MULTITARGET DETECTION/TRACKING BASED ON HIDDEN MARKOV MODELS

Monica Nicoli [1], *Vittorio Rampa* [2], *and Umberto Spagnolini* [1]

(1): Dip. Elettronica e Informazione, Politecnico di Milano, Italy
(2): C.S.T.S. - C.N.R., Politecnico di Milano, Italy

ABSTRACT

In several remote sensing applications, multitarget detection/tracking (D/T) of the backscattered wavefields is a very demanding task. Wavefield signals, sampled by an array of sensors, can be described by an hidden Markov model (HMM). As a consequence, the time of delay (TOD) profiles for each of the wavefield (or target) can be estimated by any of the known methods for state-sequence estimation such as the Viterbi (VA) and the backward/forward (BFA) algorithms. Some assumptions, that arise in the wavefield separation problem, allow one to include some additional constraints that preserve the target/tracker association. When an improved resolution is required, the choice of the multitarget Viterbi algorithm (MVA) is mandatory even if its complexity increases exponentially.

1. INTRODUCTION

In many application fields, such as remote sensing, geophysics and underwater acoustics, the characteristics of the propagating medium are inferred by sampling the backscattered wavefield with an array of sensors. To achieve delay resolution, a wideband signal is usually employed. However, to overcome the difficulties of the parametric approach due to an oversimplified propagating model adopted, a non-parametric method appeals for its simplicity and flexibility. The estimation of the delay functions for the impinging wavefronts is obtained by tracking the corresponding TODs; this is simply referred to as target tracking where each of the wavefront is one of the targets. The TODs can be described by a Markov model, signals are modelled as HMM. The tracking method for TODs can be thus adapted from the frequency-line tracking methods that also exploits the advantages of HMM formulation (see e.g., [6] and [7]). Both frequency and delay tracking methods are not different in principle but the details that refer to each application need to be carefully considered to fully take advantage of the model. In this paper, the HMM for delay tracking of wideband signals are considered by differentiating the approach from those proposed in the literature. In addition, the specific applications allow one to modify the HMM in order to keep the target/tracker association.

The wavefield sampled by the i-th sensor ($i = 1, ..., N$) is:

$$s(x_i, t) = \sum_{\ell=1}^{L_i} c_i^{(\ell)} w(t - \tau_i^{(\ell)}) + n(x_i, t), \qquad (1)$$

the ℓ-th target is characterized by the amplitude $c_i^{(\ell)}$ (for backscattered signals it is $|c_i^{(\ell)}| \leq 1$) and the delay $\tau_i^{(\ell)}$ of the corresponding echo. Moreover, x_i is the space location along the array (not necessarily uniformly spaced). L_i is the number of targets for the i-th sensor and $n(x_i, t)$ is the zero mean spatially and temporally uncorrelated Gaussian noise with known variance σ_n^2. In active systems, the waveform $w(t)$ is known and the TODs $\{\tau_i^{(\ell)}\}_{\ell=1}^{L_i}$ of the sampled wavefield must be estimated from $s(x_i, t)$. The number of targets L_i can change from one sensor to the neighboring one but it is bound by the maximum number L of targets that are allowed to be present simultaneously: $0 \leq L_i \leq L$. Recently, a multitarget detection/tracking algorithm (D/TA) for subsurface sensing that has been proposed in reference [5] as adapted from the multitarget tracking strategy employed after front-end detection [1]. However, both methods do not exploit any explicit model of the target maneuvers and show limited performances when multiple targets have closely spaced delays (as they are implicitly limited by the matched filter resolution).

In this paper we propose to circumvent these limitations. In Section 2 it is introduced a unified statistic framework to parameterize both generation and estimation of multiple targets. Each single target $\tau_i^{(\ell)}$ is described as uncorrelated random processes. The well known methods already established for the HMMs may be exploited to estimate the target $\tau_i^{(\ell)}$ from observations [4],[2]. The single target tracking methods are briefly discussed in Section 3 by introducing the Viterbi algorithm (VA) to find the optimal sequence of states (or delays) according to the statistical model. On the other hand, the backward/forward algorithm (BFA) can be employed to identify the target as composed by those echoes that, on a scan-by-scan basis, match the measurement given the assigned target model. Both MMSE and MAP criteria can be employed whenever the posterior probabilities are available. The D/TA is a simplified version of the BFA when only partial information on the data are known [3]. In a multitarget environment, two different D/T approaches may be employed (Section 4): a full multitarget algorithm and a recursive single target one. A true multitarget VA (MVA) is introduced and its performances are evaluated and compared to the ones of the other single target recursive algorithms namely recursive VA (RVA), recursive BFA (RBFA) and D/TA. Performance versus computational complexity of different D/T methods are discussed with simulations (Section 5).

