A Connective Stability Analysis of Complex System Simulation and Control via Multiagent Systems

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- We present an analysis of cooperative negotiation in the area of complex complex system simulation and control

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- We introduce a formalism to describe cooperative negotiation performed via multiagent systems
- We focus on a particular form of stability, called *connective stability*, for cooperative negotiation
- We present some results of a theoretical study of the connective stability of cooperative negotiation

What is a Complex System?

- The definition of ‘complex system’ is a debated question in literature [Astrom *et al.*, 2001]
  - Controversial definitions exist
  - A common assumption: *the behavior of a complex system depends on the interaction of several heterogeneous subsystems*
- Fields concerned with complex systems [Meyer, 1997]:
  - Physics
  - Engineering
  - Economy
  - Medicine
  - etc.

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  - Dynamic systems
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  - Robot formation management
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  - Dynamic systems
  - Mathematical models
- Examples:
  - Airplane trajectory determination [Inalhan *et al.*, 2002]
  - Resource allocation [Heiskanen, 1999]
  - Robot formation management
  - etc.
- How to face complexity?
  - Adoption of different models according to several dimensions [Astrom *et altri*, 2001]:
    - Scale
    - Operating context
    - etc.
  - How can such models be harmonized?

Classical Approaches

- Multimodeling [Fishwick, 1992]
  - Many concurrent models for a single phenomenon
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Advantages

- Coexistence of many models
- Refinement of models

Drawbacks

- There not exists integration among models
- They do not easily address highly dynamic complex systems and heterogeneous and spatial distributed models

An Emerging Approach (1)

- Decentralized optimization [Antsaklis, 1999]:
  - Decentralized multi-objective problem
  - n independent decision makers, each one embedding a model
  - The n independent decision makers are designed to cooperate to achieve a common goal

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- Formally:
  - $\mathbf{x}_0$: current state of the system
  - $\mathbf{u} = [u_1, \ldots, u_m]$: vector of variables
  - $f_i(\mathbf{u}) \leq 0$: constraints of the decision maker i

$$\begin{cases} \arg\min\limits_{\mathbf{u}} / \arg\max J_i\left(u_1(t), \ldots, u_n(t), \mathbf{x}_0\right) \;\; \forall i \\ \text{s.t. } f_i(\mathbf{u}) \leq 0 \end{cases}$$

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Open Issues

- How to face dynamicity of complex system?
  - Examples: new airplanes in an airspace, new resource users, new robots in a formation, etc.
- How to face heterogeneity and spatial distribution?
  - Examples: airplanes, remote resource users, robots, etc.

A Solution for Decentralized Optim.

- Cooperative negotiation via multiagent systems [Palm, 2004]
  - *Multiagent system* is a paradigm to model dynamic, heterogeneous, and spatial distributed entities [Weiss, 1999]
  - *Cooperative negotiation* is an optimization technique for reconfigurable and scalable systems [Bingam and Du, 2003]

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  - *Cooperative negotiation* is an optimization technique for reconfigurable and scalable systems [Bingam and Du, 2003]
- Peculiarities:
  - High formal degree (due to mathematical models)
  - High decentralization degree (due to reconfigurability and scalability)

Cooperative Negotiation

- Classic decentralized optimization:

$$\begin{cases} \arg\min_{\mathbf{u}} / \arg\max J_i \left(u_1(t), \ldots, u_n(t), \mathbf{x}_0 \right) \ \forall i \\ \text{s.t. } f_i(\mathbf{u}) \leq 0 \end{cases}$$

- Cooperative negotiation [Voos, Litz, 2000]:
  - Each agent embeds a J_i (utility $\mathcal{U}_i$ in an economic approach)
  - Each agent performs an individual optimization (*agent optimization*)
  - All the agents, via a mediator, negotiate to reach a common agreement (*agency optimization*)

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- The mediator embeds the agreement definition $\mathcal{A}(\{\mathbf{p}^t_{i \to e}\})$
- $\mathbf{p}^t_{e \to i}$ the counter-proposal of mediator e to agent i at time t
- A negotiation session is a sequence of interleaved proposals of the agents to the mediator and counter-proposals of mediator to agents:

$$\mathbf{p}^0_{i \to e} \succ \mathbf{p}^0_{e \to i} \succ \mathbf{p}^1_{i \to e} \succ \cdots \succ \mathbf{p}^\tau_{e \to i}$$

Desirable Properties

- Stability (does the negotiation always reach an agreement?)
  - Classical game theory cannot be adopted to design stable negotiations, but only to analyze them in particular simple cases [Roth, 1985]
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Desirable Properties

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  - Classical game theory cannot be adopted in presence of high decentralization
- Optimality (is the agreement optimal with respect to the agency desires?)
- Real-Time (does the agreement converge within a temporal dead-line?)
- Adaptation (can the negotiation mechanism adapt to the environmental modifications?)

