1 February 1998

ELSEVIER

OPTICS COMMUNICATIONS

Optics Communications 147 (1998) 180–186

Full length article

Solitons in fibers with lumped amplifiers

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Received 5 August 1997; revised 21 October 1997; accepted 24 October 1997

Abstract

The maximum accumulated excess compensation angle (MAECA) criterion for designing soliton transmission systems with lumped amplifiers is reviewed, extensively tested and supported by means of numerical simulations of a large number of soliton transmission systems. MAECA permits to investigate the validity domain of the ''path averaged soliton''. With respect to the $L_A/L_C<1$ criterion proposed in the literature, the MAECA criterion appears to be physically well based and less severe in case of short pulses and non-dispersion shifted fibers. © 1998 Elsevier Science B.V.

1. Introduction

In a previous paper [1] published in 1994, an approach was presented for the determination of the maximum acceptable amplifier spacing compatible with nearly-soliton transmission in a fiber with lumped amplifiers. The approach was based on a simple representation of the process by which solitons in the lossless fiber act as self-equalizing waveforms: in effect it is easy to show (see the Appendix) that, in any *infinitesimal* segment of fiber, dispersion on one hand and non-linearity of the refractive index on the other hand produce *infinitesimal modulation angles* which exactly compensate reciprocally, in the sense that their sum is an irrelevant constant phase shift. Under such conditions the pulse shape is the same everywhere. All this, provided that a soliton waveform be used with a peak power

$$P_1=\frac{1}{4\pi^2 c}\,\frac{\lambda^3 D A_{\rm eff}}{n_2 t_0^2}, \tag{1}$$

where n_2 is the Kerr coefficient, $A_{\rm eff}$ the effective area, $D=-2\pi c\beta_2/\lambda^2$ the group-velocity dispersion coefficient, c the speed of light, β_2 the chromatic dispersion and $1.763t_0$ the pulse intensity full width half maximum (FWHM).

When we consider a lossy fiber with lumped amplifiers, the compensation between the two mentioned angles cannot be anymore obtained in each infinitesimal segment of the fiber, because the signal power varies along the line. However, compensation could be obtained approximately in each amplified span provided that such a span tends to behave similarly to the infinitesimal segment of the lossless fiber. The mismatch between the two phase angles accumulates during propagation. In a generic section at a distance z from the beginning of the span the *accumulated excess compensation angle* φ_e at the pulse peak is given by (see the Appendix and Ref. [1])

$$\varphi_e(z)=-\frac{2\pi n_2}{\lambda A_{\rm eff}}\left(\int_0^z P_{1a}(z)\,{\rm d}z-P_1 z\right), \tag{2}$$

where $P_{1a}(z)$ is the pulse peak power at z. At the beginning of the span $P_{1a}(z)>P_1$ and φ_e increases up to a maximum value $\varphi_{\rm emax}$ in the section where $P_{1a}(z)=P_1$ and then decreases because $P_{1a}(z)<P_1$.

In this paper we show that φ_e *is the parameter that determines the system performances.* In order to guarantee compensation between the two angles, the condition $\varphi_e(L_A)=0$ must be imposed. In order to ensure that the amplified span tends to behave like an infinitesimal segment of fiber, the absolute value of φ_e must be a small fraction of a radian in any section of the span, particularly in the section in which it is maximum.

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PII S0030-4018(97)00636-6

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From the $\varphi_e(L_A)=0$ condition the pulse peak-power to be transmitted is found to be

$$P_{1a}(0)=P_1\frac{2\alpha L_A}{1-e^{-2\alpha L_A}} \quad (3)$$

as already derived through a different approach by Blow and Doran [2], introducing the concept of the path averaged soliton.

The maximum value of φ_e, called the *Maximum Accumulated Excess Compensation Angle* (MAECA) is

$$\varphi_{emax}=\frac{\lambda^2 D}{2\pi c t_0^2}L_A\,\rho(\alpha L_A), \quad (4)$$

where the function $\rho(\alpha L_A)$ [1] is

$$\rho(\alpha L_A)=\frac{1}{2\alpha L_A}\left[\frac{2\alpha L_A}{1-e^{-2\alpha L_A}}-1-\ln\frac{2\alpha L_A}{1-e^{-2\alpha L_A}}\right], \quad (5)$$

and is reported in Fig. 1. This function is proportional to L_A for low span attenuation and tends to unity for large αL_A values. The MAECA criterion, that is $\varphi_{emax}<1$, makes L_A dependent on the path attenuation and is therefore consistent with the physics of the involved phenomena in which the attenuation plays a fundamental role; in particular it is consistent with the fact that when the attenuation α tends to zero L_A tends to be unlimited. Ideal conditions for soliton propagation are described therefore by $\varphi_{emax}=0$.