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2. HMM FOR ECHO DELAY

For simplicity, only one wavefield is considered ($L_i = L = 1$) and superscript ℓ is neglected. The delay τ_i can be modeled as a generalized random walk $\tau_i = \tau_{i-1} + v_i$, here v_i is the driving white Gaussian process with zero mean and variance $E\left[v_i^2\right] = \sigma_v^2$ (where $E[.]$ denotes the expectation operator). The sampled wavefield is thus described by a first order HMM. The delay τ_i is discretized into M different quantized delays (or states): $\tau_i \in \{T_1 \ldots T_M\}$. For simplicity, the delay T_k in the state set $\{T_1 \ldots T_M\}$ coincides with the center of the time interval $[T_k - \Delta t/2\ ,\ T_k + \Delta t/2)$ when using the uniform sampling period Δt. By adding the zero state (T_0), the overall set of disjoint states becomes $\mathbf{T} = \{T_0\ T_1 \ldots T_M\}$. This additional state T_0 indicates that there is no target at the current scan. The target trajectory is described by the changes of states. The change from one state to the neighboring one is obtained according to the transition probability $a_{h,k} = p[\ \tau_{i+1} = T_k|\ \tau_i = T_h]$. The zero state is related to the lack of target ($a_{0,0}$), or target initiation ($a_{0,k}$) and termination ($a_{h,0}$). The $(M+1) \times (M+1)$ state transition matrix $\mathbf{A} = [a_{h,k}]$ is assumed to be known as described below. The initial state distribution is defined by assigning $\forall h = 0, 1, .., M$ the set of probabilities $\boldsymbol{\pi} = [\pi_h]$ where $\pi_h = p\left[\tau_1 = T_h\right]$.

The transition probability $g_{h,k} = p\left[\tau_{i+1} = T_k | \tau_i = T_h\right]$ for $h, k \neq 0$ can be calculated by using the delay discretization previously described and according to the Gaussian assumptions for v_i; it is $g_{h,k} = 2^{-3/2}[\mathrm{erf}(\frac{T_k - T_h + \Delta t}{\sigma_v}) - \mathrm{erf}(\frac{T_k - T_h - \Delta t}{\sigma_v})]$. The global transition probabilities $a_{h,k}$ (for $h, k = 0, .., M$) are obtained from $g_{h,k}$ by taking into account that the stochastic matrix $\mathbf{A}$ is normalized as $\sum_{k=0}^{M} a_{h,k} = 1$ for $\forall h = 0, ..., M$. Moreover, target initiation and termination probabilities (θ and ν, respectively) are independent of bin, thus $a_{0,k} = \theta/M$, $a_{h,0} = \nu$ while $a_{00} = 1 - \theta$. Probabilities θ and ν are free parameters that need to be chosen according to the specific application. However, the normalization of matrix $\mathbf{A}$, the finite bin size and the truncation effects due to the limited number of bins can cause edge effects when $a_{h,k}$ are evaluated directly from $g_{h,k}$. This causes the unbalanced gate problem [6] that can be avoided by choosing [7]: $a_{h,k} = (1 - \nu) / g_{\max}$ for $h, k = 1, ..., M$; here $g_{\max} = \max\limits_{h \geq 1} \{\sum_{j=1}^{M} g_{h,j}\}$.

The main differences with respect to other HMM-based tracking algorithms depend on the specific application and on the way the observations $\mathbf{s}_i = \left[s\left(x_i, t_1\right), \ldots, s\left(x_i, t_M\right)\right]^T$, where $(\cdot)^T$ is the matrix transpose, are taken into account in the conditional densities $b_k\left(\mathbf{s}_i\right) = p\left[\mathbf{s}_i \mid \tau_i = T_k\right]$. The observation densities $b_k\left(\mathbf{s}_i\right) = p\left[\mathbf{s}_i \mid \tau_i = T_k\right]$ denotes the probability of the observed sampled signal $\mathbf{s}_i$ conditioned to the state $\tau_i = T_k$. Observations $\mathbf{S}_N = [\mathbf{s}_1, \mathbf{s}_2, ..., \mathbf{s}_N]$ are assumed to be statistically independent. For any pulse system with a known waveform $w(t)$, the observation densities $b_k\left(\mathbf{s}_i\right)$ are derived from the likelihood ratio $\Lambda_k\left(\mathbf{s}_i\right) = b_k\left(\mathbf{s}_i\right) / b_0\left(\mathbf{s}_i\right)$ according to the model (1) [5]:

$$\Lambda_k\left(\mathbf{s}_i\right) = \begin{cases} \exp[-c_i^2 \rho/2 + \rho\ c_i\ \phi_{sw}(x_i, T_k)/E_w] & k \neq 0 \\ 1 & k = 0 \end{cases} \tag{2}$$

The SNR is defined as $\rho = E_w/\sigma_n^2$ while $\phi_{sw}(x_i, T_k) = \langle s(x_i, t)\ \ w(t - T_k)\rangle$ represents the cross-correlation between measured data and waveform delayed by T_k, E_w the energy of the waveform. In practical applications, the SNR is considered a free parameter, the amplitude c_i is estimated by using its ML estimate: $\widehat{c}_i = \phi_{sw}(x_i, T_k)/E_w$. The transition probabilities $a_{h,k}$ do not depend on SNR and describe the variability of the wavefield across the array. In the following, the compact notation $\boldsymbol{\lambda} = (\mathbf{A}, \mathbf{B}, \boldsymbol{\pi})$ indicates the complete parameter set of the model ($\mathbf{B}$ is the observation density set defined as $\mathbf{B} = \{b_k\left(\cdot\right)\}$ and π is the initial state distribution vector).

3. SINGLE TARGET D/T ALGORITHMS

We introduce here the framework for optimum state sequence estimation. Several optimality criteria may be adopted to select the optimum state sequence associated to the given observation sequence. These criteria may lead to locally or globally optimum solutions. For a given sequence of the observations $\mathbf{S}_i = [\mathbf{s}_1, ..., \mathbf{s}_i]$, $\mathcal{S}_{i+1} = [\mathbf{s}_{i+1}, ..., \mathbf{s}_N]$ and a specific set of HMM parameters $\boldsymbol{\lambda}$ (supposed known), it is possible to evaluate the likelihood function of the overall observation sequence $\mathcal{L}\left(\mathbf{S}_N \mid \boldsymbol{\lambda}\right)$ using the BFA. Let $\alpha_i\left(k\right) = p\left[\mathbf{S}_i, \tau_i = T_k \mid \boldsymbol{\lambda}\right]$ be the forward probability (i.e., the probability of the partial observation sequence $\mathbf{S}_i$ and state T_k at the i-th step) and $\beta_i\left(k\right) = p\left[\mathcal{S}_{i+1} \mid \tau_i = T_k, \boldsymbol{\lambda}\right]$ the backward probability (i.e., the probability of the partial observation sequence from the *(i+1)*-th step up to the end given the state T_k at the i-th step). The local optimum criterium depends on the a-posteriori probability $\gamma_i\left(k\right) = p\left[\tau_i = T_k \mid \mathbf{S}_N, \boldsymbol{\lambda}\right]$ according to a specified model set $\boldsymbol{\lambda}$. In BFA the forward/backward probabilities can be combined together to get $\gamma_i\left(k\right) = \alpha_i\left(k\right) \beta_i\left(k\right) / \sum_{k=0}^{M} \alpha_i\left(k\right) \beta_i\left(k\right)$. For the i-th step, the optimum state $\widehat{\tau}_i$ can be evaluated according to the maximum (MAP estimate) or mean (MMSE estimate) of $\gamma_i\left(k\right)$. In Section 5, only the MAP estimate will be considered.

The optimum state sequence, given the whole measurement set $\mathbf{S}_N$, can be obtained using a global optimization method. This can be carried out using the VA shortly revised herein. At the i-th scan, the algorithm searches for the optimum state sequence $\widehat{\tau}_1, ..., \widehat{\tau}_i \in \Gamma$ (where Γ is the set of all possible state sequences) that maximizes the joint probability $p\left[\tau_1, \tau_2, ..., \tau_i = T_k, \mathbf{S}_i \mid \boldsymbol{\lambda}\right]$ of all the state sequences and measurements: $\delta_i\left(k\right) = \max_{\tau_1, \tau_2, ..., \tau_{i-1} \in \Gamma} \{p\left[\tau_1, \tau_2, ..., \tau_i = T_k, \mathbf{S}_i \mid \boldsymbol{\lambda}\right]\}$. For each $i = 1, .., N-1$ and $k = 0, ..., M$, the previous term $\delta_i\left(k\right)$ may be written as:

