Soft Iterative Channel Estimation With Subspace and Rank Tracking

Simone Ferrara, *Student Member, IEEE*, Tad Matsumoto, *Senior Member, IEEE*, Monica Nicoli, *Member, IEEE*, and Umberto Spagnolini, *Senior Member, IEEE*

***Abstract*—This letter presents an adaptative soft-based method for channel estimation in turbo receivers. The proposed approach is based on the particular algebraic structure of multipath Rayleigh-fading channels, and it is suited for mobile systems where the multipath pattern (namely, the times of delay) changes slowly over the time. The method is implemented through a rank-and-subspace tracking algorithm that allows to adapt the estimate to the multipath variations and also to reduce the computational cost with respect to the batch implementation based on eigenvalue decomposition. A performance analysis, in terms of mean square error of the channel estimate and bit error rate, shows the advantages of the proposed technique in communications over time-varying wireless channels.**

***Index Terms*—Channel estimation, equalization, mobile communication, multipath channels, soft-iterative receiver, subspace tracking, time-varying channels, turbo processing.**

I. Introduction

PROVIDING a signal detector with accurate estimates of channel parameters is a crucial requirement, especially for iterative signal detection techniques [1], where the channel-state-information reliability makes significant influence on the convergence [2]. Recently, the use of soft feedback has been proposed for re-estimation of the channel parameters in the context of turbo equalization. This soft processing allows to improve the estimate accuracy by increasing the number of known symbols used for the estimate (i.e., by exploiting both pilot and soft-valued detected symbols). Moreover, if the code bits are first interleaved and then segmented into several bursts before transmission, a further performance improvement can be gained by jointly processing multiburst measurements, relying on the long-term properties of the channel covariance matrix in time-varying propagation environments [3]. It has been shown in [4] that the use of a multiple burst (MB) technique, in addition to soft feedback, is effective in reducing the mean-square error (MSE) of the channel estimate. The soft-based MB estimation therein proposed relies on the assumption that the second-order statistics of the channel are slowly varying, and they can be considered as constant within a time interval spanning $L \gg 1$ bursts (e.g., the time interval used for interleaving in turbo processing). However, the MB technique requires a high computational complexity when extracting the subspace spanned by the long-term channel covariance matrix as it requires an eigenvalue decomposition (EVD). Furthermore, the rank of the covariance matrix has to be estimated as well.

In this letter, we propose an adaptive version of the soft-based MB maximum-likelihood (MB-ML) technique [4] that exploits the *a priori* information on the coded bits available at the iterative receiver, and it uses a subspace tracking approach with twofold aim: 1) reducing the computational complexity of the EVD and 2) improving the tracking performance in a scenario where the multipath pattern gradually changes over the time. We assume that the path delays are slowly varying and also that the number of paths can change (due to the birth-and-death process of the paths). The low-rank adaptive filter (LORAF) [5] is employed to track the subspace spanned by the channel impulse responses associated to the varying paths; both the MSE of the estimate and the bit error rate (BER) performances are evaluated through computer simulations. A rank-tracking technique [6] is used in conjunction with subspace tracking. The sensitivity of the tracking performance to the rank estimation uncertainty is evaluated as well.

This letter is organized as follows. Section II defines the signal model and the receiver structure. Section III presents the channel model, and Section IV recalls the MB-ML soft estimator. Tracking algorithms are illustrated in Section V, and simulation results are given in Section VI. Finally, Section VII draws the concluding remarks.

II. System Model

We briefly recall the signal model from [4]. A sequence of convolutionally coded bits is interleaved, mapped into $LN'_{\rm d}$ complex symbols $\{x_{\rm d}(i)\}$, and then transmitted through L bursts over a frequency-selective burst-fading channel. The data sequence contained in each burst is denoted as $\{x_{\rm d}(i;\ell)\}$, where $i = 1,\ldots,N'_{\rm d}$ indicates the symbol index within the burst, and $\ell = 1,\ldots,L$ is the burst index. A training sequence $\{x_{\rm t}(i)\}$ of $N'_{\rm t}$ symbols is also included in each burst to allow channel estimation. At the receiver side, an iterative structure is adopted for data detection and decoding; it consists of a soft-in channel estimator, a soft-cancellation minimum-mean-square-error (SC-MMSE) equalizer [7], and a log maximum *a posteriori* (log-MAP) single-input single-output (SISO) decoder [8]. After the first iteration, the available *a priori* statistics on the information-bearing data are used to evaluate the mean value $\bar{x}_{\rm d}(i) = {\rm E}[x_{\rm d}(i)]$ and the variance $\sigma^2_{\rm d}(i) = {\rm E}[|\Delta x_{\rm d}(i)|^2]$, with $\Delta x_{\rm d}(i) = x_{\rm d}(i) - \bar{x}_{\rm d}(i)$, for each code symbol $x_{\rm d}(i), i = 1,\ldots,LN'_{\rm d}$. Within the ℓth

