Non parametric methods for multidimensional wavefront estimation

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Abstract

In active remote sensing applications the time of delay (or phase) of backscattered wavefield recorded by arrays of sensors is useful to get information about the target and/or the propagating medium. In this paper we propose two methods for non-parametric estimation of the time of delay (or phase) function. Both methods are suitable for the wavefront estimation when the arrangement of sources and receivers are rather complex and non-regularly spaced. In addition, the estimate is carried out *simultaneously* on the (n+1)-dimensional the space of measurements (n$\geq$ 1) thus avoiding the extraction of lower dimensional subsets. Examples from SAR interferometry and reflection seismology demonstrate the potentialities of the methods.

1. Introduction

In remote sensing the features of the propagating medium are investigated by measuring the backscattered wavefield for given spatial arrangements of sources and receivers, where each source-receiver pair is globally identified by the n-D set of spatial coordinates $\mathbf{r} = \{x_1, ..., x_n\}$, (n$\leq$ 6). These measurements consist of volumes of (n+1)-dimensions, $d(\mathbf{r}, t)$, where t is time; the wavefronts originated from each target (say the k-th target) can be characterized by delay (or phase) functions describing multiple *hyper-surfaces* $\tau_k(\mathbf{r})$ in $d(\mathbf{r}, t)$. Here we propose two methods to estimate $\tau_k(\mathbf{r})$ from $d(\mathbf{r}, t)$ based on a non-parametric approach (i.e., all the delays for each source-receiver pair represent free parameters). The first method (Section 2) is an extension of the reconstruction of a continuous surface from differential measurements as commonly encountered in least-squares methods for 2-D phase unwrapping [1]. The second method (Section 3) is essentially a tracking algorithm where the delays that pertain to the same backscattered wavefield are tracked according to a statistical model of $\tau_k(\mathbf{r})$.

Routinely the n-D wavefront $\tau_k(\mathbf{r})$ is obtained by estimating on subsets (e.g., 2-D slices) extracted from the (n+1)-D volume. The automatic techniques described here work directly on the global dataset by estimating the wavefront along all dimensions *simultaneously*: this avoids misties among different 2-D slices and improves performances for noisy measurements [1]. Multiple, crossing and irregularly sampled wavefronts are allowed. Discontinuous wavefronts are handled by keeping the association with the target. The examples proposed here show the application of the methods to wavefront estimation across a faulty area in reflection seismics and to phase unwrapping for irregularly spaced phase measurements as used in I-SAR to monitor the terrain motion [2].

The model: Let us consider a single target illuminated by one or more sources placed in multiple positions, the backscattered wavefield sampled by an array of sensors is $d(\mathbf{r}, t)$. The simplest model that can be adopted is:

$$d(\mathbf{r}, t) = a\left(\mathbf{r}\right) w\left(t - \tau\left(\mathbf{r}\right)\right) + n(\mathbf{r}, t) \qquad (1)$$

where: $w(t)$ is the source signature (in general it can be space-varying); $a(\mathbf{r})$ is related to the amplitude of the backscattered wavefield; $\tau(\mathbf{r})$ is the (unknown) wavefront delay; $n(\mathbf{r}, t)$ takes into account any coherent and/or incoherent additive noise. For example, in seismic acquisitions sources and geophones are spread over the (flat) land surface therefore $\mathbf{r} = \{x_s, y_s; x_g, y_g\}$ (where $\{x_s, y_s\}$ locates sources and $\{x_g, y_g\}$ geophones) and correspondingly $\tau(\mathbf{r})$ is a 4-D hyper-surface.

2. Multidimensional wavefront estimation from differential delays

Model (1) can describe either a phase modulation, when $w(t) \approx e^{j\bar{\omega}t}$ (e.g., SAR), or a pulse position modulation, when $w(t)$ is broadband (e.g., seismic). In the first case the phase takes the role of the delay and it can be retrieved from (unaliased) local phase differences. For wideband $w(t)$ the delay $\tau(\mathbf{r})$ is recovered from differential delays $\Delta\tau(\mathbf{r}_i, \mathbf{r}_j)$ estimated from cross-correlations.