In the literature [3,4] the criterion generally suggested for the determination of L_A is that the ratio of $L_A/L_C<1$, being $L_C=2\pi c t_0^2/(\lambda^2 D)$ the critical length. Taking into account that Eq. (4) can also be written as

$$\varphi_{emax}=\frac{L_A}{L_C}\rho(\alpha L_A), \quad (6)$$

it follows that when the attenuation αL_A is large, the function ρ tends to unity and the criterion $L_A/L_C<1$ coincides with the criterion $\varphi_{emax}<1$. Under these conditions L_A is proportional to the square of $t_0/\sqrt{D}$ and thus decreases rapidly as we move towards higher bit-rates or consider more dispersive fibers. Conditions are thus easily reached for which the complete expression of φ_{emax} (Eq. (4)) must be adopted, to avoid a pessimistic approach. In this case, as already remarked, ρ tends to be proportional to L_A, so that the amplifier spacing results to be proportional to $t_0/\sqrt{D}$ and not to $(t_0/\sqrt{D})^2$. This can explain why in recent published theoretical and experimental results [6–9], an amplifiers spacing equal or even greater than L_C has been achieved.

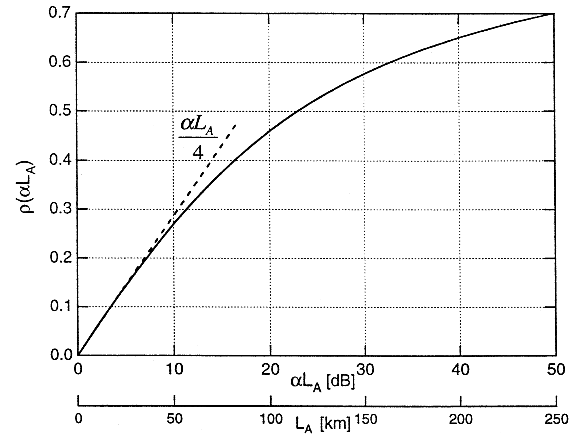

Fig. 1. Function $\rho(\alpha L_A)$ and its approximation for small αL_A. The second abscissa axis is for $\alpha=0.2$ dB/km.

In this paper a careful discussion of the MAECA approach is carried out, together with a comparison with the $L_A/L_C<1$ criterion. An extensive investigation is also performed to test the method in different situations, with the following aims: (i) to acquire further evidence that φ_{emax} is a fundamental parameter to be considered in designing soliton systems with lumped amplifiers; (ii) to determine appropriate values of φ_{emax}, and thus of L_A, compatible with an acceptable system performance; (iii) to possibly acquire new elements concerning soliton propagation in fibers with lumped amplifiers.

2. Simulation results and discussion

In the present section several examples are presented and discussed, in order to better show that the MAECA criterion sets the validity domain of the path averaged soliton. The analysis is carried out for isolated pulses:

Table 1
Amplifier spacing L_A [km] and ratio L_A/L_C (in parentheses) for different fibers and pulsewidth [ps]. $t_0/\sqrt{D}$ is in $\sqrt{\text{ps nm km}}$

$t_0/\sqrt{D}$	t_0 $D=-0.37$	t_0 $D=-15$	$L_A(L_A/L_C)$ $\varphi_{emax}=0.05$	$L_A(L_A/L_C)$ $\varphi_{emax}=0.1$	$L_A(L_A/L_C)$ $\varphi_{emax}=0.2$	$L_A(L_A/L_C)$ $\varphi_{emax}=0.3$	$L_A(L_A/L_C)$ $\varphi_{emax}=0.5$
0.82	0.5	3.18	2.13 (4.05)	3.03 (5.76)	4.28 (8.14)	5.24 (9.96)	6.77 (12.9)
1.64	1.0	6.35	4.25 (2.02)	6.05 (2.87)	8.57 (4.07)	10.5 (4.99)	13.56 (6.44)
2.46	1.5	9.53	6.40 (1.35)	9.08 (1.92)	12.86 (2.72)	15.78 (3.33)	20.41 (4.31)
3.29	2.0	12.7	8.50 (1.01)	12.15 (1.44)	17.20 (2.04)	21.11 (2.51)	27.34 (3.24)
4.11	2.5	15.9	10.7 (0.81)	15.20 (1.16)	21.55 (1.64)	26.47 (2.01)	34.38 (2.61)
8.21	5.0	31.8	21.5 (0.41)	30.75 (0.58)	44.00 (0.84)	54.55 (1.04)	72.23 (1.37)

