A Connective Stability Analysis of Complex System Simulation and Control via Multiagent Systems

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Abstract. Multiagent systems are demonstrated to be an interesting tool to simulate and control complex systems. In fact complex systems can be described by using a collection of models and combining them via multiobjective optimization by agent negotiation. The application of multiagent systems in fields such as, for example, multiple aircraft flight control and robot formation control, requires the agent negotiation paradigm to be robustly stable. We propose the adoption of the concept of connective stability for the analysis of the multiagent negotiation in order to make this robustly stable with respect to the connection and disconnection of the agents. We also introduce a theoretical analysis of the agent negotiation deriving and analytically proving, by using Lyapunov criterion, constraints that assure the connective stability.

1 Introduction

The definition of 'complex system' is a debated question in literature, but, although a number of different and controversial definitions regarding complex systems exists, they all share a common assumption: the behavior of a complex system depends from the interaction of several heterogeneous subsystems [2, 15]. In several disciplines, such as physics, chemistry, biology, psychology, economics, and political science, complex systems emerge, thus the simulation and control of these represent interesting tasks to perform. However, their intrinsic complexity makes them a class of tasks hard to be tackled [1].

Complex systems are modeled in literature by using several techniques, one of them that is emerging highly effective is based on the decentralized optimization where the system is modeled by using a dynamic set of models and these are combined by using a multiobjective optimization [2] in order to mimic the behavior of the comprehensive systems. Decentralized optimization showed itself very effective in applications where the models are spatially distributed, such as aircraft traffic and robot formation management. However, the multiobjective optimization is a computationally hard problem and usually it not can be achieved according to strict temporal constraints. In literature ad-hoc technique have been developed; one of the most interesting is inspired to the economic/market approach where initially a local optimization according to each single model is performed, then a global agreement is reached by negotiation among the models [4]. The negotiation approach provides the multiobjective optimization process with several properties: the achievement of an agreement among the models, the decentralization of these by using a mediator, and, finally, the limitation of the complexity. The recourse to negotiation among distributed and, often, autonomous entities makes multiagent systems [21] find a somehow "natural" application to achieve the simulation and control of complex systems. However, the adoption of multiagent systems in such fields rises a number of issues not deeply explored yet, such as, for instance, the agent negotiation *stability* [9], the *optimality* [11, 12], the *real-time* [3, 14], and, finally, the *adaptation*.

In this paper we concentrate on the analysis of the stability of the agent negotiation. The application fields – simulation and, particularly, control – require a formal analysis of the stability of the negotiation, but that, currently, is lacking in literature especially in the case the models are totally decentralized. Instead, many works in literature adopt flags and checks to avoid infinite loops and the divergence of the negotiation and to force its termination if it does not spontaneously end. We are interested in proposing an analytical approach to analyze stability and to determine the constraints that assure it.

We propose a novel formulation of the stability of a negotiation in a multiagent system based on the *connective stability* definition introduced in [19] for dynamical systems under structural perturbations. Connective stability definition is, in fact, very suitable in the multiagent system field. It aims at studying the constraints that assure the stability of a system that can undergo structural modification, such as a multiagent system when an agent connects and disconnects. We define the connective stability of a multiagent negotiation via a mediator and propose a preliminary study that aims at determining constraints that assure the connective stability. In the accomplishment of that, we have casted the negotiation into a dynamic system and we have adopted a geometrical approach to design candidate Lyapunov functions [13]. Then, we have forced the agent negotiation to satisfy the Lyapunov hypotheses determining, thus, constraints related to each agent that assure the connective stability. The obtained constraints can be enforced by letting a geometrical index that relates the proposal of an agent with the previous one of the same agent to vary only in a particular range of values. Then each agent can autonomously apply such constraint to its proposals according to the model and the negotiation function it embeds. In this paper we just describe the adopted geometrical index and its constraints emphasizing particularly its role when the agreement of the negotiation is the weighted average of the proposals of the agents. Finally, we report some simple examples to clarify the method we propose (we remand an interesting reader to [8]).

The paper is structured as follows. In the Section 2 we describe the complex system simulation and control problem by using multiagent paradigm. In Section 3 we introduce the connective stability issue and we define it in the case of the agent negotiation. In Section 4 we propose an analysis of the connective stability in a general agent negotiation. In Section 5 we report some simple application examples. Finally, Section 6 concludes the paper.

2 Complex System Simulation and Control via Multiagent System

Simulation and control of complex dynamical systems, such as systems exhibiting intricate interconnections, high dimensionality, multi-resolution, multi-representation, and uncertainty, are difficult tasks to tackle [1]. It is commonplace in literature the idea to face the complexity by representing complex systems using different models according to several dimensions, such as at different scales, in operating contexts, etc. [15]. In such a way simulation and control, although they are different tasks, they can be similarly addressed by combining different decision making processes embedding the respective simulation and control models.

