First-order logic - lab exercises

Computational Logic

March 15, 2010

1 Translation and equivalences

1. Translate the following formulas into natural language (English), according to their intuitive intended meaning:
   (a) $\forall x(male(x) \vee female(x))$
   (b) $\forall x((father(x) \rightarrow male(x)) \wedge (mother(x) \rightarrow female(x)))$
   (c) $\neg\exists x(father(x) \wedge mother(x))$
2. Translate the following English sentences into first-order logic formulas. Invent a suitable vocabulary.
   (a) Some students take Computational Logic
   (b) Every student who takes Computational Logic passes it
   (c) No good student flunks an exam
   (d) Brothers are siblings
   (e) (express the fact that "sibling" is a symmetric relation)
   (f) One's mother is one's female parent
   (g) A cousin is a child of a parent's sibling
   (h) Every person has only one mother
   (i) (a famous quotation): you can fool some people all of the time, and all of the people some of the time, but you cannot fool all of the people all of the time.
3. Argue why $\exists x\forall y F(x, y)$ is not logically equivalent to $\forall y\exists x F(x, y)$. Comment on the formula $F(x, y) = mother(x, y)$.
4. Use equivalences to show that $\forall y(\exists x p(x, y) \rightarrow \neg q(y))$ is logically equivalent to $\neg\exists y\exists x(p(x, y) \wedge q(y))$

2 Herbrand interpretations and models

1. Show the Herbrand universe and Herbrand base for the following formulas:
   (a) $odd(s(0)) \wedge \forall X((odd(X) \rightarrow odd(s(s(X)))))$
   (b) $has(owner(car), car) \wedge \forall X(has(X, car) \rightarrow happy(X))$
   (c) $\neg p(a) \wedge \exists X p(X)$
2. Show some Herbrand interpretations and some Herbrand models for formula 1a. Are there Herbrand interpretations that are not Herbrand models for such formula?
3. Consider formula 1c. Does it admit any Herbrand model? Does it admit any (non-Herbrand) model? Explain.

3 Normal forms

1. Transform formula 1c from Section 2 into CNF. Does its CNF admit any Herbrand model?