Manuscript received November 29, 2005; revised May 12, 2006. This work was developed in part within the IST-FP6 Network of Excellence in Wireless Communications (IST-2002-507525 NEWCOM), while S. Ferrara was visiting the Centre for Wireless Communications (CWC), University of Oulu, in 2005. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. Richard J. Kozick.

S. Ferrara was with the Dipartimento di Elettronica e Informazione (DEI), Politecnico di Milano, I-20133 Milano, Italy, and is now with the Department of Electrical and Systems Engineering, Washington University, St. Louis, MO 63130 USA (e-mail: ferrara@ese.wustl.edu).

T. Matsumoto is with the CWC, University of Oulu, FI-90014 Oulu, Finland (e-mail: tadashi.matsumoto@ee.oulu.fi).

M. Nicoli and U. Spagnolini are with the DEI, Politecnico di Milano, I-20133 Milano, Italy (e-mail: nicoli@elet.polimi.it; spagnoli@elet.polimi.it).

Digital Object Identifier 10.1109/LSP.2006.881530

burst, these quantities will be indicated as $\bar{x}_{\mathrm{d}}(i;\ell)$ and $\sigma_{\mathrm{d}}^2(i;\ell)$, respectively.

After matched filtering and symbol-rate sampling, the signals measured within the training and data fields of the ℓth burst are gathered into the vectors $\mathbf{y}_{\mathrm{t}}(\ell) \in \mathbb{C}^{N_{\mathrm{t}}\times 1}$ and $\mathbf{y}_{\mathrm{d}}(\ell) \in \mathbb{C}^{N_{\mathrm{d}}\times 1}$ that can be written as

$$\begin{cases} \mathbf{y}_{\mathrm{t}}(\ell) = \mathbf{X}_{\mathrm{t}}\mathbf{h}(\ell) + \mathbf{w}_{\mathrm{t}}(\ell), & \text{Training} \\ \mathbf{y}_{\mathrm{d}}(\ell) = \bar{\mathbf{X}}_{\mathrm{d}}(\ell)\mathbf{h}(\ell) + \Delta\mathbf{w}_{\mathrm{d}}(\ell) + \mathbf{w}_{\mathrm{d}}(\ell), & \text{Data.} \end{cases} \tag{1}$$

The vector $\mathbf{h}(\ell) \in \mathbb{C}^{W\times 1}$ denotes the discrete-time impulse response of the channel (including also the filters at the transmitter and receiver). Since its temporal support is $W > 1$, the first $W-1$ samples at the beginning of each field are not considered in (1), to avoid overlapping between training and data symbols, thus leading to the reduced field lengths $N_{\mathrm{t}} = N'_{\mathrm{t}} - W + 1$ and $N_{\mathrm{d}} = N'_{\mathrm{d}} - W + 1$. The convolution matrices $\mathbf{X}_{\mathrm{t}} \in \mathbb{C}^{N_{\mathrm{t}}\times W}$ and $\bar{\mathbf{X}}_{\mathrm{d}}(\ell) \in \mathbb{C}^{N_{\mathrm{d}}\times W}$ are built from the transmitted sequences according to the Toeplitz structures: $[\mathbf{X}_{\mathrm{t}}]_{m,n} = x_{\mathrm{t}}(W+m-n)$ and $[\bar{\mathbf{X}}_{\mathrm{d}}(\ell)]_{m,n} = \bar{x}_{\mathrm{d}}(W+m-n;\ell)$. The vectors $\mathbf{w}_{\mathrm{t}}(\ell) \sim \mathcal{CN}(\mathbf{0}, \sigma_w^2\mathbf{I}_{N_{\mathrm{t}}})$ and $\mathbf{w}_{\mathrm{d}}(\ell) \sim \mathcal{CN}(\mathbf{0}, \sigma_w^2\mathbf{I}_{N_{\mathrm{d}}})$ collect uncorrelated complex-valued Gaussian noise samples with zero mean and variance σ_w^2. The additional term $\Delta\mathbf{w}_{\mathrm{d}}(\ell) = \Delta\mathbf{X}_{\mathrm{d}}(\ell)\mathbf{h}(\ell)$ depends on the soft estimate error matrix $\Delta\mathbf{X}_{\mathrm{d}}(\ell) = \mathbf{X}_{\mathrm{d}}(\ell) - \bar{\mathbf{X}}_{\mathrm{d}}(\ell)$ obtained from the sequence $\{\Delta x_{\mathrm{d}}(i;\ell)\}$. This sequence is treated as uncorrelated zero mean with variance $\sigma_{\mathrm{d}}^2 = (1/LN'_{\mathrm{d}})\sum_i \sigma_{\mathrm{d}}^2(i)$, while $\Delta\mathbf{w}_{\mathrm{d}}(\ell)$ is modeled as a complex white Gaussian noise vector, independent of $\mathbf{w}_{\mathrm{d}}(\ell)$, with zero mean and variance $\Delta\sigma_w^2 = \sigma_{\mathrm{d}}^2\sigma_w^2\rho$ [4], where $\rho = \mathrm{E}[\|\mathbf{h}(\ell)\|^2]/\sigma_w^2$ is the signal-to-noise ratio (SNR).