The multimodeling paradigm – introduced in [6, 7] – was one of the earlier methodologies based on the adoption of a set of models to represent a single phenomenon. However, the adoption of a multiplicity of models of an unique phenomenon rises several issues regarding the coexistence of them. In particular the models can be overlapping (different models can be effectively applied at the same time) and, as consequence, conflicts can occur due to the overlaps. Traditional techniques such as weighted average, model selection (according to a confidence index of the model), and fuzzy combination, do not solve the conflicts since they do not take into account any inter-effects between the models. In addition, several distributed processes can be addressed only by using a set of models.

In literature optimal decentralized approach [2] is emerging as the most effective technique to simulate/control systems by using different models. An optimal decentralized simulation/control task is formulated as a decentralized multi-objective problem where n independent decision makers embedding, each one of them, a model are forced to cooperate to achieve a common goal [12, 20]. More specifically, each decision maker i determines the optimal variable $\mathbf{u}$ as the argument that minimizes/maximizes an objective function J_i under local and global constraints. Formally, called $\mathbf{x}_0$ the current state of the system, t the time, and $\mathbf{u} = [u_1, \ldots, u_n]$:

$$\begin{cases} \min\limits_{\mathbf{u}} / \max \; J_i \left(u_1(t), \ldots, u_n(t), \mathbf{x}_0\right) \;\; \forall i \\ \text{s.t. } f_i(\mathbf{u}) \leq 0 \end{cases} \tag{2.1}$$

where $f_i(\mathbf{u}) \leq 0$ represents the constraints of the decision maker i. The solution of (2.1) is a computationally non-trivial problem. In some special cases represented by linear or linearizable systems, traditional techniques of decentralized optimization can be adopted [18]. In all other cases, including complex non-linear systems, ad-hoc techniques are required.

In a large number of works in literature, such as [4, 11, 16], market-based/economic techniques are adopted. Each decision maker i operates a local minimization/maximization of its own utility $\mathcal{U}_i$ (the economic analogue of the J_i) and a global optimum is achieved via a cooperative negotiation among them. The term ‘cooperative negotiation’ is in fact adopted in literature to identify a process where several decision makers negotiate among them without knowing the utility functions of the other decision makers (decentralization hypothesis) in order to maximize the utility of the comprehensive system. Thus cooperative negotiation [17] tries to solve the conflicts risen by the different objectives of the decisional makers in order to, in an economic perspective, maximize social utility $\mathcal{U}$ of the system, and, correspondingly, in an optimization perspective, maximize a global objective function J. The function $\mathcal{U}$ is implicitly defined in the negotiation function of the decisional makers, in other words such functions are designed on the purpose of maximizing the social utility function. Cooperative negotiation is thus a way to solve the problem (2.1). We note that $\mathcal{U}$ can be not known *a priori* in the design phase of the simulation/control system, in this case we need to propose parametric negotiation function and calibrate the parameter in order to mimic at best the phenomenon.

Nowadays, another issue is considered interesting to exhibit for complex systems: the dynamical model reconfigurability. Two kinds of model reconfigurability can be taken into account: the substitution of a particular model with another and the insertion and removing of a model in/from the system. However, we note that the former can be traced back to the latter: a model can be substituted by removing it and inserting the new model. Thus the complex systems can be modeled by using a set of models that can change during time. Some interesting applications refer to: the multiple aircraft route control, where a variable number of aircrafts can coexist in a particular flight area, and, similarly to, the robot formation control.

The properties exhibited by multiagent systems, such as the multiparadimgaticy, the autonomy, the mobility, the cooperativity (in particular addressed by negotiation techniques), make the multiagent approach [21] to find a somehow "natural" application within decentralized optimization. The applications of multiagent systems for decentralized optimization currently regards mainly the resource allocation field. For example, in [14] a cooperative negotiation approach for real-time control of cellular network coverage is proposed. Each base station is an agent that negotiates the antenna radiation pattern around traffic spot with the neighboring agents. In [3] a cooperative negotiation approach is adopted to solve a distributed resource allocation problem in radar tracking of targets in an environment. Each agent embeds a model and all the agents periodically negotiate in order to achieve an agreement. However methodologies to achieve a formal analysis of the properties of the cooperative negotiation have not been completely developed yet.

