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Cooperative fading regions in wireless ad-hoc networks

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Telecommunication Research Center - FTW, Vienna, 6/3/2009



Motivations... how to study the impact of fading impairments on distributed transmissions?

Cooperative Diversity in Wireless Network Efficient Protocols and Outage Behavior

J. Nicholas Laneman, *Member, IEEE*, David N. C. Tse, *Member, IEEE*,
and Gregory W. Wornell, *Senior Member, IEEE*

For **Rayleigh fading**, i.e. $|a_{i,j}|^2$ independent and exponentially distributed with parameters $\sigma_{i,j}^{-2}$, analytic calculation of the outage probability becomes involved, but we can approximate its high SNR behavior as

$$P_{AF}^{out}(SNR, R) := \Pr[I_{AF} < R]$$

IEEE TRANSACTIONS ON COMMUNICATIONS, VOL. 51, NO. 11, NOVEMBER 2003

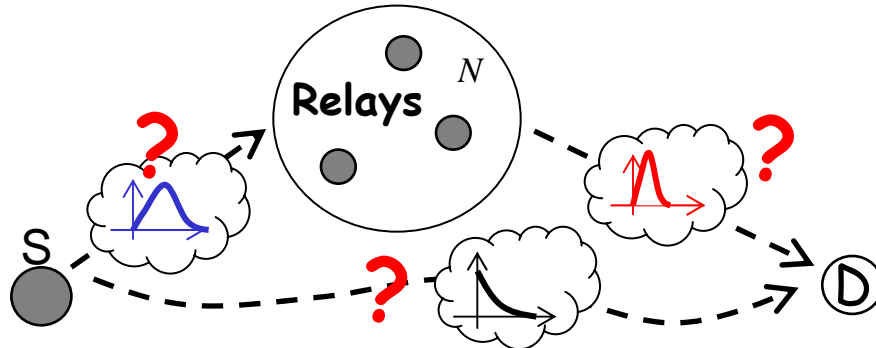
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User Cooperation Diversity—Part I: System Description

Andrew Sendonaris, *Member, IEEE*, Elza Erkip, *Member, IEEE*, and Behnaam Aazhang, *Fellow, IEEE*

the **fading coefficients K_{ij} are zero-mean complex Gaussian** random variables with variance ξ_{ij}^2 (which corresponds to Rayleigh fading). We also assume that the BS can track the

Rayleigh fading... is experienced when the dimension of the cluster of scatterers surrounding the transmitter or the receiver is much larger than the signal wavelength...



$$SNR = \frac{\rho}{N_0} |h|^2 \sim f_{|h|^2}(w)$$

Short-range communication (for cooperative tx)... Is Rayleigh fading model still effective?

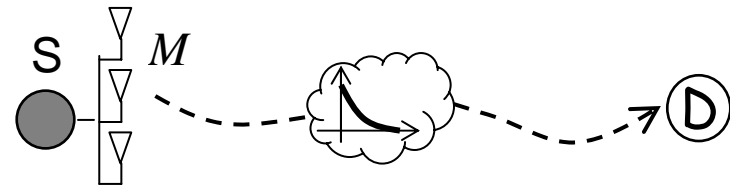
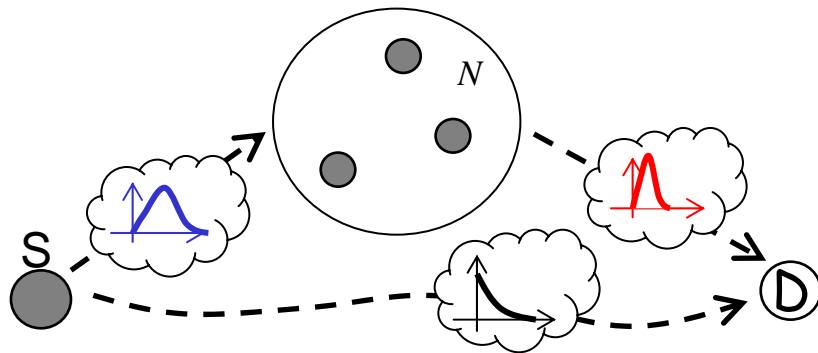
How does the propagation environment impact on the performances of (distributed) wireless transmissions?