III. Channel Model

A multipath propagation scenario is considered with $d(\ell)$ paths, delays $\{\tau_k(\ell)\}_{k=1}^{d(\ell)}$, and mean powers $\{A_k(\ell)\}_{k=1}^{d(\ell)}$, where fading complex envelope stays the same during the burst and changes burst by burst. According to the wide sense stationary uncorrelated scattering (WSSUS) and the Rayleigh fading assumptions, the channel is modeled as $\mathbf{h}(\ell) \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_h(\ell))$, with covariance $\mathbf{R}_h(\ell)$ that varies slowly with respect to the fading amplitudes. It can be easily shown that it is $\mathbf{R}_h(\ell) = \mathbf{G}(\ell)\mathbf{R}_\alpha(\ell)\mathbf{G}^{\mathrm{T}}(\ell)$, where $\mathbf{G}(\ell) = [\mathbf{g}(\tau_1(\ell)), \ldots, \mathbf{g}(\tau_{d(\ell)}(\ell))] \in \mathbb{C}^{W\times d(\ell)}$ contains the delayed pulse waveforms (convolution of the transmitter and receiver filter responses) and $\mathbf{R}_\alpha(\ell) = \mathrm{diag}\{A_1(\ell), \ldots, A_{d(\ell)}(\ell)\}$.

In many practical situations, the $d(\ell)$ paths can be grouped into a small set of $r(\ell) \le d(\ell)$ clusters or macro-paths, each gathering paths with comparable delays (i.e., with delay difference below the system resolution). This consideration implies that the columns of $\mathbf{G}(\ell)$ are not necessarily independent, being $r(\ell) = \mathrm{rank}(\mathbf{R}_h(\ell)) \le W$. Thus, the channel can be rewritten using a model similar to that proposed in [3] in terms of the new parameters

$$\mathbf{h}(\ell) = \mathbf{U}(\ell)\mathbf{b}(\ell) \tag{2}$$

where $\mathbf{U}(\ell) \in \mathbb{C}^{W\times r(\ell)}$ is a full-column-rank matrix whose columns represent the slowly changing modes of $\mathbf{h}(\ell)$, while $\mathbf{b}(\ell) \in \mathbb{C}^{r\times 1}$ collects the fast-changing fading amplitudes. The channel modes $\mathbf{U}(\ell)$ can be evaluated by the eigenvalue decomposition (EVD) of the long-term channel covariance matrix $\mathbf{R}_h(\ell)$ (i.e., through a modal analysis). Moreover, during the transmission, paths can appear/disappear, due to the obstacles between the mobile station and the base station, as illustrated in Fig. 1, where it is $r(\ell) = 2$ in the interval $[A, B] \cup [C, D]$ and $r(\ell) = 1$ in the interval $[B, C]$. We assume that this shadowing affects only the eigenvalues, leaving unchanged the temporal modes $\mathbf{U}(\ell)$. The slight variations of the modes from burst to burst are related to the slow and continuous change of the delay times due to the terminal movement.