3 A Connective Stability Formulation for Multiagent Systems

The achievement of the simulation/control of a complex system via multiagent rises issues that must be explored to make the control effective, robust, and reconfigurable. In particular, the negotiation mechanism is requested to exhibit several properties: the stability [9] (e.g., does the negotiation always reach an agreement?), the optimality of the result obtained from the negotiation [11, 12] (e.g., is the agreement optimal with respect to the agency desires?), the real-time convergence of the agreement [3, 14] (e.g., does the agreement converge within a temporal dead-line?), the adaptation of the negotiation to the environmental modifications (e.g., can the multiagent system adapt the negotiation mechanism to the environmental modifications).

In this paper we concentrate on the first issue we have introduced: the analysis of the stability of the negotiation, in particular on the provision of constraints that assure the stability of the negotiation. In fact a formal analysis of the stability of cooperative negotiation (as introduced in Section 2) has not been deeply explored yet and flags and checks are usually adopted to verify at runtime the convergence of the negotiation and to force its termination if that does not spontaneously end. Conversely, formal analysis have been proposed in game theory, where differently from cooperative negotiation, the knowledge about utility of the agents is not decentralized. Formal examples of negotiations can be found in [11] and [9], where the negotiation functions change the proposals of the agents, respectively, keeping these tangent to the Pareto optimality surface and combining the direction toward the agreement with the direction tangent to the optimality surface. In both the two cases the achievement of the agreement not can be assured, since many counter-examples of negotiation sessions can be found where the agreement diverges. This is due to the fact that formal analysis of the stability represents a research field not deeply explored yet in multiagent systems. Furthermore, we remark that the stability of the negotiation process just implies the achievement of an agreement not the optimality of the agreement with respect to a criterion. Hence the achieved agreement can not be an equilibrium in the sense of game theory for the system.

Formally, for the purposes of this paper, a negotiation involves n contracting agents and a mediator agent. Each agent i embeds an utility function $\mathcal{U}_i$ referring to a space of variables $A_i \in \mathbb{R}^{N_i}$ and its state is represented by a vector $\mathbf{p}_i \in A_i$; thus $\mathcal{U}_i : A_i \rightarrow \mathbb{R}^{l_i}$ where $l_i = 1$ if it is a single-objective optimization function and $l_i > 1$ if it is a multiobjective optimization function. The spaces A_i can overlap (i.e., they can have common dimensions) and the global parameter space A of the system is $A = \bigcup_{i=1}^{n} A_i$. In addition, each agent i exhibits a negotiation function $\mathcal{F}_i$ that defines the negotiation strategy of the agent i.

We call $\mathbf{p}_{i\rightarrow e}^t$ the *offer* (or *proposal*) formulated by agent i at time t to the mediator e. $\mathbf{p}_{i\rightarrow e}^t$ is a proposed state in the space A_i. We call $\mathbf{p}_e^t \in A$ the counter-offer formulated by the mediator at time t, and $\mathbf{p}_{e\rightarrow i}^t$ the projection of $\mathbf{p}_e^t$ to the space A_i of the agent i. $\mathbf{p}_{e\rightarrow i}^t$ is the expression of the last negotiation step. A *negotiation session* is a sequence of interleaved offers of the agents to equalizer and counter-offers of mediator to the agents, starting at time 0 and ending at time $\tau \in \mathbb{N}$. For example, the portion of a negotiation session regarding agent i can be represented as follows:

$$\mathbf{p}_{i\rightarrow e}^0 \succ \mathbf{p}_{e\rightarrow i}^0 \succ \mathbf{p}_{i\rightarrow e}^1 \succ \cdots \succ \mathbf{p}_{e\rightarrow i}^\tau$$

After each i agent has expressed its proposal $\mathbf{p}_{i\rightarrow e}^t$, the mediator calculates the agreement $\mathbf{a}^t$ of the negotiation as $\mathbf{a}^t = \mathcal{A}\left(\mathbf{p}_{1\rightarrow e}^t, \ldots, \mathbf{p}_{n\rightarrow e}^t\right)$ and assign $\mathbf{p}_e^t = \mathbf{a}^t$, generating its counter-offer.

Thus, each agent i calculates its offer $\mathbf{p}_{i\rightarrow e}^{t+1}$ by using its negotiation function $\mathcal{F}_i$, where $\mathcal{F}_i : \mathbb{R}^{N_i} \times \mathbb{R}^{N_i} \rightarrow \mathbb{R}^{N_i}$ and $\mathbf{p}_{i\rightarrow e}^{t+1} = \mathcal{F}_i\left(\mathbf{p}_{i\rightarrow e}^t, \mathbf{a}^t\right)$. In the linear case, we can assume $\mathcal{F}_i = \mathbf{p}_{i\rightarrow e}^t + \mathcal{S}_i\left(\mathbf{p}_{i\rightarrow e}^t, \mathbf{a}^t\right)$, where $\mathcal{S}_i$ is the movement that an agent i accomplishes from a proposal $\mathbf{p}_{i\rightarrow e}^t$ given such proposal and the agreement $\mathbf{a}^t$. We introduce some definitions.