soliton interaction, jitter and in-line soliton controls are not considered, although included in the program of our future activities. The investigation has been carried out in the range of $t_0/\sqrt{D}$ that goes from 0.8 to 8 $\sqrt{\text{ps nm km}}$, as reported in Table 1. Here some values of t_0 are listed for $D = -15$ ps/(nm km), which corresponds to step index fibers, and $D = -0.37$ ps/(nm km), which corresponds to dispersion shifted fibers. Table 1 reports the amplifiers spacing and the correspondent ratio L_A/L_C (in parentheses) for the six values of $t_0/\sqrt{D}$ considered and for φ_{emax} from 0.05 up to 0.5 radians. Some short values of the amplifiers spacing may have no practical interest, but they are important to understand the system behaviour when φ_{emax} tends to zero.

Numerical results are obtained by solving the non-linear Schrödinger equation, representing the propagation in the fiber, with the technique described in Ref. [5]. A suitable absorber has been added to avoid reflections from the edges of the computational time window.

In order to allow also the observation of microperturbations of the pulse propagating along the fiber, we report, instead of the evolution of the overall pulse temporal shape, a single parameter of the pulse itself, i.e. the pulsewidth. (The amplitude can be approximately recovered from the pulsewidth because, as results from our numerical investigations, the pulse area is nearly constant during the propagation).

A first example of pulsewidth evolution is shown in Fig. 2 which reports the relative FWHM in percent at the output of each amplifier in the case of an input soliton with $t_0/\sqrt{D} = 0.82$ (i.e. for instance $t_0 = 1$ ps and $D = -1.5$) propagating in a 300 km long fiber. The FWHM is shown for values of the parameter φ_{emax} equal to 5×10^{-4}, 0.01 and 0.05, corresponding to a ratio L_A/L_C of 0.4, 1.9 and 4, respectively.

As expected, as φ_{emax} tends to zero the pulse is less perturbed and changes in FWHM become negligible. However, several other interesting features arise from this figure. First of all an initial transient is noted, mainly a positive step whose entity rapidly increases by increasing φ_{emax}. A smooth trend follows, characterized by a ripple superimposed to a monotonically increasing average value. These three observed behaviours determine the system performances: the initial transient is determinant for relatively short transmission systems, the continuous monotonic pulse spread has to be considered for long transmission systems and the ripple must be in any case small. In the following these three behaviours are studied in detail and related, when possible, with the parameter φ_{emax}.

Fig. 3 shows the FWHM relative broadening (in percent) as a function of the number of amplified spans, for the situation corresponding to the sixth column of Table 1. The angle φ_{emax} is equal to 0.2 in each case and L_A/L_C varies from 0.82 to 8.2. The transient magnitude, as defined in the inset of Fig. 3, is approximately the same for each considered pulse and it is reported by triangles in Fig. 4 for $\varphi_{emax} = 0.2$. By extending the observation to all the columns of Table 1, the dependence of the initial transient magnitude on φ_{emax} is determined and shown in Fig. 4. The transient magnitude increases rapidly by increasing φ_{emax} and apart from very long systems, as discussed below, may be the main cause of pulse broadening. Under