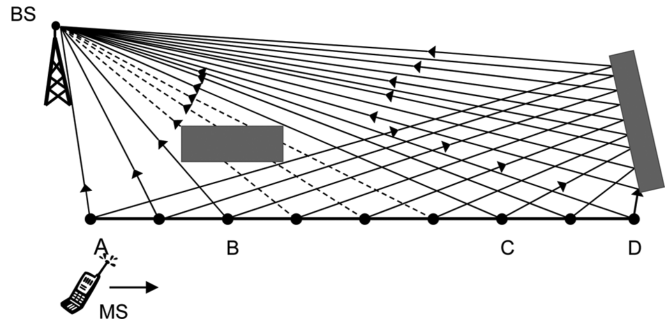

Fig. 1. Burst-varying propagation scenario with a mobile station (MS) moving in the direction of the arrow. During the position intervals A-B and C-D, the channel is composed of two macro-paths (a main one generated by a cluster of scatterers nearby the MS and a secondary one due to reflections on a far-away obstacle), while from the position B to C, there is only one macro-path (reflected) due to the presence of an obstacle (here depicted as a black box) between MS and the base station (BS).

IV. Soft Iterative Channel Estimation

We consider the ML estimation of $\mathbf{h}(\ell)$ from the MB ensemble of measurements $\{\mathbf{y}_{\mathrm{t}}(\ell), \mathbf{y}_{\mathrm{d}}(\ell)\}_{\ell=1}^{L}$ under the constraint (2). Notice that L can correspond to the number of interleaved bursts used in the iterative structure as described in Section II. The ML solution is here recalled from [4] for the case of channel modes and rank being constant within the L-bursts interval, i.e., for $\mathbf{U}(\ell) = \mathbf{U}$ and $r(\ell) = r, \forall\ell$. The extension to time-varying scenarios will be then proposed in the next section. We indicate by $\mathbf{R}_{\mathrm{t}} = \mathbf{X}_{\mathrm{t}}^{\mathrm{H}}(\ell)\mathbf{X}_{\mathrm{t}}(\ell)$ the training-sequence correlation matrix, which is the same for all the bursts. The information-bearing data symbols $\{x_{\mathrm{d}}(i)\}$ are considered as statistically independent, and their number is large enough so that $\mathbf{R}_{\mathrm{d}} = \mathbf{X}_{\mathrm{d}}^{\mathrm{H}}(\ell)\mathbf{X}_{\mathrm{d}}(\ell) \approx N_{\mathrm{d}}\mathbf{I}_W$ and $\bar{\mathbf{R}}_{\mathrm{d}} = \bar{\mathbf{X}}_{\mathrm{d}}^{\mathrm{H}}(\ell)\bar{\mathbf{X}}_{\mathrm{d}}(\ell) \approx \tilde{N}_{\mathrm{d}}\mathbf{I}_W$. Here, $\tilde{N}_{\mathrm{d}} = N_{\mathrm{d}}(1-\sigma_{\mathrm{d}}^2)$ represents the *effective* number of known data symbols depending on the average symbol variance σ_{d}^2.

Soft channel estimation based on the model (1) is performed in two steps. In step 1, the unconstrained ML estimate of $\mathbf{h}(\ell)$ is obtained burst by burst from the single-burst (SB) measurement $\{\mathbf{y}_{\mathrm{t}}(\ell), \mathbf{y}_{\mathrm{d}}(\ell)\}$ based on the white Gaussian assumption $\Delta\mathbf{w}_{\mathrm{d}}(\ell) \sim \mathcal{CN}(\mathbf{0}, \Delta\sigma_w^2\mathbf{I}_{N_{\mathrm{d}}})$. The SB-ML estimate reads

$$\hat{\mathbf{h}}_{\mathrm{SB}}(\ell) = \bar{\mathbf{R}}^{-1}\left(\mathbf{X}_{\mathrm{t}}^{\mathrm{H}}(\ell)\mathbf{y}_{\mathrm{t}}(\ell) + \gamma\bar{\mathbf{X}}_{\mathrm{d}}^{\mathrm{H}}(\ell)\mathbf{y}_{\mathrm{d}}(\ell)\right) \tag{3}$$