The problem of the reconstruction of surfaces from differential heights (leveling) originates from topography and geodesy [3]. In these application fields, the estimated surface $\hat{\tau}(\mathbf{r})$ (where $\mathbf{r} = \{x, y\}$) is constrained to replicate the observed elevation differences $\Delta\tau(\mathbf{r}_i, \mathbf{r}_j)$, i.e. $\hat{\tau}(\mathbf{r}_i) - \hat{\tau}(\mathbf{r}_j) \sim \Delta\tau(\mathbf{r}_i, \mathbf{r}_j)$ for any couple of points $(\mathbf{r}_i, \mathbf{r}_j)$. Given a set of measurements for differential heights (vector $\Delta\boldsymbol{\tau}$), the model $\hat{\boldsymbol{\tau}}$ is fitted to observations through a linear system of equations:

$$\mathbf{A}\hat{\boldsymbol{\tau}} = \Delta\boldsymbol{\tau} \ . \qquad (2)$$

Notice that the mean value of $\tau(\mathbf{r})$ cannot be retrieved from $\Delta\boldsymbol{\tau}$. Eq.(2) is solved in a least squares (LS) sense as $\mathbf{A} \in \mathrm{R}^{N\times M}$ and $N > M$:

$$\hat{\boldsymbol{\tau}} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\Delta\boldsymbol{\tau} \ . \qquad (3)$$

In remote sensing applications the unknown delay (or phase) can be estimated by globally fitting the model $\hat{\boldsymbol{\tau}}$ to all the differential delays (or phases) $\Delta\tau(\mathbf{r}_i, \mathbf{r}_j)$. The extension to n-D (n> 2) is straightforward: it is only required to find the neighbors to each source-receiver pair (or pixel) regardless of the regularity (or irregularity) of their locations [1]. Only the pairing scheme of the points $(\mathbf{r}_i, \mathbf{r}_j)$ is needed to set up eq.(2). The incidence matrix $\mathbf{A}$ is extremely sparse (it has only two non-zero elements

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per row) thus the use of iterative algorithms (e.g., conjugate gradient) in (3) is mandatory. Matrix $\mathbf{A}^T\mathbf{A}$ has almost the same sparsity as $\mathbf{A}$ (provided that the symmetry of $\mathbf{A}^T\mathbf{A}$ is exploited) and it is strongly structured when $\Delta\tau(\mathbf{r}_i, \mathbf{r}_j)$ are evaluated over a regular grid [1]. In this case (3) can be efficiently solved by using DCT [4].

The degree of reliability is far from being the same for all measurements $\Delta\tau(\mathbf{r}_i, \mathbf{r}_j)$ due to noise and/or variations of $w(t)$ across the array. If for each point $\mathbf{r}$ the set of neighboring points is denoted as $\mathcal{V}(\mathbf{r})$, the most reliable measurements are expected when $\mathbf{r}_i \in \mathcal{V}(\mathbf{r}_j)$ and it becomes mandatory to limit differential measurements to neighboring (but not necessarily nearest neighboring) source-receiver pairs. However, a weighted LS (WLS) solution can achieve the same results by choosing the weighting matrix accordingly. Without an analytic model for errors of $\Delta\boldsymbol{\tau}$ a practical and reasonable choice seems to be the inverse distance weighting scheme, i.e. $(\hat{\tau}(\mathbf{r}_i) - \hat{\tau}(\mathbf{r}_j))/d_{i,j} = \Delta\tau(\mathbf{r}_i, \mathbf{r}_j)/d_{i,j}$ where $d_{i,j} = \|\mathbf{r}_i - \mathbf{r}_j\|$. The WLS solution becomes:

$$\hat{\boldsymbol{\tau}} = (\mathbf{A}^T\mathbf{W}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{W}\Delta\boldsymbol{\tau} \tag{4}$$

where $\mathbf{W} = diag\{d_{i,j}^{-1}\}$. Matrix $\mathbf{A}^T\mathbf{W}\mathbf{A}$ is still sparse and structured but DCT cannot be used to solve eq.(4) directly [4]. Similarly, DCT cannot be used when spatial sampling is non-uniform as the number of differential measurements varies from pixel to pixel and the regular structure in $\mathbf{A}^T\mathbf{A}$ is lost.

The probability density function of errors of $\Delta\boldsymbol{\tau}$ is hardly Gaussian due either to phase aliasing or to cycle skipping when the differential delays are estimated by picking cross-correlation maxima. The l_p Norm (with $1 \leq p < 2$) is known to be more robust than l_2 Norm with respect to outliers and it can be conveniently exploited through the iteratively reweighted LS (IRLS) algorithm [5]. IRLS converges to the l_p Norm solution by solving a WLS problem at each iteration where weights are computed from the residuals at the previous step. The computational cost of each IRLS-step increases faster as M raises; a considerable reduction in the number of operations ($\sim$ 30%) can be achieved by using the preconditioned conjugate gradient algorithm with the preconditioning matrix given by the incomplete QR decomposition of $\mathbf{A}$. This is the numerical solution chosen for the application herein. Figure 1 shows an example of 2-D phase unwrapping (i.e., wavefront estimation for n=2) for irregular pixel location as obtained from the monitoring of terrain motion by using few stable natural reflectors (or permanent scatterers, PS), see ref.[2] for further details. After the selection of the PSs, the problem reduces to the estimation of a continuous phase surface for sparse targets. The left part of figure 1 shows the wrapped phase before PS's selection while the right part is the result of the unwrapping only for PSs (the neighboring selection criteria is based on Delaunay triangulation).