Definition of Stability

- A succession of offers $\mathbf{p}^t_{i \to e}$ converge to $\bar{\mathbf{p}}_i \in A_i$ when $\exists \bar{t} > 0$ such that $\mathbf{p}^t_{i \to e} = \bar{\mathbf{p}}_i$ for $t > \bar{t}$

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- A succession of agreements $\mathbf{a}^t$ converge to $\bar{\mathbf{a}} \in A$ when $\exists \bar{t} > 0$ such that $\mathbf{a}^t = \bar{\mathbf{a}}$ for $t > \bar{t}$
- A negotiation session is *strongly stable* when the succession of offers $\mathbf{p}^t_{i\rightarrow e}$ converge to $\bar{\mathbf{p}} \in A_i$, for all agents i
  - If a negotiation session is strongly stable, then the agents involved are guaranteed to converge to an agreement

Elements Affecting Stability

- The stability of a negotiation session depends on:
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  - Negotiation functions $\{\mathcal{F}_i\}$
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- Additional problem: agents can continuously connect to and disconnect from the system
  - The stability must be studied at *runtime* whenever a *modification of the network of agents* occurs
  - In complex systems, dynamic systems, and systems that require a low latency the stability usually cannot be studied at runtime

Introduction to Connective Stability

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- *Connective stability* is studied for dynamic systems subject to structural perturbations [Siljak, 1972]
- Multiagent systems can be considered as particular dynamic systems subject to structural perturbations (agents can connect and disconnect)
- *A cooperative negotiation is connective stable if it is stable independently from the connected agents*
  - More formally: a negotiation session is *connective strongly stable* when the succession of proposals $\mathbf{p}^t_{i \rightarrow e}$ converge to $\bar{\mathbf{p}} \in A_i$, for all agents i independently from the connected agents

Connective Stability Study

- Our aim:
  - Finding general constraints that assure the connective stability of the negotiation for both analysis and design

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- Our proposed methodology:
  1. Cast a negotiation session into an equivalent dynamic system with discrete time t, state $\mathrm{P}^t = [\mathbf{p}^t_{1\rightarrow e} \;\; \mathbf{p}^t_{2\rightarrow e} \;\; \ldots]$, and transfer function:
  $$\mathbf{p}^{t+1}_{i\rightarrow e} = \mathcal{F}_i\left(\ldots\right)$$
  2. Use the Lyapunov criterion [Lasalle and Lefschetz, 1961]

Lyapunov Criterion

- Lyapunov criterion: *if there exists a smooth positive (equal to 0 in the equilibrium) monotonically decreasing function $\mathcal{L}$ – called Lyapunov function – of the state $\mathbf{x}$ of a dynamic system, then the system is asymptotically stable*

  1. $\mathcal{L}(\mathbf{x}) = 0$ if and only if $\mathbf{x}$ is the equilibrium of the dynamic system
  2. $\mathcal{L}(\mathbf{x}) > 0$ for all other $\mathbf{x}$
  3. $\mathcal{L}(\mathbf{x}^t) > \mathcal{L}(\mathbf{x}^{t+1})$ for all $\mathbf{x}$

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- To study stability with the Lyapunov criterion, we have to find a function that satisfies the Lyapunov hypotheses for a given dynamic system
  1. We propose Lyapunov candidate functions $\mathcal{L}$
  2. We determine the constraints for the negotiation that allow $\mathcal{L}$ to satisfy the Lyapunov hypotheses

Connective Stability Criterion

- Given a generic $\mathbf{o}_i^t \in A_i$, the scalar $\Gamma_i^t = \dfrac{\|\mathbf{p}_{i \to e}^{t+1} - \mathbf{o}_i^t\|}{\|\mathbf{p}_{i \to e}^{t} - \mathbf{o}_i^t\|}$ is called *contraction factor* for the agent i at the time t

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- **Theorem:** Called n the number of agents, if $\forall i \ \Gamma_i^t < \sqrt{\frac{n-1}{n}}$ for all $t > \bar{t}$, then the negotiation is connective strongly stable