with $\bar{\mathbf{R}} = R_{\mathrm{t}} + \gamma\bar{\mathbf{R}}_{\mathrm{d}}$ and $\gamma \triangleq (1 + \Delta\sigma_w^2/\sigma_w^2)^{-1}$. In step 2, a reduced-rank ML approach is applied to the MB measurements $\{\mathbf{y}_{\mathrm{t}}(\ell), \mathbf{y}_{\mathrm{d}}(\ell)\}_{\ell=1}^{L}$ under the constraint (2), yielding the MB-ML estimate. For ideal training sequence, i.e., for $\mathbf{R}_{\mathrm{t}} \approx N_{\mathrm{t}}\mathbf{I}_W$, this estimate is given by

$$\hat{\mathbf{h}}_{\mathrm{MB}}(\ell) = \hat{\mathbf{P}} \cdot \hat{\mathbf{h}}_{\mathrm{SB}}(\ell) \tag{4}$$

where $\hat{\mathbf{P}} = \hat{\mathbf{U}}\hat{\mathbf{U}}^{\mathrm{H}}$ denotes the estimate for the projector onto the channel subspace obtained from the r leading eigenvectors $(\hat{\mathbf{U}})$ of the sample correlation matrix

$$\mathbf{R}_{\mathrm{MB}} = \frac{1}{L}\sum_{\ell=1}^{\mathrm{L}} \mathbf{R}_{\mathrm{SB}}(\ell) \tag{5}$$

with $\mathbf{R}_{\mathrm{SB}}(\ell) = \hat{\mathbf{h}}_{\mathrm{SB}}(\ell)\hat{\mathbf{h}}_{\mathrm{SB}}^{\mathrm{H}}(\ell)$.

V. Subspace and Rank Tracking Algorithm

When the multipath delays vary slightly from burst to burst, the estimate of the modes $\hat{\mathbf{U}}(\ell)$, as well as the projector $\hat{\mathbf{P}}(\ell) = \hat{\mathbf{U}}(\ell)\hat{\mathbf{U}}^{\mathrm{H}}(\ell)$ and the rank $\hat{r}(\ell)$, need to be adapted to the channel variations. This can be accomplished by updating burst by burst the matrix (5), using an exponential weighting

$$\mathbf{R}_{\mathrm{MB}}(\ell) = \sum_{t=1}^{\ell} \beta^{\ell-t}\mathbf{R}_{\mathrm{SB}}(t) = \beta\mathbf{R}_{\mathrm{MB}}(\ell-1) + \mathbf{R}_{\mathrm{SB}}(\ell) \tag{6}$$

where $\beta \in (0,1)$ denotes the forgetting factor.

The LORAF method, proposed in [5] for tracking the subspace spanned by the eigenvectors of the covariance matrix of a parameter vector, has been adapted here to our problem. First, let $\hat{\mathbf{R}}_{\mathrm{MB}}(\ell) = \hat{\mathbf{U}}(\ell)\hat{\mathbf{\Lambda}}(\ell)\hat{\mathbf{U}}^{\mathrm{H}}(\ell)$ be the EVD of the matrix (6) truncated to the $r(\ell)$ leading eigenvalues. The algorithm is based on the consideration that, since at convergence is $\hat{\mathbf{U}}(\ell-1) \approx \hat{\mathbf{U}}(\ell)$, the projection of $\hat{\mathbf{R}}_{\mathrm{MB}}(\ell)$ onto the column-space of the matrix $\hat{\mathbf{R}}_{\mathrm{MB}}(\ell-1)$, defined as $\mathbf{A}(\ell) = \hat{\mathbf{R}}_{\mathrm{MB}}(\ell)\hat{\mathbf{U}}(\ell-1)$, tends by iterations to $\mathbf{A}(\ell) \approx \hat{\mathbf{U}}(\ell)\hat{\mathbf{\Lambda}}(\ell)$. The algorithm may be described by two steps:

1) *Estimation step:* QR decomposition of the matrix $\mathbf{A}(\ell)$ to obtain the estimate $\hat{\mathbf{U}}(\ell)$;
2) *Tracking step:* updating the matrix $\mathbf{A}(\ell)$ as follows:

$$\mathbf{A}(\ell) = \beta\mathbf{A}(\ell-1)\mathbf{\Theta}(\ell-1) + \mathbf{R}_{\mathrm{SB}}(\ell)\hat{\mathbf{U}}(\ell-1) \tag{7}$$

where $\mathbf{\Theta}(\ell) = \hat{\mathbf{U}}(\ell-1)^{\mathrm{H}}\hat{\mathbf{U}}(\ell)$ is a rotation matrix that realigns the axes of the matrix $\mathbf{A}(\ell-1)$ to those of the modes $\hat{\mathbf{U}}(\ell-1)$, the same that the current channel covariance matrix is projected onto.