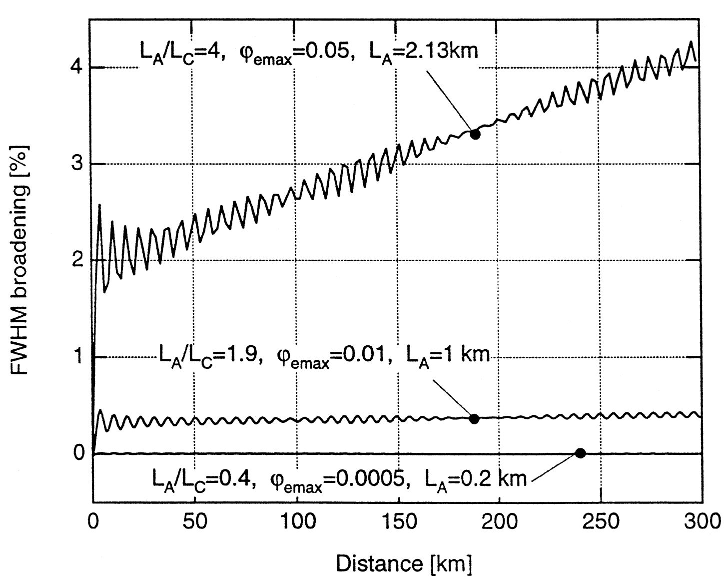

Fig. 2. FWHM broadening versus propagated distance. $t_0/\sqrt{D} = 0.82$ $\sqrt{\text{ps nm km}}$.

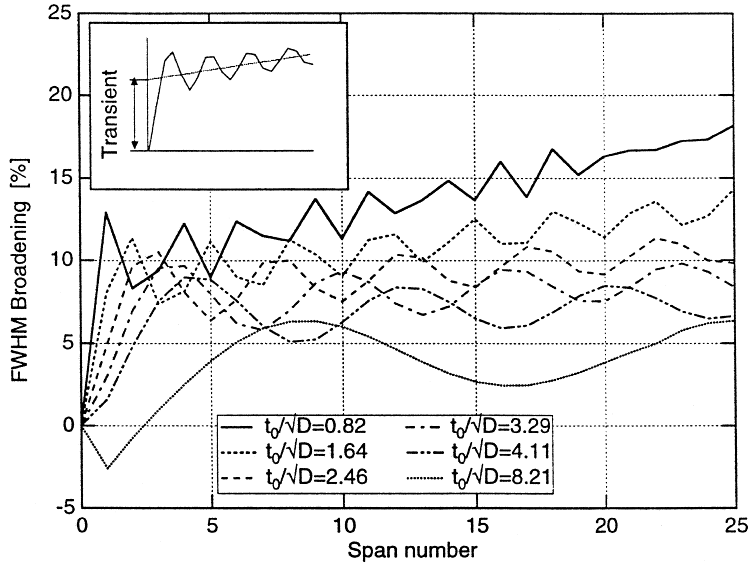

Fig. 3. FWHM broadening versus propagated distance ($\varphi_{emax} = 0.2$, sixth column of Table 1).

such conditions a noticeable improvement can be obtained by using the pulse after the transient, as input waveform. This is perhaps in the same line of research reported in Refs. [6–8].

The second investigated behaviour is the smooth, monotonically increasing average FWHM value. Fig. 5(a) and 5(b) show the FWHM evolution over 5000 km for pulses of the fourth and seventh column of Table 1, i.e. for $\varphi_{emax} = 0.05$ and $\varphi_{emax} = 0.3$. The average value increases monotonically with distance, as already remarked, very slowly in the case $\varphi_{emax} = 0.05$ (Fig. 5(a)) and much more rapidly in the case $\varphi_{emax} = 0.3$ (Fig. 5(b)). (Notice the large difference in the scales adopted for the two figures!). The ratio $t_0/\sqrt{D}$ is marked on each line. Instead of considering such curves we can refer to their slope, i.e. to the percentage pulse broadening per kilometer as a function of distance. This slope decreases very slowly by increasing the distance, and increases noticeably by increasing φ_{emax}. It depends also on the pulsewidth, but on this relation some further considerations are needed. We deal with complex phenomena, part of which depends on the total traveled distance (total dispersion, total attenuation, etc.) and part on the number of times the propagation irregularities are passed on, i.e. the number of spans. By reporting the relative pulse broadening *per span* as a function of the *distance*, the results of Fig. 6 are obtained, from which further evidence of the importance of the parameter φ_{emax} is derived.

