper bound)
  15 Gray (upper bound)
  10 Gray (upper bound



Ideal Intensity Profile

Given: Dose bounds $L_1 \geq 0$ and $U_k \geq 0,\ k=2, \dots , K$

Find: Intensity vector $x \geq 0$ satisfying the system of linear inequalities

$$D_1 = P_1 x \geq L_1 e \qquad \textit{(target condition)}$$

$$D_k = P_k x \leq U_k e,\ k = 2, \dots , K, \quad \textit{(risk conditions)}$$

Such an x does in general not exist !

Approximate Intensity Profile using Penalties

minimize $\mu_1 F_1(x) + \dots + \mu_K F_K(x)$ for given weights $\mu_1,\dots, \mu_K > 0$

- Bortfeld, Schlegel, Brahme, Gustafsson ...
  $F_1(x) := \| L_1 e - P_1 x \|_2$
  $F_k(x) := \| (P_k x - U_k e)_+ \|_2,\ k = 2, \dots , K$
  „Least square approach "
- Holmes, Mackie, Burkard, ...
  $F_1(x) := \| (L_1 e - P_1 x)_+ \|_\infty$
  $F_k(x) := \| (P_k x - U_k e)_+ \|_\infty,\ k = 2, \dots , K$
  „Minimax approach "

Disadvantage: time consuming
unsatisfying results

Parto Intensity Profile

Hamacher, Küfer: Discrete Applied Mathematics, 2002

minimize $(t_1, t_2, \dots , t_K)$

such that

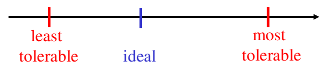
- $P_1\, x + t_1\, \mathbf{L}_1\, e \geq L_1\, e$
- $P_k\, x - t_k\, \mathbf{U}_k\, e \leq U_k\, e, \quad k = 2, \dots ,K$
- $t = (t_1, t_2, \dots , t_K) \geq 0$
- $x \geq 0$

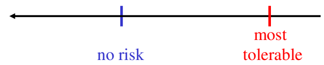
Too many Pareto solutions !

Compute a representative set of app. 100 Pareto solutions and search within the data base using graphical tool

Multi Criteria Data Base: Graphical Search Tool

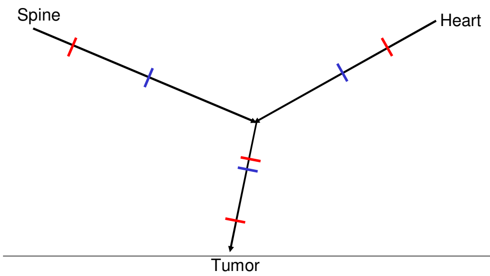
Patent: ITWM - Küfer and Trinkaus, 2002



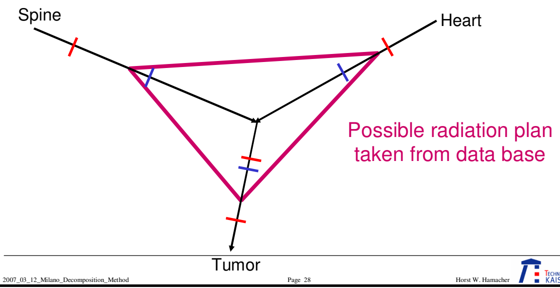


Multi Criteria Data Base: Graphical Search Tool

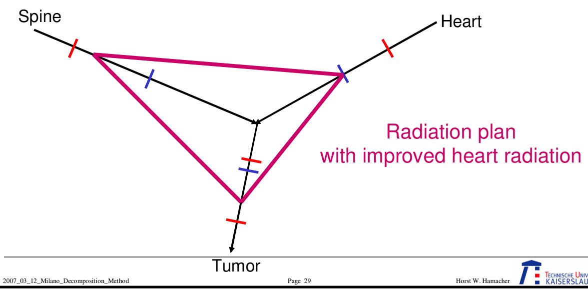
Example:



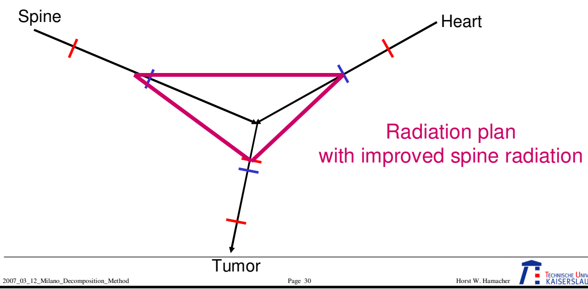
Representative System for Multicriteria Optimization