Definition 3.1. A succession of offers $\mathbf{p}_{i\rightarrow e}^t$ converge to $\bar{\mathbf{p}}_i \in A_i$ when $\exists \bar{t} > 0$ such that $\mathbf{p}_{i\rightarrow e}^t = \bar{\mathbf{p}}_i$ for $t > \bar{t}$.

Definition 3.2. A succession of agreements $\mathbf{a}^t$ converge to $\bar{\mathbf{a}} \in A$ when $\exists \bar{t} > 0$ such that $\mathbf{a}^t = \bar{\mathbf{a}}$ for $t > \bar{t}$.

Definition 3.3. A negotiation session is *stable* when the succession of agreements $\mathbf{a}^t$ converge to a $\bar{\mathbf{a}} \in A$.

If a negotiation session is stable, then the agents involved are guaranteed to eventually reach an agreement if they are given enough time.

Definition 3.4. A negotiation session is *strongly stable* when the succession of offers $\mathbf{p}_{i\rightarrow e}^t$ converges to $\bar{\mathbf{p}}_i \in A_i$, for all agents i.

Remark 3.5. It is easy to see that, if a negotiation session is strongly stable, then it is also (simply) stable.

Several issues are involved in the determination of the stability of the negotiation: the models embedded by the agents ($\mathcal{U}_i$), the negotiation strategy adopted by each agent ($\mathcal{S}_i$), the agreement definition ($\mathcal{A}$), and the dinamicity of the agent network (e.g., agents can dynamically connect and disconnect the agent network). The dynamic connection and disconnection of the agents play a relevant role in stability constraint determination, in fact they make the determination of stability constraints to be context dependent, where the context is described by the composition of the network. Thus, whenever a modification of the network occurs the stability must be studied, but this task is computationally hard and it not can be usually accomplished according to the time constraints of the applications. In addition in order to analyze the stability by using classical technique an entity is required to have knowledge of the models of all the agents, violating, thus, the hypothesis of decentralization. Hence we propose the adoption of the concept of *connective stability*, a particular kind of stability developed to the study of interconnected systems under structural perturbations [10, 19]. The connective stability property is very suitable for the multiagent systems since these are composed of interconnected systems (i.e., the agents) and are subject to structural perturbations (i.e., the agent connection and disconnection). Connective stability techniques assign constraints to each single component of the network according to a decentralized approach, thus it does not require a centralized determination of the stability constraints at each time a network modification occurs. According to that we can define the connective stability for a multiagent system.

Definition 3.6. A negotiation session is *connective stable* when the succession of agreements $\mathbf{a}^t$ converge to a $\bar{\mathbf{a}} \in A$ independently from the agent network composition.

Definition 3.7. A negotiation session is *connective strong stable* when the succession of offers $\mathbf{p}^t_{i \to e}$ converges to $\bar{\mathbf{p}}_i \in A_i$, for all agents i independently from the agent network composition.

Remark 3.8. It is easy to see that, if a negotiation session is connective stable, then it is also stable. The reciprocal is not verified.

4 A Connective Stability Analysis

In literature, as we said previously, the connective stability is studied for dynamical systems composed of several interconnected subsystems. The analysis of connective stability is based on the adoption of a vector of Lyapunov functions, one function for each component, and a combination of these for the whole system. In particular the Lyapunov function related to the whole system is the weighted sum of the Lyapunov functions of each component. However, in literature studies on connective stability assume two hypotheses that in the case of multiagent systems are not verified: the *a priori* knowledge about what are the components that can connect to and disconnect from the system, and when such components connect and disconnect. In such a way we need to develop a novel methodology that distributes the stability constraints to each component independently from each other.

To formally study the stability of a session of our cooperative negotiation protocol, it is convenient to cast problem in the framework of dynamical systems [13]. Given the equations introduced in Section 3, the negotiation process can be formulated as:

$$\mathbf{p}^{t+1}_{i \to e} = \mathbf{p}^t_{i \to e} + \mathcal{S}_i\left(\mathbf{p}^t_{i \to e}, \mathbf{a}^t\right) \tag{4.1}$$

This can be seen as a dynamical system corresponding to the negotiation session, whose state at time t is the collection of the $\mathbf{p}^t_{i \to e}$ (for all i). This dynamical system is asymptotically stable if and only if it exists a $\bar{t}$ such that $\mathbf{p}^{t+1}_{i \to e} = \mathbf{p}^t_{i \to e}$, for $t > \bar{t}$ and for all i. Hence, *the dynamical system above is asymptotically stable if and only if the corresponding negotiation session is strongly stable*, the equilibrium of the dynamical system being the collection of $\bar{\mathbf{p}}_i$ of Definition 3.4.