It is easy to see that, for $\hat{\mathbf{U}}(\ell-2) = \hat{\mathbf{U}}(\ell-1)$, the tracking step (7) can be written as

$$\mathbf{A}(\ell) = \beta\mathbf{A}(\ell-1) + \mathbf{R}_{\mathrm{SB}}(\ell)\hat{\mathbf{U}}(\ell-1) \tag{8}$$

which is fully equivalent to (6) projected onto $\hat{\mathbf{U}}(\ell-1)$.

The availability of the eigenvalue diagonal matrix $\hat{\mathbf{\Lambda}}(\ell) \in \mathbb{R}^{r(\ell)\times r(\ell)}$ in $\mathbf{A}(\ell)$ allows us to employ the minimum description length (MDL) algorithm [6] to track the variations of the rank $r(\ell)$. In order to perform subspace tracking with varying subspace dimension, we assume that $r(\ell)$ is always upper-bounded by a known value $r_{\max}$. It is understood that the subspace has to be tracked not only on the main $r(\ell)$ eigenvalues but also on all the $r_{\max}$ dimensions.

A. Implementation Issues

We recall that the iterative method proposed above for subspace and rank tracking has to be used in a turbo equalizer; thereby, there are two nested loops of iterations: the equalization-decoding turbo loop and, within each detection-decoding iteration, the channel estimation loop over the bursts contained in the frame. This implies that all the variables needed for subspace tracking have to be initialized at the beginning of each frame and also before starting each turbo iteration within the frame. Since the modes vary burst by burst and since the subspace estimate for the last burst of the previous frame (at the last turbo iteration) is available, when initiating the channel estimation for the new frame, we propose the following initialization: 1) for the first frame, at the beginning of each iteration, we set $\hat{\mathbf{U}}(0) = [\mathbf{I}_{r_{\max}}\mathbf{0}_{r_{\max}\times(W-r_{\max})}]^{\mathrm{T}}$, $\mathbf{\Theta}(0) = \mathbf{I}_{r_{\max}}$, and $\mathbf{A}(0) = \mathbf{0}_{W\times r_{\max}}$; 2) from the second frame, at each iteration, we use the values obtained at the last iteration in the last burst of the previous frame.

The implementation of the adaptive channel estimation technique requires the selection of the forgetting factor value β, which affects both the memory and the convergence speed of the tracking algorithm. The forgetting factor defines in fact the effective length of the temporal window used for multiblock averaging. This can be expressed in number of blocks as $L_w \simeq 1/(1-\beta)$ [5]. Usually, delays are characterized by slow variations over the blocks, calling thereby for large values of L_w (i.e., long memory length) so as to reduce the MSE of the channel estimate. On the other hand, sudden changes on the number of paths (i.e., on the channel rank) can occur due to the birth-and-death path process, requiring small L_w values to allow a fast convergence to the new multipath pattern. The optimal value for the parameter β has to be selected as tradeoff between estimate accuracy and convergence speed.

The complexity order of the straightforward EVD implementation is $\mathcal{O}(W^3)$. In the LORAF approach, the order is reduced to $\mathcal{O}(W^2 r_{\max})$ [i.e., the complexity required by the updating of the matrix $\mathbf{A}(\ell)$, according to (7)] providing a complexity gain of $W/r_{\max}$. Moreover, a more efficient implementation for the QR decomposition, the matrix $\mathbf{\Theta}(\ell)$ and $\mathbf{A}(\ell)$ updating processes, provides a further computational cost reduction to $\mathcal{O}(W r_{\max})$ [5].