Representative System for Multicriteria Optimization



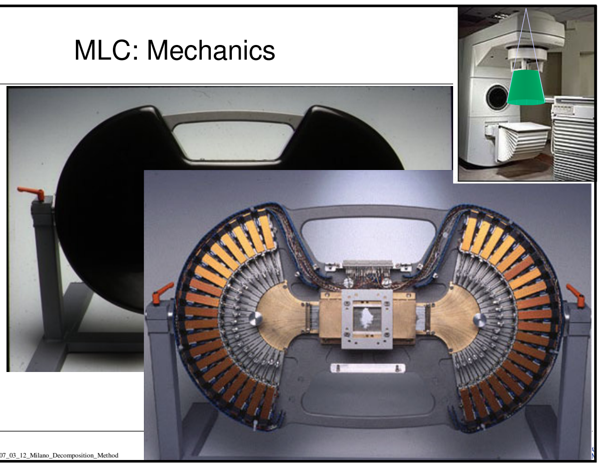
Representative System for Multicriteria Optimization



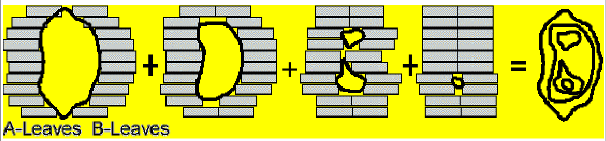
Intensity Modulated Radiation Therapy

- Geometry Problem
- Intensity Problem
- Realization Problem

MLC: Mechanics

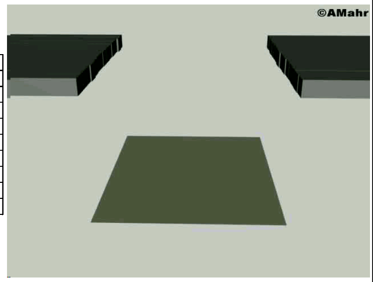


Application of C1-Dec: MLC



MLC in Action - a Simple Example

1	1	1	1	1	1	1	1	1	1
1	2	2	2	2	2	2	2	2	1
1	2	3	3	3	3	3	3	2	1
1	2	3	4	4	4	4	3	2	1
1	2	3	4	5	5	4	3	2	1
1	2	3	4	5	5	4	3	2	1
1	2	3	4	4	4	4	3	2	1
1	2	3	3	3	3	3	3	2	1
1	2	2	2	2	2	2	2	2	1
1	1	1	1	1	1	1	1	1	1

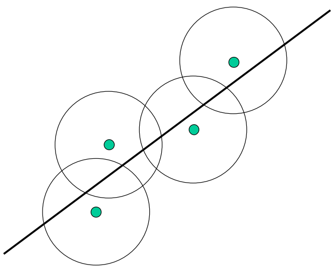


back

Stops in Public Transportation

Project with German Railway, DB

(Schöbel, Ham, Liebers, Wagner, 2003, Poetranto, Ham, Horn, Schöbel, 2006)

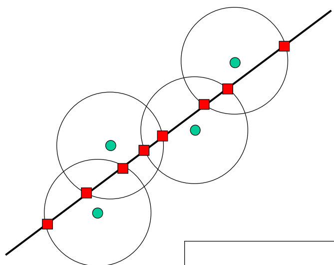


- Customers

Stops in Public Transportation

Project with German Railway, DB

(Schöbel, Ham, Liebers, Wagner, 2003, Poetranto, Ham, Horn, Schöbel, 2006)



- Customers
- Discretization: Finite set of Candidates for location of new stops

$$A = (a_{ps}) \text{ with } a_{ps} := \begin{cases} 1, \text{ if } dist(p,s) \le r \\ 0, \text{ otherwise} \end{cases}$$

Stops in Public Transportation

Covering Problem (CP): minimize $c^T x$
subject to $Ax \geq 1$
x (0,1) - vector

- A may be a C1 matrix ...