$$\begin{aligned} \delta_{i+1}\left(k\right) &= b_k\left(\mathbf{s}_{i+1}\right) \max_{0 \leq h \leq M} \{\delta_i\left(h\right)\ a_{h,k}\} = \\ &= \mu_{i+1} \Lambda_k\ \left(\mathbf{s}_{i+1}\right) \max_{0 \leq h \leq M} \{\delta_i\left(h\right)\ a_{h,k}\} \end{aligned} \tag{3}$$

where $\mu_{i+1} = b_0\left(\mathbf{s}_{i+1}\right)$ is a multiplicative term for the i-th iteration. Since the optimum state path needs to be optimized, the true value $\delta_i\left(k\right)$ is of no interest and it can be normalized (this reduce some of the numerical problems that arise in practical implementations). For $i = 1, ..., N-1$, the value of the index k that maximizes the term in brackets of equation (3) keeps track of the optimum state sequence $\zeta_{i+1}\left(k\right) = \arg\max\limits_{0 \leq h \leq M} \{\delta_i\left(h\right)\ a_{h,k}\}$. The algorithm stops when the N-th iteration is performed: $\widehat{\tau}^*(x_N) = \arg\max_{0 \le h \le M} \delta_N(h)$. The optimum state sequence, $\forall i = N-1, ..., 1$, is obtained from the backtracking equation $\widehat{\tau}_i^* = \zeta_{i+1}(\widehat{\tau}_{i+1}^*)$.

A remark is in order: if the model $\boldsymbol{\lambda}$ is not given but it needs to be estimated from the observations, a re-estimation procedure similar to the one described in references [4] and [2] needs to be implemented. This topic is not covered in this paper.

4. VITERBI ALGORITHM FOR MULTITARGET D/T (MVA)

The VA can be extended to the tracking of multiple wavefronts (or multitargets). In remote sensing applications, it is mandatory to maintain the association of the wavefield with the backscattering target. This can be considered the main difference with respect to the multitarget method proposed in reference [7]. The sampled wavefield is described by equation (1) with $L \geq 1$ and the state-space is defined as the L-dimensional set $\{\mathbf{T}\}^L$. As introduced previously, we suppose that each target is described by the uncorrelated generalized random walk $\tau_i^{(\ell)} = \tau_{i-1}^{(\ell)} + v_i^{(\ell)}$. Targets are uncorrelated: $E[\tau_i^{(m)}\tau_i^{(\ell)}] = 0$ for $\ell \neq m$. Sometimes, it can be useful to model different targets with different roughness (or equivalently different variances $\sigma_{v(\ell)}^2$).

The target behavior depends on the $(M+1)^L \times (M+1)^L$ transition probability matrix $\mathcal{A}$ for the set of L states, each element of $\mathcal{A}$ is $a_{h_1,h_2,...,h_L;k_1,k_2,...,k_L} = p[\ \tau_{i+1}^{(1)} = T_{k_1}, \tau_{i+1}^{(2)} = T_{k_2}, ..., \tau_{i+1}^{(L)} = T_{k_L} |\ \tau_i^{(1)} = T_{h_1}, \tau_i^{(2)} = T_{h_2}, ..., \tau_i^{(L)} = T_{h_L}]$. Since here the set of states $(\tau_i^{(1)}, \tau_i^{(2)}, ..., \tau_i^{(L)})$ is ordered according to the targets, the transition matrix $\mathcal{A}$ is a compound of L transition matrixes $\mathcal{A} = \mathbf{A}_1 \otimes \mathbf{A}_2 \otimes ... \otimes \mathbf{A}_L$, where $\mathbf{A}_\ell$ is the transition probability matrix for the ℓ-th target only and $\otimes$ denotes the Knonecker product. When this ordering is not needed, the state sets become indistinguishable with respect to any permutation; for instance $(\tau_i^{(1)} = T_{h_1}, \tau_i^{(2)} = T_{h_2}, ..., \tau_i^{(L)} = T_{h_L})$ coincides with $(\tau_i^{(1)} = T_{h_2}, \tau_i^{(2)} = T_{h_1}, ..., \tau_i^{(L)} = T_{h_L})$. This means that it is not possible to maintain the correct target association.