VI. Simulation Results

For simulations, we assume the following transmission system. Each frame is obtained from 4000 randomly chosen equiprobable information bits, which are coded by a four-state convolutional code with generators $(7,5)_o$ and then permuted by a random interleaver. The coded bits are mapped onto 4000 QPSK symbols and arranged into $L = 20$ bursts with $N'_d = 200$ symbols each. A training sequence of $N'_t = 46$ QPSK symbols is added to each burst. The frame of L bursts is then transmitted over a burst-faded Rayleigh channel with temporal support $W = 10$. A total number of ten frames is sent. The multipath structure is simulated according to the double-cluster channel model in Fig. 1: the multipath pattern is composed of four paths, with power delay profile $\mathbf{R}_\alpha(0) = \mathrm{diag}\{1/3, 1/3, 1/6, 1/6\}$ and $\tau(\ell)$ being linearly varying over ℓ from $\tau(0) = [0, 0.25, 3.25, 3.5]$ μs to $\tau(10L) = [0.13, 0.33, 3.33, 3.58]$ μs. The paths are gathered into $r = 2$ clusters, each composed of two paths having similar delays. The first cluster is shadowed from the 50th to the 150th burst. The ratio between the bit energy and the noise power spectral density is defined as $E_{\mathrm{b}}/N_0 = \mathrm{E}[||\mathbf{h}(\ell)||^2]/\sigma_w^2$, while the SNR is defined as in Section II.

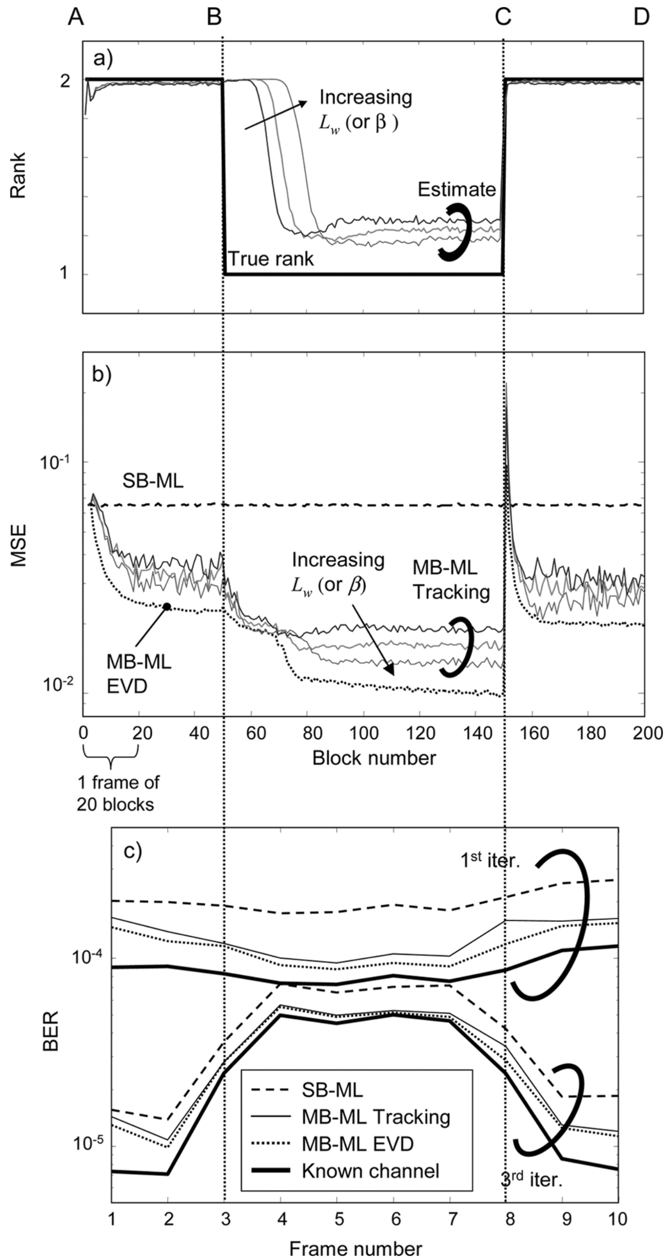

Fig. 2. According to the scenario in Fig. 1: (a) Rank (real and estimated) versus the *block* number. (b) MSE of the channel estimate versus the *block* number. (c) BER versus the *frame* number at first and third iterations of turbo equalization. For MSE simulation, we set SNR $= 6$ dB, $\beta \in \{0.83, 0.86, 0.89\}$, and $\sigma_{\rm d}^2 \approx 0.55$. For BER simulation, $E_{\rm b}/N_0 = 6$ dB and $\beta = 0.89$.