We have adopted a decentralized approach to the determination of the stability constraints using an approach based on the Lyapunov criterion [13]: if we can find a "smooth" positive (equal to 0 in the equilibrium) monotonically decreasing function (called Lyapunov function) of the state of a dynamical system, then the system is asymptotically stable. To apply this criterion in our case, we need to determine a function that is positive and that decreases at each negotiation step. In our analysis we propose Lyapunov candidate functions and we determine constraints on the systems that satisfy Lyapunov hypotheses; we applied such approach to several Lyapunov candidate functions and for each one of them we determined different constraints, we report in this paper the most expressive constraints.

Initially, we define geometrical indexes that we adopt in the study of the Lyapunov candidate functions.

Definition 4.1. Given a generic $\mathbf{o}_i^t \in A_i$, the scalar $\Gamma_i^t = \dfrac{\|\mathbf{p}_{i\to e}^{t+1} - \mathbf{o}_i^t\|}{\|\mathbf{p}_{i\to e}^{t} - \mathbf{o}_i^t\|}$ is called *contraction factor* for the agent i at the time t. Since agents share only $\mathbf{a}^t$, usually $\mathbf{o}_i^t = \mathbf{p}_{e\to i}^t$.

Definition 4.2. Given a generic $\mathbf{o}^t \in A$ and $\mathbf{o}_i^t \in A_i$ the projection of $\mathbf{o}^t$ on A_i, the vector $\mathbf{\Gamma}^t = [\Gamma_1^t, \ldots, \Gamma_n^t]$ is called *contraction vector* for the negotiation at the time t.

We remark that in our proposal a vector $\mathbf{u}$ is less than ($<$) a vector $\mathbf{k}$ if and only if each i*-th* element of $\mathbf{u}$ is less than ($<$) i*-th* element of $\mathbf{k}$.

Remark 4.3. The expression of Γ_i^t in the case of linear/linearizable negotiation function is: $\Gamma_i^t = \dfrac{\|\mathbf{p}_{i\to e}^t + \mathcal{S}_i^t - \mathbf{o}_i^t\|}{\|\mathbf{p}_{i\to e}^t - \mathbf{o}_i^t\|}$.

Definition 4.4. The competition space is a subset of A containing the variables that are in conflicts for the agents. For example, given two agents i and j the variables that satisfy $A_i \bigcap A_j$ compose the competition space for the two agents.

Definition 4.5. The matrix P^t is the collection of the $\mathbf{p}_{i\to e}^t$ projected in the competition space of the variables involved in the negotiation and shared by agents.

Theorem 4.6. *Given n the number of agent and a vector $\mathbf{k} = \sqrt{\dfrac{n-1}{n}}$, the constraint $\mathbf{\Gamma}^t < \mathbf{k}$ for all $t > \bar{t}$ of the negotiation is sufficient to assure the connective strong stability of the negotiation.*

Proof. We call $\mathbf{p}_i^t$ the projection of $\mathbf{p}_{\to e}^t$ to the competition space. It is easy to see that if the system composed of $\mathbf{p}_i^t$ converges then the system composed of $\mathbf{p}_{i\to e}^t$ converges. We assume that all the agents have the same competitive space. We adopt the following Lyapunov candidate function:

$$\mathcal{L}\left(\mathrm{P}^t\right) = \sum_{i=1}^{n}\sum_{j=1}^{n} \|\mathbf{p}_i^t - \mathbf{p}_j^t\|^2 \tag{4.2}$$

It is equal to zero only in the case where $\forall i, j: \ \mathbf{p}_i^t = \mathbf{p}_j^t$, in other words it is equal to zero only when an agreement has been achieved.

It is elsewhere strictly positive since it is a sum of positive term.

In order to prove the monotonically decreasing property, we must determine constraints that satisfy:

$$\mathcal{L}\left(\mathrm{P}^{t+1}\right) < \mathcal{L}\left(\mathrm{P}^t\right) \tag{4.3}$$

We can write $\mathcal{L}(\mathrm{P}^t)$, introducing a vector $\mathbf{o}^t$ defined in the competition space that can change at any negotiation iteration, as:

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\|\mathbf{p}_i^t-\mathbf{p}_j^t\|^2=\sum_{i=1}^{n}\sum_{j=1}^{n}\|(\mathbf{p}_i^t-\mathbf{o}^t)-(\mathbf{p}_j^t-\mathbf{o}^t)\|^2= \tag{4.4}$$

$$=\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\|\mathbf{p}_i^t-\mathbf{o}^t\|^2+\|\mathbf{p}_j^t-\mathbf{o}^t\|^2-2(\mathbf{p}_i^t-\mathbf{o}^t)\cdot(\mathbf{p}_j^t-\mathbf{o}^t)\right)= \tag{4.5}$$

$$=\sum_{j=1}^{n}\sum_{i=1}^{n}\|\mathbf{p}_i^t-\mathbf{o}^t\|^2+\sum_{i=1}^{n}\sum_{j=1}^{n}\|\mathbf{p}_j^t-\mathbf{o}^t\|^2-2\sum_{i=1}^{n}\sum_{j=1}^{n}(\mathbf{p}_i^t-\mathbf{o}^t)\cdot(\mathbf{p}_j^t-\mathbf{o}^t)= \tag{4.6}$$

$$=n\sum_{i=1}^{n}\|\mathbf{p}_i^t-\mathbf{o}^t\|^2+n\sum_{j=1}^{n}\|\mathbf{p}_j^t-\mathbf{o}^t\|^2-2\sum_{i=1}^{n}(\mathbf{p}_i^t-\mathbf{o}^t)\cdot\sum_{j=1}^{n}(\mathbf{p}_j^t-\mathbf{o}^t)= \tag{4.7}$$

$$=2n\sum_{i=1}^{n}\|\mathbf{p}_i^t-\mathbf{o}^t\|^2-2\|\sum_{i=1}^{n}(\mathbf{p}_i^t-\mathbf{o}^t)\|^2 \tag{4.8}$$

Taking into account $\mathcal{L}\left(\mathrm{P}^{t+1}\right)$, we can express this similarly to $\mathcal{L}\left(\mathrm{P}^{t}\right)$ introducing again $(\mathbf{o}^t-\mathbf{o}^t)$. Similarly we obtain:

$$\mathcal{L}\left(\mathrm{P}^{t+1}\right)=2n\sum_{i=1}^{n}\|\mathbf{p}_i^{t+1}-\mathbf{o}^t\|^2-2\|\sum_{i=1}^{n}(\mathbf{p}_i^{t+1}-\mathbf{o}^t)\|^2 \tag{4.9}$$

The monotonically decreasing of Lyapunov function $\mathcal{L}\left(\mathrm{P}^{t}\right)$ is satisfied if:

$$\mathcal{L}\left(\mathrm{P}^{t+1}\right)-\mathcal{L}\left(\mathrm{P}^{t}\right)= \tag{4.10}$$

$$=2n\sum_{i=1}^{n}\left(\|\mathbf{p}_i^{t+1}-\mathbf{o}^t\|^2-\|\mathbf{p}_i^{t}-\mathbf{o}^t\|^2\right)-2\|\sum_{i=1}^{n}(\mathbf{p}_i^{t+1}-\mathbf{o}^t)\|^2+2\|\sum_{i=1}^{n}(\mathbf{p}_i^{t}-\mathbf{o}^t)\|^2= \tag{4.11}$$

writing $\|\mathbf{p}_i^{t+1}-\mathbf{o}^t\|$ as $\Gamma_i^t\|\mathbf{p}_i^t-\mathbf{o}^t\|$ (we use Γ_i^t, for simplicity, to identify the projection of Γ_i^t on the competition space), by Definition 4.1, we have:

$$=2n\sum_{i=1}^{n}\left(((\Gamma_i^t)^2-1)\|\mathbf{p}_i^t-\mathbf{o}^t\|^2\right)-2\|\sum_{i=1}^{n}(\mathbf{p}_i^{t+1}-\mathbf{o}^t)\|^2+2\|\sum_{i=1}^{n}(\mathbf{p}_i^{t}-\mathbf{o}^t)\|^2\leq \tag{4.12}$$

we minimize $\|\sum_{i=1}^{n}(\mathbf{p}_i^{t+1}-\mathbf{o}^t)\|^2$ (we note that: $\min\ \|\sum_{i=1}^{n}(\mathbf{p}_i^{t+1}-\mathbf{o}^t)\|^2=0$):

$$\leq 2n\sum_{i=1}^{n}\left(\left((\Gamma_i^t)^2-1\right)\|\mathbf{p}_i^t-\mathbf{o}^t\|^2\right)+2\|\sum_{i=1}^{n}(\mathbf{p}_i^{t}-\mathbf{o}^t)\|^2\leq \tag{4.13}$$

maximizing $\|\sum_{i=1}^{n}(\mathbf{p}_i^{t}-\mathbf{o}^t)\|^2$ by adopting triangular inequality:

$$\leq 2n \sum_{i=1}^{n} \left(((\Gamma_i^t)^2 - 1) \|\mathbf{p}_i^t - \mathbf{o}^t\|^2 \right) + 2 \sum_{i=1}^{n} \|\mathbf{p}_i^t - \mathbf{o}^t\|^2 \leq \tag{4.14}$$

$$\leq 2 \sum_{i=1}^{n} \|\mathbf{p}_i^t - \mathbf{o}^t\|^2 \left(n \left((\Gamma_i^t)^2 - 1 \right) + 1 \right) < 0 \tag{4.15}$$

inequality (4.15) is satisfied if and only if:

$$\forall i, t: \; n \left((\Gamma_i^t)^2 - 1 \right) + 1 < 0 \; \rightarrow \; \max_{t > \bar{t}} \mathbf{\Gamma}^t < \sqrt{\frac{n-1}{n}} \tag{4.16}$$