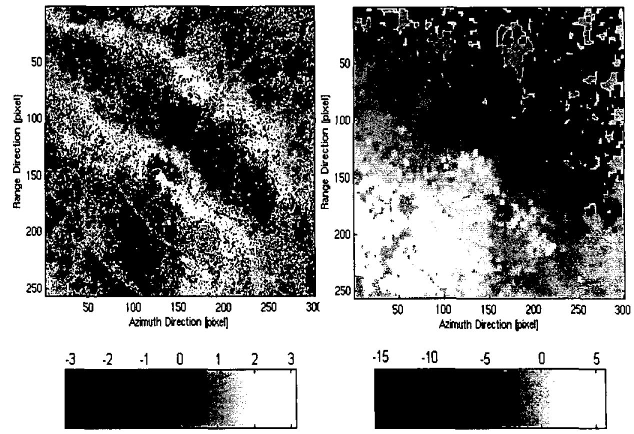

Fig. 1. Wrapped phase on regular grid (left) and unwrapped phase (right) for PS only. For visualization purposes the unwrapped phase for each PS has been assigned to a larger pixel (size 8x8). Image courtesy of A. Ferretti.

3. Probabilistic wavefront tracking

Here the estimation of $\tau_k(\mathbf{r})$ in $d(\mathbf{r}, t)$ is obtained by tracking the times of delay, this is also referred to as horizon tracking as each wavefront appears in $d(\mathbf{r}, t)$ as one horizon. The delays $\tau_k(\mathbf{r})$ are modelled by a Markov model, the signals $d(\mathbf{r}, t)$ are thus described as an hidden Markov model (HMM). As a consequence, the delays of each wavefront can be estimated by any of the known methods for state-sequence estimation, such as the Viterbi and the backward-forward algorithms proposed in ref.[6] for 2-D volumes. A simplified (and somewhat sub-optimum but computationally efficient) version of the forward algorithm when only partial information on the data are known has been proposed for GPR applications, this is the detection/tracking algorithm (D/TA) [7]. In the original form, the D/TA is performed according to a predefined sequence of signals (i.e., the 2-D data volume). In order to handle the n-D source/receivers positioning simultaneously and to have a natural data ordering, here we propose to use a region growing method. The estimation algorithm is the same as D/TA but the ordering of the signals is obtained from the data while tracking. Similarly to the D/TA, the source signature $w(t)$ is assumed to be known (or estimated separately) and multiple target is employed by a recursive approach [7].

The estimation is performed by starting from one (or more) seed pick placed on the wavefront of interest, the delay function $\tau_k(\mathbf{r})$ is obtained by tracking the horizon along the largest continuity paths (that is, the paths where the wavefront can be predicted more easily) till the entire wavefront is covered. At each step of the region growing algorithm, the estimated target grows from any point of the boundary to one of its neighboring point. The choice of the transition depends on a cost-function, here we propose to select the growing-path that maximizes the continuity of the target (i.e., for each $\mathbf{r}_i$ the smaller are the delay variations in $\mathcal{V}(\mathbf{r}_i)$ and the smaller is the cost-function). The main advantage arises by modelling $\tau_k(\mathbf{r})$ as a first-order Markov random process, the a-posteriori probability of detection for each signal in $\mathcal{V}(\mathbf{r}_i)$ can be used as cost-function for the choice of the growing-path along the boundary. This model minimizes the probability of error as the estimation in $\mathbf{r}_j \in \mathcal{V}(\mathbf{r}_i)$ is obtained by using the information derived from the analysis of the consistency for neighboring observations.

To be more specific, let $\mathbf{r}_i^{(j)}$ be any pixel on the boundary at j-th step of the region growing algorithm (see Fig. 2), and $[\mathbf{r}_1^{(j)}, ..., \mathbf{r}_i^{(j)}]$ the pixels selected during previous growing steps ordered into the path that led to $\mathbf{r}_i^{(j)}$, the algorithm selects the transition $\mathbf{r}_i^{(j)} \rightarrow \mathbf{r}_{i+1}^{(j)}$ that maximizes the cost-function, say $C(\mathbf{r}_i^{(j)}, \mathbf{r}_{i+1}^{(j)})$, for all the pixels $\mathbf{r}_i^{(j)}$ in the boundary and for all the corresponding neighboring pixels (i.e., $\mathbf{r}_{i+1}^{(j)} \in \mathcal{V}(\mathbf{r}_i^{(j)})$). Once the transition has been chosen among all the allowed transitions, the delay $\tau(\mathbf{r}_{i+1}^{(j)})$ can be estimated. $C(\mathbf{r}_i^{(j)}, \mathbf{r}_{i+1}^{(j)})$ and the estimate of $\tau(\mathbf{r}_{i+1}^{(j)})$ are based on the a-posteriori probability density function (pdf) of the delay. The a-posteriori pdf is obtained by applying the D/TA algorithm to the sequence of signals associated to the growing path. Let $d(\mathbf{r}_i^{(j)}, t)$ be the signal at $\mathbf{r}_i^{(j)}$, $\mathbf{d}_i^{(j)}$ the time-observations ordered into a vector and $\mathbf{D}_i^{(j)} = [\mathbf{d}_1^{(j)}, ..., \mathbf{d}_i^{(j)}]$ the sequence of observations up to the position $\mathbf{r}_i^{(j)}$, the a-posteriori pdf $p(\tau(\mathbf{r}_{i+1}^{(j)})|\mathbf{D}_{i+1}^{(j)})$ is obtained according to Bayes formula:

$$p(\tau(\mathbf{r}_{i+1}^{(j)})|\mathbf{D}_{i+1}^{(j)}) \equiv p(\mathbf{d}_{i+1}^{(j)}|\tau(\mathbf{r}_{i+1}^{(j)}))\, p(\tau(\mathbf{r}_{i+1}^{(j)})|\mathbf{D}_i^{(j)});$$

where the conditional pdf $p(\mathbf{d}_{i+1}^{(j)}|\tau(\mathbf{r}_{i+1}^{(j)}))$ depends on the cross-correlation between $d(\mathbf{r}_i^{(j)}, t)$ and the known waveform $w(t)$ [6]. The a-priori pdf $p(\tau(\mathbf{r}_{i+1}^{(j)})|\mathbf{D}_i^{(j)})$ is obtained from the a-posteriori pdf for the previous pixel $\mathbf{r}_i^{(j)}$ of the selected path by using the transition probabilities of the Markov model of the horizon. Once the a-posteriori pdf has been determined, the delay is estimated by picking the lag where $p(\tau(\mathbf{r}_i^{(j)})|\mathbf{D}_i^{(j)})$ peaks (MAP criteria).

The probabilistic technique handles discontinuities (e.g., faults) or low SNR areas. If an horizon is connected in the (n+1)-D volume, the tracking grows along the largest continuity paths that go around the discontinuities as the a-posteriori probability in those areas is usually very low. This approach has no proof of global optimality, however the horizon is obtained as a collection of locally optimum choices. The growing algorithm can be efficiently implemented by using an height-balanced binary tree as data structure.

Figure 3 shows an example of discontinuous horizon tracking on a 3-D seismic data. Since the target consists of five disjointed horizons, the estimation is performed by placing one seed point on each surface.

4. Conclusions

Two methods have been proposed for the estimation of delay (or phase) for backscattered wavefield in (n+1)-D dataset (n$\geq$ 1). The first integrates differential measurements, the second one is based on a probabilistic model for the wavefront to be tracked. Both techniques work directly on the global (n+1)-D dataset by estimating the delay along all dimensions simultaneously. Examples of application have been proposed derived from reflection seismology and I-SAR.

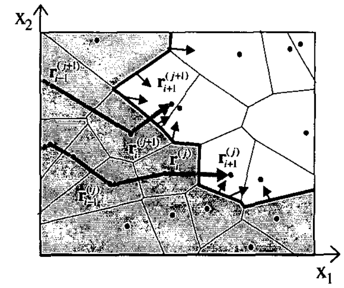

Fig. 2. Region growing algorithm: selection of the growing path (gray line) that maximizes the cost function at j-th step and $(j+1)$-th step.

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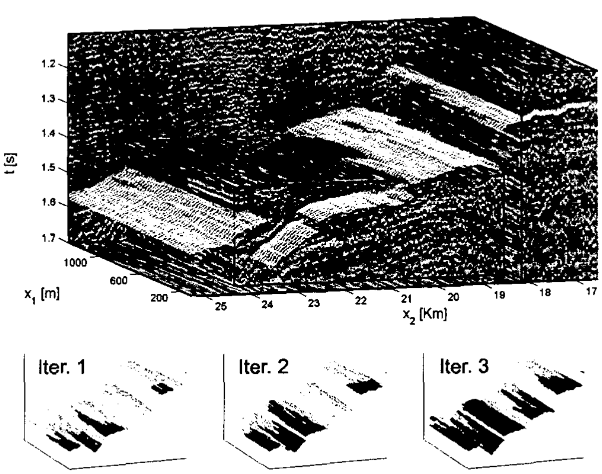

Fig. 3. Tracking of a faulted wavefront on a 3-D data volume. Top figure: estimated wavefront (white mesh) superimposed to $d(x_1, x_2, t)$ (graycolor scale, here sliced for visualization purpose). Botton figure: region growing iterations (dark color).