The likelihood ratio for multitarget system becomes:

$$\Lambda_{\mathbf{h}}(\mathbf{s}_i) = \exp\left\{ \frac{1}{\sigma_n^2}\ \mathbf{C}_i^T \phi_{sw} - \frac{1}{2\sigma_n^2}\ \mathbf{C}_i^T \mathbf{\Phi}_{ww} \mathbf{C}_i \right\}, \quad (4)$$

where $\mathbf{C}_i = [c_i^{(j)}]$, $\phi_{sw} = [\phi_{sw}(x_i, T_{h_j})]$ and $\mathbf{\Phi}_{ww} = [\phi_{ww}(T_{h_m} - T_{h_j})]$ for $\forall \mathbf{h} \in \{(h_1, ..., h_L) : h_j = 1, ..., M$ for $\forall j = 1, ..., L\}$. The likelihood ratio depends on the $2L$ parameters $c_i^{(j)}$ and T_{h_j} ($\forall j = 1, ..., L$). The ML amplitude estimate $\widehat{\mathbf{C}}_i = \mathbf{\Phi}_{ww}^{-1} \phi_{sw}$ let the likelihood ratio depend only on the L parameters T_{h_j}: $\Lambda_{\mathbf{h}}(\mathbf{s}_i) = \exp\{\frac{1}{2\sigma_n^2} \phi_{sw}^T \mathbf{\Phi}_{ww}^{-1} \phi_{sw}\}$. If one or more target disappear, equation (4) is modified accordingly.

The computational complexity and the computer memory of MVA increase exponentially (approx. $N(M+1)^{2L}$ and $N(M+1)^L$, respectively). If $L \gg 1$, the MVA can be used only for off-line processing in remote sensing and geophysical applications. In all these cases, sub-optimal approaches may be employed by estimating recursively few ($L = 2 \div 3$) targets at a time when these can be separated simply by time windowing.

5. PERFORMANCE EVALUATION

Recursive methods are sub-optimum but reduce the computational complexity of the MVA. In D/TA it is proposed to track only one target at a time, the target signature is thus removed from the input data to prevent successive estimation of the same target (this procedure is iterated for all L targets or until no more targets are found). Similarly, the VA or the BFA can be used in a recursive manner; in this case, the multitarget D/T algorithms are referred to as recursive VA (RVA) or recursive BFA (RBFA), respectively.

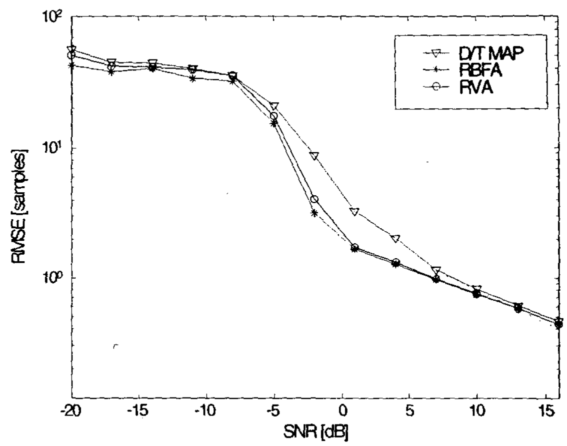

Figure 1: Performance evaluation in terms of root mean square error (RMSE) for estimated delays of single target algorithms (RVA, RBFA and D/TA) evaluated for optimal parameter selection.

The RMSE of the estimated delays for single target algorithms (VA, BFA, D/TA) are shown in Figure 1 for varying SNR and assuming that the HMM parameters $\boldsymbol{\lambda}$ are known (for the D/TA all free parameters are chosen accordingly without the tracking filter [5]). The waveform $w(t)$ is the second-order derivative of a Gaussian pulse $g(t) = \exp(-t^2/2D_w^2)$ as this signal is representative of the waveforms used in remote sensing applications (i.e., $w(t) = \ddot{g}(t)$, here $D_w = 4$ samples). The simulations show that the tracking methods based on joint probability for the overall measurements (RBFA and RVA) have better performances for SNR$\simeq -5dB \div 15dB$ (the threshold region); for larger SNR all the methods have comparable RMSE and attain the Cramer-Rao bound (dotted line).

