

Propositional logic - lab exercises

Computational Logic

March 3, 2010

1 Equivalence

Show that two formulas F and G are semantically equivalent ($F \equiv G$) iff $F \leftrightarrow G$ is a tautology.

2 Connectives

- Prove that $\{\neg, \wedge\}$ is a functionally complete set of logical connectives.
- Show what other smallest sets of connectives, among those we have seen for propositional logic, are functionally complete and prove also that none of their proper subsets is functionally complete.
- Can you define another boolean connective that, alone, is functionally complete?

3 Normal forms

Transform the following propositional formulas into CNF:

- (a) $(a \wedge b) \vee (c \wedge d) \vee (e \wedge f)$
- (b) [Use the result from (a) and immediately predict how many conjuncts there will be in the resulting CNF formula] $(a \wedge b) \vee (c \wedge d) \vee (e \wedge f) \vee (g \wedge h)$
- (c) [Be smart and look at the structure of the formula - the CNF is much simpler] $dog_barks \wedge grass_wet \wedge \neg sprinkler_on \wedge \neg storm \vee$
 $dog_barks \wedge grass_wet \wedge \neg sprinkler_on \wedge rain \vee$
 $dog_barks \wedge grass_wet \wedge grass_wet \wedge \neg storm \vee$
 $dog_barks \wedge grass_wet \wedge grass_wet \wedge rain \vee$
 $dog_barks \wedge \neg rained_last_night \wedge \neg sprinkler_on \wedge \neg storm \vee$
 $dog_barks \wedge \neg rained_last_night \wedge \neg sprinkler_on \wedge rain \vee$
 $dog_barks \wedge \neg rained_last_night \wedge grass_wet \wedge \neg storm \vee$
 $dog_barks \wedge \neg rained_last_night \wedge grass_wet \wedge rain$

4 Definite programs

Check that the CNF formula you obtained above in point (c) is a definite program, write it in clausal form, and conclude whether *grass.wet* holds by looking at its least model.

5 Natural deduction

Carry out these proofs with natural deduction, indicating what rules you use at each step:

- (a) $\emptyset \vdash \neg(P \wedge \neg P)$
- (b) $\{\neg P\} \vdash P \rightarrow Q$