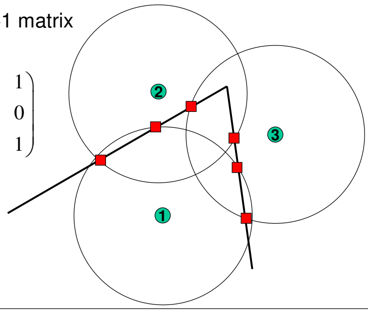
  ... then CP is as easy as Linear Programming

- A may not be a C1 matrix

Stops in Public Transportation

- A may not be a C1 matrix

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$



Goal:
Decompose A into
a small number
of C1 matrices

Stops in Public Transportation

$A_{m.}$ m^{th} row of A
bl(m) number of blocks of consecutive ones in $A_{m.}$

A has the almost consecutive ones property, if $\sum_{m=1}^{M} blm \; \square \; MN$

Covering problem with almost consecutive ones property can be solved more efficient than general covering problems

Schöbel (2003), Ruf and Schöbel (2003)

KC1 Linear Programs and Semi-Simultaneous Flows

Engau, Ham 2006 *(Report)*

KC1-LP (IP) min $c_1 x^K + \ldots + c_K x^K$
subject to
$A^1 x^1 + \ldots + A^K x^K = b$
$x^1, \ldots, x^K \geq 0$ (integer)
and $A^1, \ldots, A^K$ C1 matrices

- Every binary-constraint LP (or IP) is a KC1-LP (or IP)
- KC1-LP does not have the integrality property for K > 1
- KC1-LP can be tackled using semi-simultaneous flows

Algorithms for Decomposition Time (DT) Problem

minimize $DT(\alpha) := \sum_{t \in T'} \alpha_t$

such that

$$\sum_{t \in T'} \alpha_t Y_t = A$$
$$\alpha_t \geq 0$$
$$Y_t \text{ C1-matrix}$$
$$T' \subseteq T$$

DT is easy if T'=T

Ahuja, Hamacher (*Networks* 2005)

- C1-Dec is separable into M independent row problems
- Each row problem is equivalent to a min cost network flow problem solvable in linear time O(N)
- DT is solvable in O(NM) time

Optimization and Medical Community

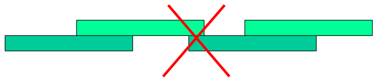
Bortfeld and Boyer (1993): Sweep algorithm

back

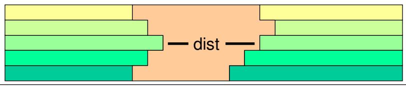
Polynomial Time Algorithm if T' ≠ T

Boland, Hamacher, Lenzen (*Networks* 2004)

- No collision (interleaf motion) between adjacent leaf pairs



- Minimal radiation width: dist $\geq \delta$



Feasible C1 Network

101	102	103	112	113	123
201	202	203	212	213	223
301	302	303	312	313	323
401	402	403	412	413	423

Graph $G = (V,E)$ with

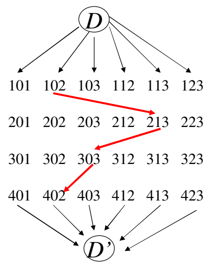
- nodes $V = \{(k,l,r) : r \geq l+\delta+1,\ k = 1,\dots M,\ l = 0,\dots,N,\ r = 1,\dots,N+1\}$
- edges E satisfying interleaf motion constraints

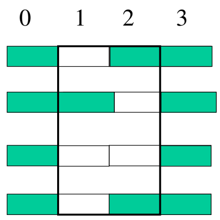
  $(k,l,r) \rightarrow (k+1,p,q)$ if $p \leq r-1$ and $q \geq l+1$

plus super source and super sink

C1P-matrix network (with $O(M\,N^2)$ nodes)

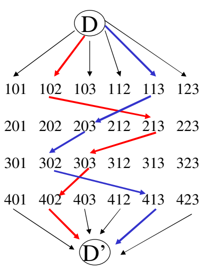
Source-sink paths ⇔ feasible C1 matrices





$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{pmatrix}$$

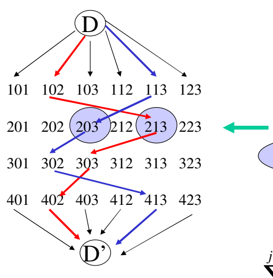
Example



$$\Leftrightarrow \quad 3\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{pmatrix} + 2\begin{pmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 2 & 5 \\ 5 & 3 \\ 3 & 2 \end{pmatrix}$$

$\alpha=3$ $\alpha=2$

Which nodes contribute to a_{ij}?