Under such constraints $\mathcal{L}$ introduced in Equation 4.2 satisfies Lyapunov hypotheses, so the negotiation is strongly stable. In addition the determined constraints refer to the single agents. In other words if each agent satisfies the constraint expressed in (4.16), the negotiation is stable independently of the models embedded by agents, the negotiation strategies, the agreement, and the connected agents and do not require the agent share information about their model satisfying, thus, decentralization hypothesis. (In Table 1 we report constraints according to n value. In Figure 1 constraints on Γ are showed.) □

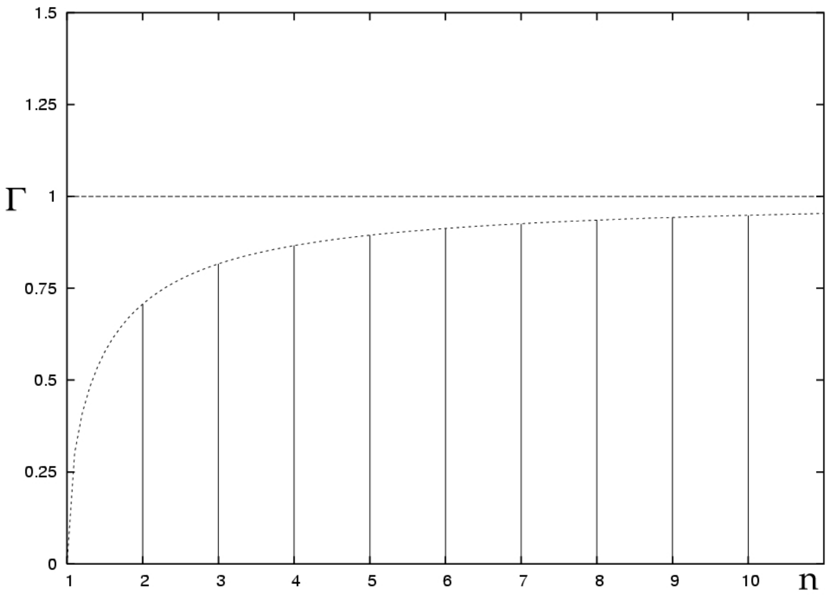

Figure 1: Relation between Γ_i and the number of agents n

	n										
	2	3	4	5	6	7	8	9	10	20	100
$\sqrt{\frac{n-1}{n}}$	0.707	0.816	0.866	0.894	0.912	0.925	0.935	0.942	0.948	0.974	0.995

Table 1: Constraints on $\mathbf{\Gamma}^t$ to assure stability

Remark 4.7. The function $\sum_{i=1}^{n} \sum_{j=1}^{n} \|\mathbf{p}_i^t - \mathbf{p}_j^t\|$ has a particular geometric meaning: it is twice the sum of the length of the vectors that compose the skeleton of the polyhedral whose vertexes are $\mathbf{p}_i^t$ (see Figure 2).

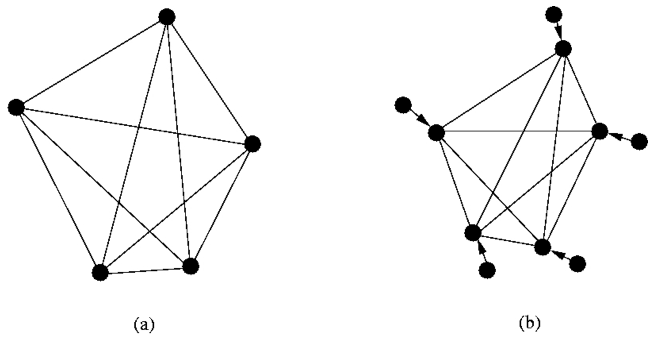

Figure 2: Reduction of the skeleton from case *(a)* to case *(b)*

Remark 4.8. Theorem 4.6 expresses that if at any negotiation iteration from the time $\bar{t}$ all the n agents share a common point in the competition space to converge in, and all n the agents satisfy with constraints on $\Gamma_i = \Gamma_i(n)$ then the negotiation is connective strongly stable.