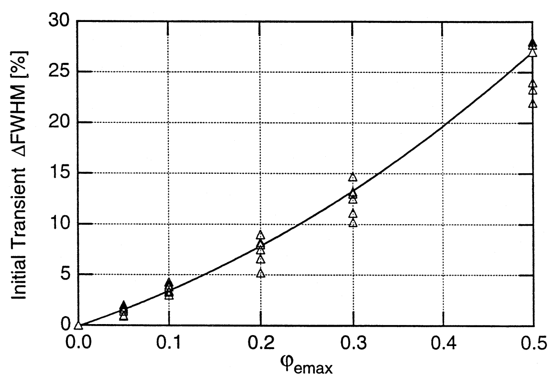

Fig. 4. Initial transient magnitude versus φ_{emax}. Marks correspond to the various columns of Table 1 and the continuous line is the fitting.

The last observed parameter is the ripple. The maximum magnitude of the ripple encountered in 5000 km of propagation is obtained from Fig. 5(a) and 5(b) and reported by triangles in Fig. 7. By extending the analysis to the other columns of Table 1, Fig. 7 is completed. The point dispersion for each φ_{emax} increases by increasing φ_{emax} and for $\varphi_{emax} = 0.5$ this dispersion is so large that the fitting curve has no meaning and transmission could become impossible.

From this analysis it appears that φ_{emax} may have an upper limit between 0.2 and 0.3, at least for the systems investigated in this study. An interesting experiment successfully carried out by Nakazawa in 1994 [9], appears to be in agreement with the above recommendation. In the experiment an 80 Gbit/s soliton transmission system (FWHM = 2.8 ps, i.e. $t_0 = 1.6$ ps) in a dispersion shifted fiber with $D = -0.19$ ps/(nm km) at $\lambda = 1552$ nm and amplifier spacing $L_A = 25$ km was realized and tested. In such a system $t_0/\sqrt{D} = 3.67$ $\sqrt{\text{ps nm km}}$ and $\varphi_{emax} = 0.33$. Note that in this case $L_C = 10.5$ km. In the experiment, solitons propagate over 500 km with an ‘‘acceptable

