# Decomposition Methods in Optimization and Applications in Practical Problems
# Exercises and Projects

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## 1 Exercises

Please submit your solutions of the following exercises until

**May 31, 2007**

in electronic form to

**hamacher@mathematik.uni-kl.de**.

If you worked out your solutions on paper, please scan your solutions and send the scanned pages (please check for readability). If anything else fails, you can also send them on paper with regular mail, but this makes communiation between Kaiserslautern and Milano more difficult.

Each of the exercises should be done individually and if you used any information from literature etc. you should quote this information correspondingly.

**Assignment 1**: Minimize the decomposition time $DT(\alpha)$ for the matrix

$$A = \begin{pmatrix} 2 & 1 & 3 & 4 & 7 & 1 \\ 4 & 0 & 0 & 3 & 8 & 2 \\ 1 & 3 & 0 & 0 & 7 & 8 \\ 9 & 3 & 2 & 4 & 0 & 7 \end{pmatrix}$$

**Assignment 2**: Solve the 1C1 Row Problem

$$\begin{array}{rll} \min & \mathbf{cx} & \\ \text{s.t.} & A\mathbf{x} & = b \\ & \mathbf{x} & \geq \mathbf{0} \text{ integer} \end{array}$$

using the network flow approach applied to the data

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 2 \\ 1 \\ 3 \\ 4 \end{pmatrix}, \quad \mathbf{c} = (\mathbf{2}, \mathbf{1}, \mathbf{3}, \mathbf{7}, \mathbf{4})$$

**Assignment 3**: Solve the 2C1 Row Problem

$$\begin{array}{lll} \min & \mathbf{c^1x^1} + \mathbf{c^2x^2} & \\ \text{s.t.} & A^1\mathbf{x^1} + A^2\mathbf{x^2} & = \mathbf{b} \\ & \mathbf{x^1}, \mathbf{x^2} & \geq \mathbf{0} \text{ integer} \end{array}$$

using the se-sim network flow approach applied to the data

$$A = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}, \quad \mathbf{c} = (\mathbf{2}, \mathbf{1}, \mathbf{3}, \mathbf{1}, \mathbf{2}, \mathbf{1})$$

**Assignment 4**: Set up the two network flow formulations for finding the minimal constraint decomposition time for

$$A = \begin{pmatrix} 2 & 4 & 7 & 0 & 1 \\ 1 & 0 & 3 & 5 & 0 \\ 2 & 0 & 1 & 3 & 2 \end{pmatrix}$$

Solve the constraint DT using these networks by inspection. Try in particular a cycle canceling approach in the second network, i.e. find iteratively improving cycles which maintain the edge matching property. Any suggestion for combinatorial algorithms to solve the constrained network flow problems (both in the first and the second network are welcome)!

**Assignment 5**: Design and draw and example of MOLIP (similar to Example 3.11) with two variables which has unsupported, nondominated objective vectors.

**Assignment 6**: Develop an *a priori* version of the box method: Compute in advance the necessary number of iterations to obtain an accuracy $\Delta$.

**Assignment 7**: Solve the planar CovLocStop problems presented in class (Example 4.3) using (semi-simultaneous) flows.

**Assignment 8**: Prove that the median location problem in networks always has an optimal location in nodes (Hakimi result, Theorem 4.20). (Hint: Start with the case of a single new location and then conclude that the result is also correct, if $k > 1$ new facilities have to be located.) Is the node optimality result also true if the median objective function is replaced by the *center objective function* $\max_{v \in V} w_v d(v, \mathcal{S})$?

## 2 Projects

Choose from the subsequent list of project one, which you want to work on. It requires more work than the exercises. You are allowed to work on these projects in teams of at most two. For the submission of your reports the same applies as for the exercises.

**Project 2.1** *Search the literature for applications of* weak consecutive ones *matrices, i.e., matrices which can be transformed to C1 matrices by columns permuations. (Note, that in the literature, these matrices are often denoted as "consecutive ones" matrices, in contrast to the denotation used in our class!) Discuss the literature using in this context optimization methods. Discribe the application problems as well as the optimization methodologies used.*

**Project 2.2** *The first project involves implementation.*

- *Implement the SeSim Flow Algorithm*
- *Compare with standard software*

**Project 2.3** *(Posed by Sandro Bosio, Milan, 2007-03-19) According to our results in class the decomposition cardinality problem DC can be solved for binary matrices in polynomial time. Can one combine this result with transformation results of weak C1 matrices to solve the cardinality problem if column permuation is also allowed?*

**Project 2.4** *Instead of starting with coefficient matrices which have the row C1 property, one may want to consider a binary matrix* $A \in \mathbb{B}^{m \times n}$ *into matrices that are column consecutive one,* $A = (A^1 \ \dots \ A^L)$ *with* $A^l \in \mathbb{B}^{m_l \times n}$, $l = 1, \dots, L$ *and* $\sum_{l=1}^{L} m_l = m$. *The resulting LP is equivalent to*

$$\begin{aligned} &\textit{minimize} \sum_{l=1}^{L} c^{\mathrm{T}} x \\ &\textit{subject to } A^l x = b^l \textit{ for all } l = 1, \dots, L, \\ &\qquad\qquad x \in \mathbb{R}^n_+. \end{aligned} \tag{1}$$

*and the equality constraints* $A^l x = b^l$ *can directly be transformed into L systems of flow conservation constraints in L underlying networks* $G^l$, $l = 1, \dots, L$. *A solution of problem (1) then corresponds to a vector* $x \in \mathbb{Z}^n_+$ *so that for each* $l = 1, \dots, L$, *the vector x establishes a feasible flow in each of the networks* $G^l$ *while minimizing the objective* $c^{\mathrm{T}} x$. *In contrast to the semi-simultaneous flows considered in this paper, in this approach there are no individual flows. We can therefore call x a* simultaneous flow.
*Which results an you derive for simultaneous flows?*

**Project 2.5** *In the box method we used as stopping criterion, that the area of the largest box is less than a given accuracy bound* $\Delta$. *Consider as alternative stopping criterion, that none of the two side lengths of any of the boxes is larger than* $\Delta$. *How do you have to modify the box method? Can you derive a relation between the number of solutions of a lexicographical* $\epsilon-$*constraint problem and the accuracy as we did for the case of the area stopping criterion.*

**Project 2.6** *Consider stop location problems for the Euclidean distances. Which of the results presented in class can be duplicated from the case of rectilinear distances which we considered mostly in class, which can not? Do the results get wrong or are just new proofs needed?*