Proof (1)

- Lyapunov candidate function:

$$\mathcal{L}\left(\mathsf{P}^t\right) = \sum_{i=1}^{n}\sum_{j=1}^{n} \|\mathbf{p}_i^t - \mathbf{p}_j^t\|^2$$

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  2. It is elsewhere strictly positive since it is a sum of positive terms
  3. In order to prove that it is monotonically decreasing, we must determine the constraints under which:

$$\mathcal{L}\left(\mathsf{P}^{t+1}\right) < \mathcal{L}\left(\mathsf{P}^t\right)$$

Proof (2)

- Introducing a vector $\mathbf{o}^t$:

$$\mathcal{L}\left(\mathsf{P}^t\right) = \sum_{i=1}^{n}\sum_{j=1}^{n}\|\mathbf{p}_i^t - \mathbf{p}_j^t\|^2 = \sum_{i=1}^{n}\sum_{j=1}^{n}\|(\mathbf{p}_i^t - \mathbf{o}^t) - (\mathbf{p}_j^t - \mathbf{o}^t)\|^2 = \ldots$$

$$\ldots = 2n\sum_{i=1}^{n}\|\mathbf{p}_i^t - \mathbf{o}^t\|^2 - 2\|\sum_{i=1}^{n}(\mathbf{p}_i^t - \mathbf{o}^t)\|^2$$

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$$\dots = 2n\sum_{i=1}^{n}\|\mathbf{p}_i^t - \mathbf{o}^t\|^2 - 2\|\sum_{i=1}^{n}(\mathbf{p}_i^t - \mathbf{o}^t)\|^2$$

- Similarly:

$$\mathcal{L}\left(\mathsf{P}^{t+1}\right) = 2n\sum_{i=1}^{n}\|\mathbf{p}_i^{t+1} - \mathbf{o}^t\|^2 - 2\|\sum_{i=1}^{n}(\mathbf{p}_i^{t+1} - \mathbf{o}^t)\|^2$$

Proof (3)

- After some math, we obtain:

$$\mathcal{L}\left(\mathsf{P}^{t+1}\right) - \mathcal{L}\left(\mathsf{P}^{t}\right) \leq 2\sum_{i=1}^{n} \|\mathbf{p}_i^t - \mathbf{o}^t\|^2 \left(n\left((\Gamma_i^t)^2 - 1\right) + 1\right)$$

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- To have $\mathcal{L}\left(\mathsf{P}^{t+1}\right) - \mathcal{L}\left(\mathsf{P}^{t}\right) < 0$ ($\mathcal{L}$ monotonically decreasing):

$$\forall i, t: \; n\left((\Gamma_i^t)^2 - 1\right) + 1 < 0 \;\rightarrow\; \max_{t > \bar{t}} \mathbf{\Gamma}^t < \sqrt{\frac{n-1}{n}} \;\square$$

	n						
	2	3	4	5	10	20	100
$\sqrt{\frac{n-1}{n}}$	0.707	0.816	0.866	0.894	0.948	0.974	0.995

VALUES OF THE CONSTRAINT VALUE IN FUNCTION OF THE NUMBER n OF AGENTS

A Particular Case

- The agreement $\mathbf{a}^t$ is the weighted average $\mathbf{m}^t$

$$\mathbf{a}^t = \mathbf{m}^t = \frac{\sum_{i=1}^{n} \mathbf{p}_{i \to e}^t \cdot \omega_i \left(\mathbf{p}_{i \to e}^t\right)}{\sum_{i=1}^{n} \omega_i \left(\mathbf{p}_{i \to e}^t\right)}$$

- A negotiation among n agents is equivalent to a negotiation among $n+1$ agents where the agent $n+1$ has $\omega_{n+1} = 0$ during the negotiation
- **Corollary:** A negotiation among any number of agents with agreement $\mathbf{a}^t = \mathbf{m}^t$ is connective strongly stable if $\mathbf{\Gamma}^t < 1$ for all $t > \bar{t}$

Conclusions

- Cooperative negotiation via multiagent systems can effectively address reconfigurable and scalable decentralized optimization problems
- Connective stability theory is significant for cooperative negotiation via multiagent systems
- The proposed methodology can be generalized to other negotiation protocols
- Future works:
  - Apply the criterion to other negotiation protocols
  - Study the real-time properties of our negotiation protocol