Example: $a_{22}=5$

i=2

j=2

Constraint:

$$\sum_{l=0}^{j-1}\sum_{r=j+1}^{n+1}\sum_{e\in E_{-}(i,l,r)} x_e = a_{ij}$$

$\alpha=3$ $\alpha=2$

Optimal C1 decomposition ⇔ minimal network flow problem

sending α_t units on path P_t
⇔
use C1 matrix α_t times

flow value
⇔
decomposition time

minimal flow value = minimal decomposition time

Polynomial Solvability

Minimum C1-Decomposition Time problem
is network flow problem with side constraints

$$\sum_{l=0}^{j-1} \sum_{r=j+1}^{n+1} \sum_{e \in E\_(i,l,r)} x_e = I_{ij}$$

Optimal decomposition times in polynomial time!
Computation times small for 10 x 10 matrices

There is always an alternative optimal solution which is integer.

Interval Version of Solution Algorithm for DT Problem

Baatar, Ehrgott, Ham, Woeginger, *Discrete Applied Mathematics*, 2005

Avoid network flows and deal directly with intervals corresponding to blocks of ones.

Purely combinatorial algorithm solves large scale integer program corresponding to decomposition for M ≈ 100 and N ≈ 80 within a minute CPU time.

Decomposition Cardinality (DC) Problem

minimize $\quad DC(\alpha) := \left|\{\alpha_t : \alpha_t > 0\}\right|$

such that

$$\sum_{t \in T'} \alpha_t Y_t = A$$
$$\alpha_t \geq 0$$
$$Y_t \text{ C1}-\text{matrix}$$
$$T' \subseteq T$$

DC is NP-hard

$$\min \; DC(\alpha) := \left|\{\alpha_t : \alpha_t > 0\}\right|$$

Proof (Burkard, Oberwolfach 2002):

Reduction from

Subset Sum:
Given: Integers $a_1,\dots,a_N$, and integer B
Question: Exists subset $J \subseteq \{1,\dots,N\}$ such that $\sum_{j \in J} a_j = B$?

DC is NP-hard

$$A = \begin{pmatrix} a_1 & a_2 & \cdots & a_N \\ B & B & \cdots & B \end{pmatrix}$$

- A can always be partitioned into (N+1) C1-matrices:

$$A = \sum_{i=1}^{N} a_i \begin{pmatrix} & e_i & \\ 0 \;\; 0 & \cdots & 0 \end{pmatrix} + B \begin{pmatrix} 0 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \end{pmatrix}$$

DC is NP-hard

- A can be partitioned into N C1-matrices if and only if there exists a solution to the subset sum problem:

$$A=\sum_{j\in J} a_j \begin{pmatrix} 1 & 1 & \overset{e_j}{\cdots} & 1 \end{pmatrix} + \sum_{j\notin J} a_j \begin{pmatrix} 0 & 0 & \overset{e_j}{\cdots} & 0 \end{pmatrix}$$

DC is strongly NP-hard

Baatar, Ehrgott, Hamacher, Woeginger (*DAM, 2005*)

Problem is NP-hard in the strong sense, even for single-row decomposition.

Proof: Reduction by 3-Partition

Solution approaches:
Heuristics: Langer et al. (1996-), Baatar et al (*DAM, 2005*)
Integer Programming: Langer et al. (1996-), Baatar (Dissertation, 2005)
Enumeration: Chen et al. (2003-2005), Engel (*DAM, 2005*), Kalinowski (*DAM, 2005*), Kamath et al. (*2002-2004*)

Polynomially solvable special cases of DC

DC is polynomially solvable if A is a binary matrix
Proof: Any decomposition time algorithm solves DC

DC is polynomially solvable for matrices A with bounded entries
Proof:

1. Kalinowski (*Discrete Applied Math 2005*) solves lexmin(DT,DC) in polynomial time.
2. Modify algorithm for varying values of DT (Nussbaum *Diploma thesis, Univ. of Kaiserslautern 2006)*

Kalinowski Algorithm is very fast
Kalinowksi *(in General Theory …Combinatorics, 2005)*
Nussbaum *(Diploma thesis, Univ. of Kaiserslautern 2006)*