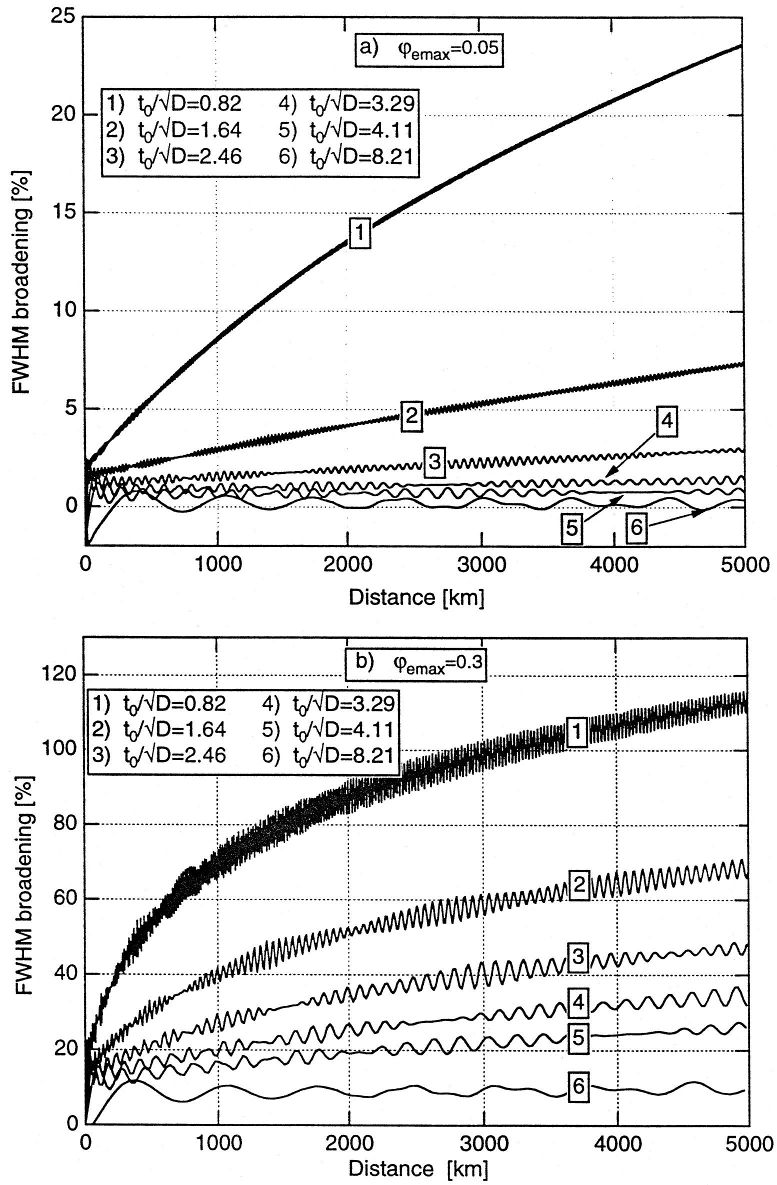

Fig. 5. FWHM evolution for long distance transmission: (a) $\varphi_{\text{emax}} = 0.05$, (b) $\varphi_{\text{emax}} = 0.3$.

power penalty'' in a digital system performance. Similar performances should be achieved with the same amplifier spacing in a 10 Gbit/s soliton transmission system on a standard fiber with $D = -15$ ps/(nm km).

Extension of the present work will be carried out and will include the use of precorrected input waveforms, in particular of the pulse after the transient, as already mentioned.

3. Conclusion

Several interesting phenomena have been observed with the simulator, which gives further evidence on the importance of φ_{emax} in determining the performances of soliton systems with lumped amplifiers. From this analysis it appears that the MAECA criterion is a suitable approach for the design of high bit rate soliton transmission systems.

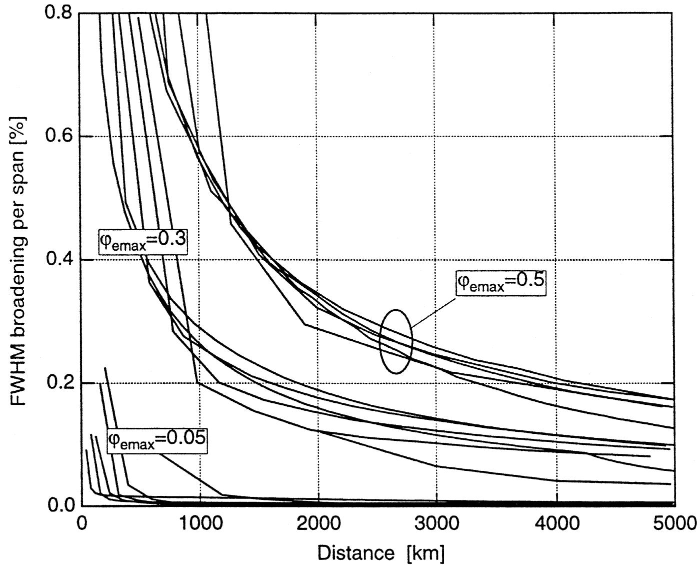

Fig. 6. FWHM relative broadening per span versus propagated distance.

The upper limit of φ_{emax} may be between 0.2 and 0.3.

Acknowledgements

The authors wish to thanks Dr. Cocito, Dr. Cisternino and Dr. Roccato of CSELT for fruitful discussions.

Appendix A

In this appendix the maximum excess compensation angle is defined and derived as in Ref. [1]. The baseband transfer function $H(f)$ of a lossless fiber of infinitesimal segment $\mathrm{d}z$, with second order dispersion is

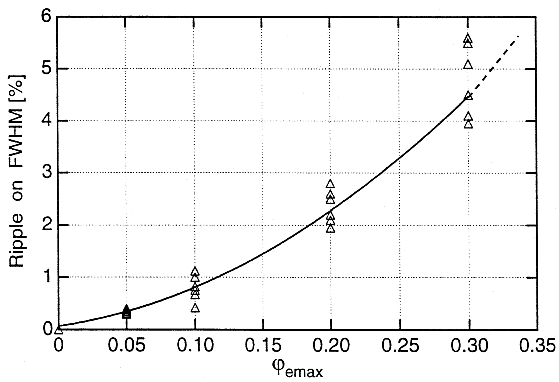

Fig. 7. FWHM ripple magnitude versus φ_{emax}. Marks correspond to the various columns of Table 1 and the continuous line is the fitting.

$$H(f) = \mathrm{e}^{j\frac{1}{2}\beta_2(2\pi)^2 f^2\,\mathrm{d}z} \approx 1 + j\tfrac{1}{2}\beta_2(2\pi)^2 f^2\,\mathrm{d}z. \tag{A.1}$$