Remark 4.9. In the case the competition space is different for each agent it is enough to substitute equation (4.8) with $2n \sum_{i=1}^{n} \|\mathbf{p}_i^t - \mathbf{o_i}^t\|^2 - 2\| \sum_{i=1}^{n}(\mathbf{p}_i^t - \mathbf{o_i}^t)\|^2$ where $\mathbf{o}_i$ is the projection of $\mathbf{o}$ on the competition space of agent i. We note that such formula looses the geometrical meaning described in Remark 4.7.

We assume that the agreement $\mathbf{a}^t$ is the weighted average $\mathbf{m}^t$ of the proposals of the agents with weights $\omega_i\left(\mathbf{p}_{i\rightarrow e}^t\right)$ that can change at every t iteration of the negotiation session:

$$\mathbf{a}^t = \mathbf{m}^t = \frac{\sum_{i=1}^{n} \mathbf{p}_{i\rightarrow e}^t \cdot \omega_i\left(\mathbf{p}_{i\rightarrow e}^t\right)}{\sum_{i=1}^{n} \omega_i\left(\mathbf{p}_{i\rightarrow e}^t\right)} \tag{4.17}$$

We note that agreement $\mathbf{a}^t = \mathbf{m}^t$ is not affected by the proposals of the agents with $\omega_i = 0$ during all the negotiation session.

Remark 4.10. If $\mathbf{a}^t = \mathbf{m}^t$, a negotiation of n agents is equivalent to a negotiation of $n+1$ agents where n agents are equal to the agents of the previous negotiation and one agent j with $\omega_j = 0$ during all the negotiation.

Theorem 4.11. *Given a negotiation of n agents with agreement $\mathbf{a}^t = \mathbf{m}^t$, such negotiation is connective strongly stable if $\mathbf{\Gamma}^t < 1$ for all $t > \bar{t}$.*

Proof. From Remark 4.10, any number of agents with $\omega_i = 0$ during all the negotiation can be added to a negotiation of n agents without affecting the negotiation process and agreement. Then by Theorem 4.6 the sufficient condition to assure the stability of a negotiation with ∞ agents is:

$$\max_{t>\bar{t}} \Gamma^t < \lim_{n\rightarrow\infty} \sqrt{\frac{n-1}{n}} = 1 \square \tag{4.18}$$

Remark 4.12. The constraints determined in Theorem 4.6 are not affected by the number of agents that take part in the negotiation.

5 Application Fields

We cite just some application examples where the proposed theoretic methods can be applied. We take into account three simple cases: a negotiation between seller and buyer as formulated by Jennings *et al.* in [5], the negotiation paradigm adopted in [9] by Amigoni and Gatti to combine physiological models, and the negotiation paradigm adopted in [8] to solve Cornout game by using a decentralized approach.

In the negotiation seller-buyer introduced in [5], the seller and the buyer try to adjust their proposals to achieve a common agreement. It easy to see that, if max buyer price is lower than min seller buyer, there exists a time t after which the buyer always offers its max and the seller always offers its min independently from the agreement calculated in t (formally: $\Gamma^t_{buyer} = 1$ and $\Gamma^t_{seller} = 1$). Hence their proposals do not converge violating the constraints $\Gamma_i < 1$.

In [9] a negotiation paradigm is adopted to combine different physiological models relating the same phenomenon. The proposed negotiation paradigm exhibits a set of parameters that must be tailored in order to mimic at best the physiological phenomenon. The application of the constraints $\Gamma_i < 1$ to this negotiation allows the determination of the ranges of the parameters that assure the stability of the negotiation. Once such ranges are determined the process of calibration operates to tailor the parameters belonging to the respective ranges.

In [8] $\Gamma_i < 1$ is adopted to design a stable negotiation among agents to satisfy the Pareto optimality. In particular each agent is an actor of Cornout game and does not know anything about the utility of the other actors. The designed negotiation paradigm allows to achieve a Pareto optimal agreement assuring the converge of the agreement by adopting the criterion proposed in this paper.

6 Conclusions

In this paper we proposed a novel approach for the stability analysis of the cooperative negotiation. We introduced the concept of connective stability relating the negotiation of a dynamic set of agents and we proposed an approach based on Lyapunov criterion to assure the connective stability of the negotiation. Furthermore it has been analytically proved. The result obtained is general and can be applied to every kind of negotiation.

In future we intend to apply the proposed constraints to several different applications to map such constraints to a specific set of negotiation functions. Moreover we will explore the real-time aspects according to the formalization proposed in this paper in order to assure the convergence of the agreement within a given temporal dead-line. Other issues we are interested to explore are the determination of the optimality of the achieved agreement (Pareto optimality and its peculiarities, such as Kalai and Smodorov optimal, and Nash equilibrium).

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