Fig. 2(a) shows the behavior of the rank estimator averaged over 2000 realizations. While for a disappearing path the tracking response is not immediate, due to the memory effect of the algorithm, when a new path appears, the increasing rank is quickly updated. This is because the eigenvalue generated by a newly appearing path substitutes into the matrix $\mathbf{R}(\ell)$ an eigenvalue that corresponds to a noise component. For high SNR, the difference between these two eigenvalues is large enough to allow the algorithm to adapt itself to the new rank.

The comparison, in terms of MSE, between the SB-ML estimator (3) and the MB-ML (4) is shown in Fig. 2(b). The MB method is simulated both with the tracking algorithm for $\beta \in \{0.83, 0.86, 0.89\}$ (i.e., $L_w \in \{6, 7, 9\}$) and with the EVD implementation (as originally proposed in [4]) for $\beta = 0.89$. From the MSE comparison, we can conclude that the LORAF approach allows to reduce the computational cost of the EVD implementation, still preserving almost the same estimate accuracy. The proposed approach is also effective in tracking the changes of the number of paths. Fig. 2(a) and (b) shows how the forgetting factor β affects the convergence speed of the tracking algorithm. The smaller the value of β (or, equivalently, the effective time window L_w defined in Section V-A), the faster the convergence and the higher the MSE at convergence.

Fig. 2(c) shows the BER performance of the complete turbo receiver (as described in Section II). These results confirm that the performance of the adaptive version of the MB-ML method is very close to that of the EVD implementation. Furthermore, the turbo equalizer with the MB method is shown to provide remarkable gains for increasing number of iterations: this is particularly evident (already at third iteration) in the interval $[A, B] \cup [C, D]$, where the intersymbol interference (due to $r(\ell) = 2$ macro paths) is successfully cancelled by the iterative processing. The low BER values reached in these conditions are a combined result of the path diversity achieved by the equalizer and the time diversity of the code (coded bits are allocated over several frames having different fading variations). Notice also that the path disappearance occurs in the middle of frame 3, and each BER value in Fig. 2 denotes the error rate measured over *all* the 20 blocks included in the frame. The BER in frame 3 (average of the performances over the double- and the single-cluster channels) is thereby higher than in frame 2 and lower than in frames 4-5-6-7.

VII. Conclusion

The results presented in this letter show that the MB method, combined with soft feedback provided by iterative equalization, can be efficiently implemented by means of a subspace-tracking technique in scenarios with either fast or slowly changing channel features. This tracking technique allows a reduction of the computational complexity of the MB method with negligible performance loss. Though the method is here developed for a single-carrier SISO system, the extension to multiple carrier and/or multiple antenna systems is straightforward.

References

[1] R. Koetter, A. C. Singer, and M. Tuchler, "Turbo equalization," *IEEE Signal Process. Mag.*, vol. 21, no. 1, pp. 67–80, Jan. 2004.

[2] M. Kobayashi, J. Boutros, and G. Caire, "Successive interference cancellation with SISO decoding and EM channel estimation," *IEEE J. Sel. Areas Commun.*, vol. 19, no. 8, pp. 1450–1460, Aug. 2001.

[3] M. Nicoli, O. Simeone, and U. Spagnolini, "Multi-slot estimation of frequency-selective fast-varying channels," *IEEE Trans. Commun.*, vol. 51, no. 8, pp. 1337–1347, Aug. 2003.

[4] M. Nicoli and U. Spagnolini, "A subspace method for channel estimation in soft-iterative receivers," in *Proc. 13th Eur. Signal Processing Conf.*, Sep. 2005.

[5] P. Strobach, "Low-rank adaptive filters," *IEEE Trans. Signal Process.*, vol. 44, no. 12, pp. 2932–2947, Dec. 1996.

[6] M. Wax and T. Kailath, "Detection of signals by information theoretic criteria," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. ASSP-33, no. 2, pp. 387–392, Apr. 1985.

[7] T. Abe and T. Matsumoto, "Space-time turbo equalization in frequency selective MIMO channels," *IEEE Trans. Veh. Technol.*, vol. 52, no. 3, pp. 469–475, May 2003.

[8] L. R. Bahl, J. Cocke, F. Jelinek, and J. Ravin, "Optimal decoding of linear codes for minimizing symbol error rate," *IEEE Trans. Inf. Theory*, vol. IT-20, no. 2, pp. 284–287, Mar. 1974.

[9] B. Yang, "Projection approximation subspace tracking," *IEEE Trans. Signal Process.*, vol. 43, no. 1, pp. 95–107, Jan. 1995.