According to Eq. (A.1), after propagation in the segment $\mathrm{d}z$, a signal $u(t)$ becomes

$$u_0(t) = u(t) - j\frac{\beta_2}{2}\frac{\mathrm{d}^2 u(t)}{\mathrm{d}t^2}\,\mathrm{d}z, \tag{A.2}$$

that is the chromatic dispersion produces a modulation angle

$$\mathrm{d}\varphi_d(t) = -\frac{\beta_2}{2}\frac{\mathrm{d}^2 u(t)/\mathrm{d}t^2}{u(t)}\,\mathrm{d}z, \tag{A.3}$$

as illustrated in Fig. 8.

Similarly the non-linearity of the refractive index produces another modulation angle, depending on the peak power P_1 of $u(t)$ [1],

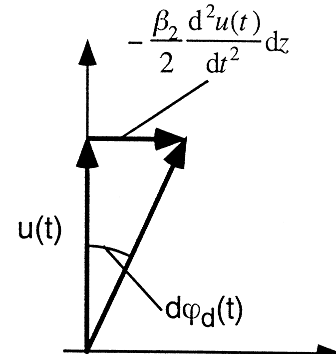

Fig. 8. Dispersion effect on the infinitesimal fiber segment.

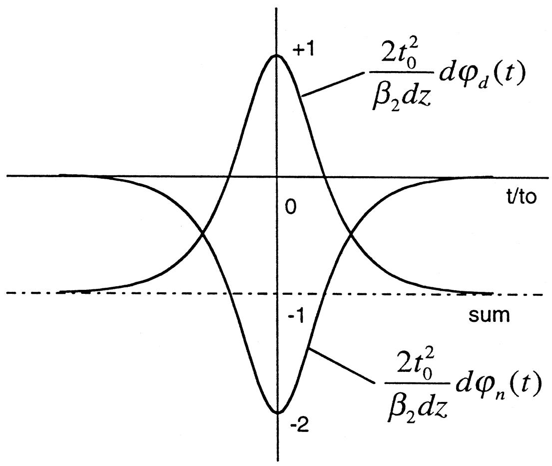

Fig. 9. Compensation between the two modulation angles.

$$\mathrm{d}\varphi_n(t) = -\frac{2\pi n_2}{\lambda A_{\mathrm{eff}}} P_1 |u(t)|^2 \,\mathrm{d}z. \tag{A.4}$$

The two angles (A.3) and (A.4) can compensate, giving rise to a perfectly equalized line, if the sum of the two angles is a constant, i.e.

$$-\frac{\beta_2}{2}\frac{\mathrm{d}^2u(t)/\mathrm{d}t^2}{u(t)}\,\mathrm{d}z - \frac{2\pi n_2}{\lambda A_{\mathrm{eff}}} P_1|u(t)|^2\,\mathrm{d}z = k\,\mathrm{d}z, \tag{A.5}$$

because this implies a constant phase shift of the carrier, which is irrelevant for recovering the modulating signal. Notice that Eq. (A.5) is just the non-linear wave equation in the case of a pulse that preserves its shape. As is well known Eq. (A.5) is satisfied if $u(t)$ has hyperbolic secant form of a soliton (Fig. 9), provided that $k = -\beta_2/(2t_0^2)$ and P_1 is given by Eq. (1).

In the case of a lossy fiber with lumped amplifiers the compensation (11) does not exactly occur in any infinitesimal segment of the fiber. In fact, the signal power is no more constant with z. Under such conditions, an excess compensation angle is produced in each infinitesimal segment of fiber, given by

$$\mathrm{d}\varphi_{\mathrm{e}}(t,z) = -\frac{2\pi n_2}{\lambda A_{\mathrm{eff}}}\left(P_{1\mathrm{a}}(z) - P_1\right)|u(t)|^2\,\mathrm{d}z, \tag{A.6}$$

where $P_{1\mathrm{a}}(z)$ is the pulse peak power at z. By assuming that the modulus of $u(t)$ does not change with z, as in Fig. 8, in a generic section the accumulated excess compensation angle φ_{e} is calculated by integrating Eq. (A.6) over z, resulting in Eq. (2) of the text.

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