Decomposition Sequence (DS) Problem

$$\sum_{t=1}^{T} \alpha_t + \sum_{t=1}^{T-1} s_{\sigma(t)\sigma(t+1)}$$

minimize decomposition time plus sequence dependent set-up time, where $s_{kl} := \mathrm{dist}\,(Y_k, Y_l)$

Sequential approach:
- Solve DT with output $Y_1,\ldots,Y_K$
- Use $s_{s(t)s(t+1)}$ as data of a (small !) T-node TSP

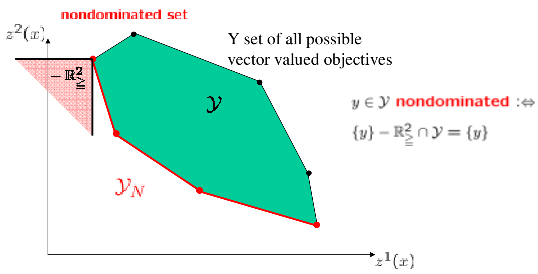
Integrated approach:
- Solve decomposition problem with combined objective

(current approaches: multicommodity flow, column generation)

Multicriteria Approach

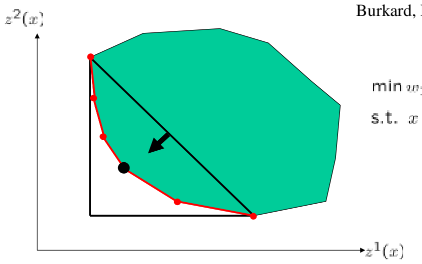
- Minimize the objectives simultaneously
- Recall: objectives are contradictory
- Use multicriteria approach starting with 2 objectives

Non-Dominated Solutions (supported)



Approximation Algorithm: Sandwich Procedure

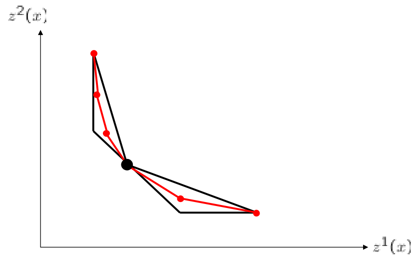
Burkard, Hamacher, Rote (1989)



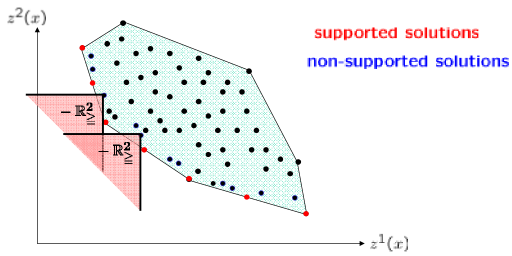
$\min\ w_1 z^1(x) + w_2 z^2(x)$

s.t. $x \in \mathcal{P}$

Approximation Algorithm: Sandwich Procedure



Non-Dominated Solutions (non-supported)



Quality Features of a Representative System

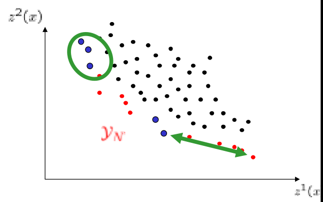
- Accuracy

  $\max_{y \in Y_N} \min_{z \in Rep} \|y - z\|_\infty$

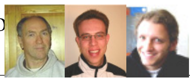
- Number of approximating points
- Clusters

  $\min_{w,z \in Rep,\ w \neq z} \|w - z\|_\infty$

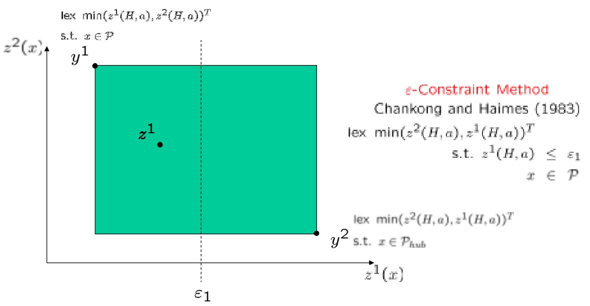
- Approximating points should be nondominated
- Maintaining of properties of nondominated set