The multitarget approach based on MVA is compared (for $L = 2$) with the recursive RVA. The example of Figure 2 shows the combination of interfering effects that arise when the targets are closely spaced in time. The HMM model is used to generate two targets with SNR= 12dB, same amplitude $c_i^{(\ell)} = 1$ and standard deviation $\sigma_{v(\ell)} \simeq 1.6$ samples for both targets ($\ell = 1, 2$). The MVA has better resolution capabilities with respect to the RVA that show higher estimation errors (other recursive methods are not shown here as they would have achieved a behavior similar

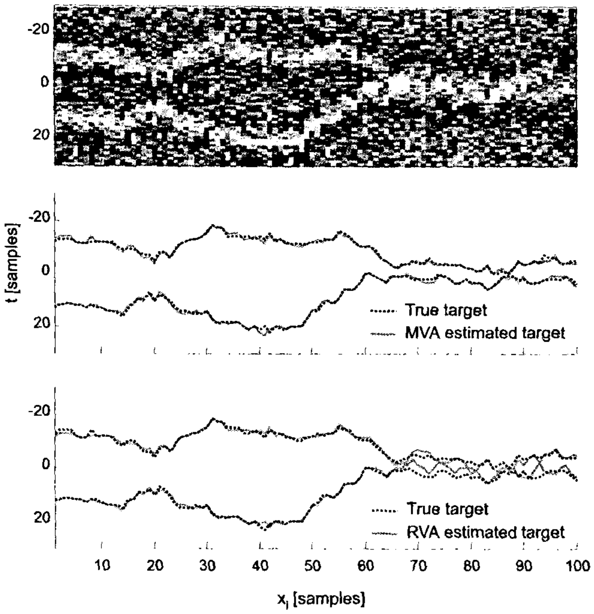

Figure 2: Example of true vs. recursive multitaget D/T methods (from top to bottom) simulated data (SNR= 12dB), MVA and RVA.

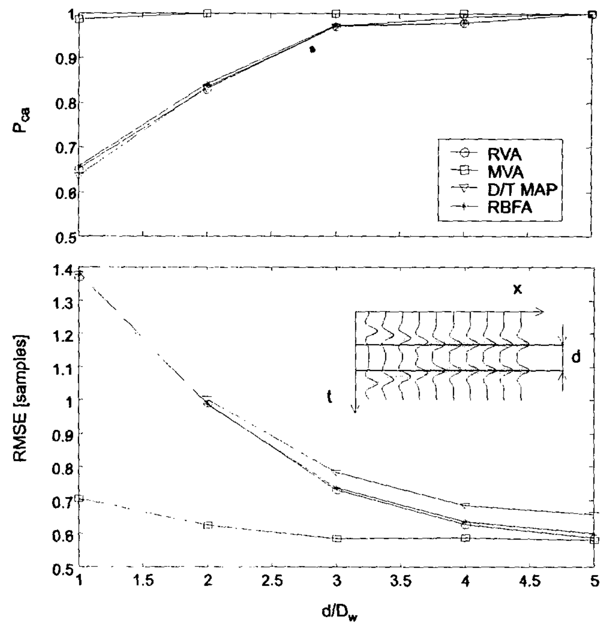

Figure 3: Performance evaluation of different multitarget D/T algorithms: probability of correct target association (top) and RMSE (bottom) (SNR=12dB).

to the RVA one).

The resolution performances of the multitarget algorithms for varying differential delays $\tau_i^{(1)} - \tau_i^{(2)}$ are compared in terms of probability of correct target association P_{ca} and RMSE value (upper and lower part of Figure 3) when the steady configuration $\tau_i^{(1)} - \tau_i^{(2)} = d$ is reached. The RMSE value is computed only for the correctly associated targets. The simulation test set is the same as Figure 2. The correct association depends on a validation interval centered around the true delay (the range of this interval is $\pm D_w/2$ as sketched in the upper frame of Figure 3). The MVA has higher P_{ca} and lower RMSE with respect to the other recursive methods (RVA, RBFA, D/TA), this difference is remarkable when d is small ($d \simeq D_w$). In addition, recursive methods have similar behavior as the resolution on each pass basically depends on $\phi_{sw}(x_i, T_k)$ only (matched filter).

6. CONCLUSIONS

Tracking of delays of backscattered wavefields sampled by an array of sensors can be efficiently handled by using HMMs. MVA is better suited than recursive single target algorithm when a true multitarget application is considered and high P_{ca} is needed. The recursive algorithms may be effectively adopted when the resolution and target association is not an issue. Since all the recursive single target tracking algorithms considered in this paper have comparable behavior, the D/TA remains the most attractive in terms of complexity/resolution trade-off.

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