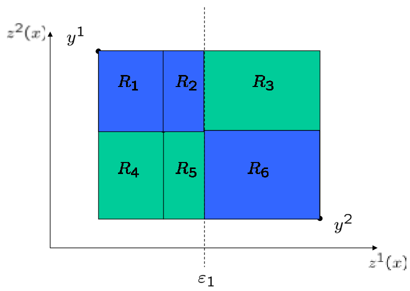
Approximation Algo… Box Method



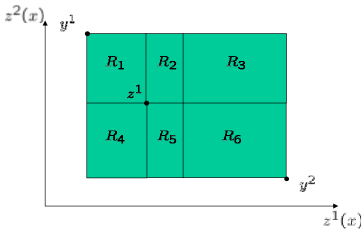
Hamacher, Pedersen, Ruzika (2005)



Approximation Algorithm

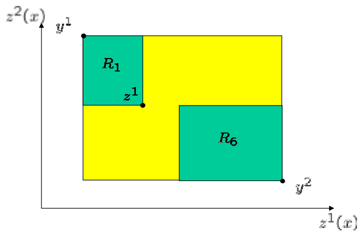


Approximation Algorithm



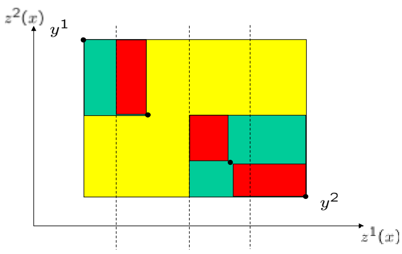
Theorem:
There are no non-dominated points in R_2, R_3, R_4, and R_5

Approximation Algorithm

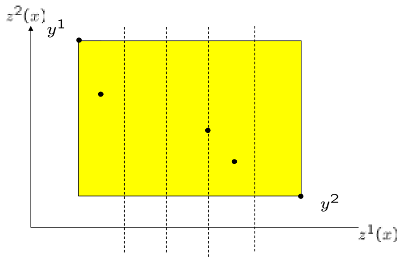


Theorem:
The remaining area to be considered is reduced by at least 50%

A Posteriori-Algorithm



A Priori-Algorithm



Quality Features of the Representation Systems

Algorithms:

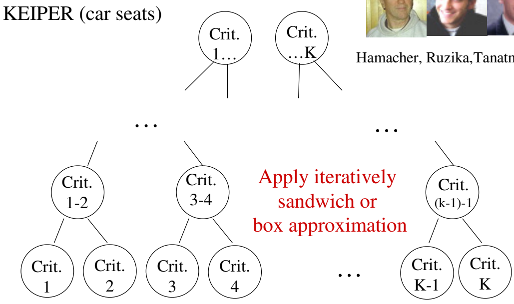
- Accuracy: Δ
- Size of Representation: $\leq \frac{a(box(z^1,z^2))}{\Delta} - 1$ (tight)
- Cluster Density: ≤ 2

Representative Systems:

Quality features for more than two criteria?

Decision support system for KEIPER (car seats)

Hamacher, Ruzika,Tanatmis (2006)



back

KC1 and Semi-Simultaneous Flows

$A^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$ $A^2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$ $b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Engau, Ham 2006

$(c^1, c^2) = \begin{bmatrix} 1 & 3 & 1 & 3 \end{bmatrix}$

Primal Problem:

1	3	1	3	→ min	
0	1	1	1	=	1
1	0	1	1	=	1
1	1	0	1	=	1

Dual Problem:

1	1	1	→ max	
0	1	1	≤	1
1	0	1	≤	3
1	1	0	≤	1
1	1	1	≤	3
π_1	π_2	π_3		

KC1 and Semi-Simultaneous Flows

Dual Problem:

1	1	1	→ max	
0	1	1	≤	1
1	0	1	≤	3
1	1	0	≤	1
1	1	1	≤	3
π_1	π_2	π_3		

Dual Problem:

1	1	1	0	0	0	0	→ max	
0	1	1	1	0			=	1
1	0	1	0	1			=	3
1	1	0			1	0	=	1
1	1	1			0	1	=	3
π_1	π_2	π_3	α_1	α_2	β_1	β_2		

KC1 and Semi-Simultaneous Flows

Dual Problem:

1	1	1	0	0	0	0	→ max	
0	1	1	1	0			=	1
1	0	1	0	1			=	3
1	1	0			1	0	=	1
1	1	1			0	1	=	3
π_1	π_2	π_3	α_1	α_2	β_1	β_2		

Dual Problem:

1	1	1	0	0	0	0	→ max	
0	1	1	1	0			=	1
1	0	1	0	1			=	3
0	0	0	0	0			=	0
1	1	0			1	0	=	1
1	1	1			0	1	=	3
0	0	0			0	0	=	0
π_1	π_2	π_3	α_1	α_2	β_1	β_2		

KC1 and Semi-Simultaneous Flows

Dual Problem:

1	1	1	0	0	0	0	→ max	
0	1	1	1	0			=	1
1	0	1	0	1			=	3
0	0	0	0	0			=	0
1	1	0			1	0	=	1
1	1	1			0	1	=	3
0	0	0			0	0	=	0
π_1	π_2	π_3	α_1	α_2	β_1	β_2		

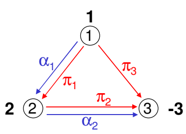
Dual Problem:

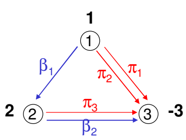
1	1	1	0	0	0	0	→ max	
0	1	1	1	0			=	1
1	-1	1	-1	1			=	2
-1	0	-1	0	-1			=	-3
1	1	0			1	0	=	1
0	0	1			-1	1	=	2
-1	0	-1			0	-1	=	-3
π_1	π_2	π_3	α_1	α_2	β_1	β_2		

KC1 and Semi-Simultaneous Flows

Dual Problem:

1	1	1	0	0	0	0	→ max	
0	1	1	1	0			=	1
1	-1	1	-1	1			=	2
-1	0	-1	0	-1			=	-3
1	1	0			1	0	=	1
0	0	1			-1	1	=	2
-1	-1	-1			0	-1	=	-3
π_1	π_2	π_3	α_1	α_2	β_1	β_2		



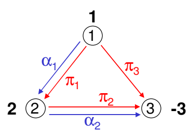


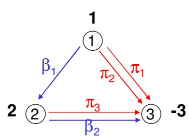
KC1 and Semi-Simultaneous Flows

Semi-Simultaneous
Network Flow
(Se-Sim Flows)

on two individual networks:

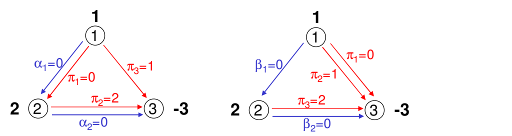
- identical on read arcs
- indifferent and ≥ 0 on blue arcs





KC1 and Semi-Simultaneous Flows

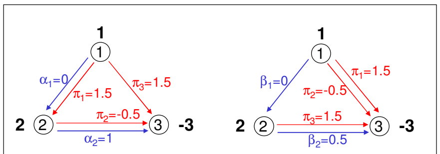
Se-sim flow exists if and only if each individual flow exists



1. Solve individual feasibility problem using only red arcs
2. Set $\pi_i := \min \{ \pi_i^1, \pi_i^2 \}$
3. Adapt in each network the flow values on the blue arcs

KC1 and Semi-Simultaneous Flows

Se-Sim Flows do not have the integrality property



Optimal fractional solution with value 2.5

KC1 and Semi-Simultaneous Flows

- Se-Sim is optimal if and only if there exists no improving residual se-sim flow (analog to negative cycle theorem)
  - more than one cycle may be needed for improvement
  - no polynomial algorithm is known

More results

Bits and Pieces

Current topics in cancer modeling in Kaiserslautern (and partners):

IRMT:

- Simultaneous solution of geometry, intensity and realization problem by allowing only a small supply of C1 matrices
  → network flow problem on a small set of paths
- Find good representative system for Q=2
- Tackle Q>2 representative system question
- Polyhedral approach for DC problem (multicommodity !?)
- Aggregation of voxels
- Model new technology (Shuttle multileaf collimators, vertical plus horizontal MLCs, …)

Radiosurgery

- Covering of 3-dim bodies by spheres
- Polyhedral analysis
- computational results

(Dissertation of Olena Gavriliouk)

Bits and Pieces

(Se-)Sim Flows:

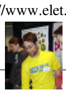
- Show practical efficiency of combinatorial approach
- Establish complexity
- Develop simultaneous graph theory (paths, spanning trees, matchings,…)

Circular ones matrices (instead of consecutive ones):

- Develop theory (partial results are available)
- Apply to smart antenna problems: Amaldi, Capone, Malucelli, 2002


http://www.elet.polimi.it/upload/malucell/ricerca/slides/luciditwente.pdf

- Gijswijt, 2003




THE END