Outline

Part 1 Outage probability analysis for arbitrary fading channels

- Moment Generating Function based approach
- Outage probability analysis for arbitrary fading channels



S. Savazzi, U. Spagnolini "Cooperative fading regions for decode and forward relaying" IEEE Trans. on Info. Theory, vol. 54, no.11, November 2008

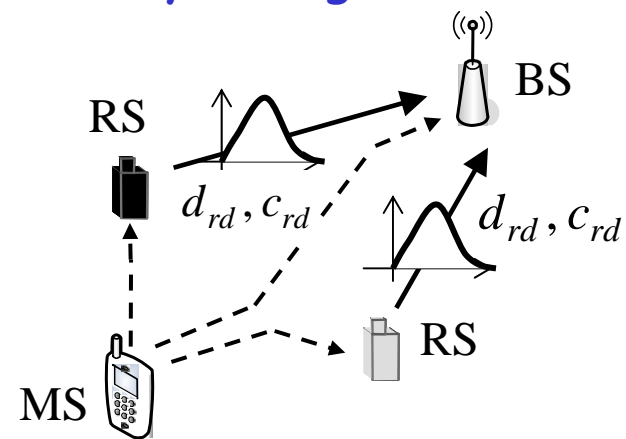
Part 2 Quantifying the price of cooperation in arbitrary fading

- Cooperative fading regions

Part 3 Cooperative Networks design

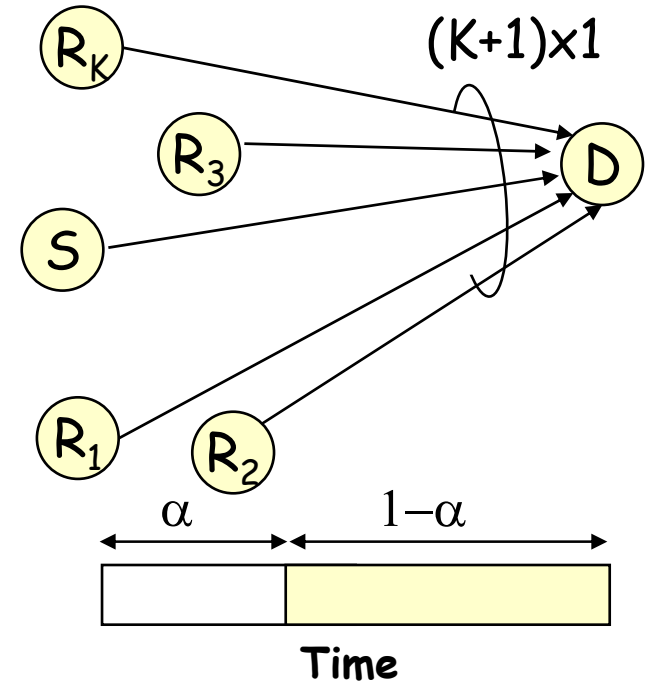
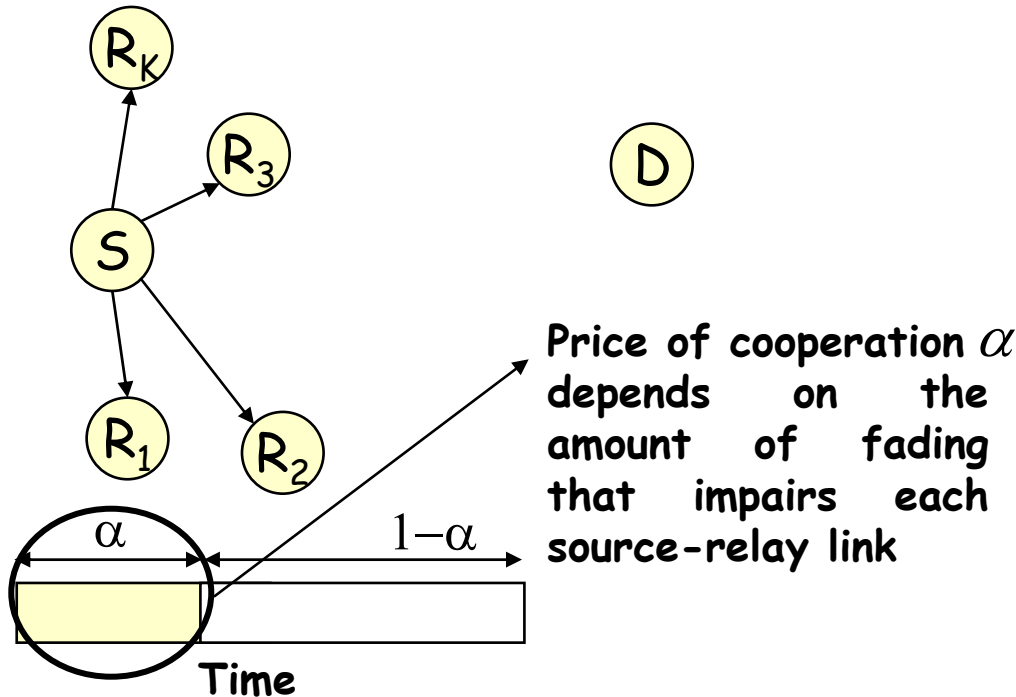
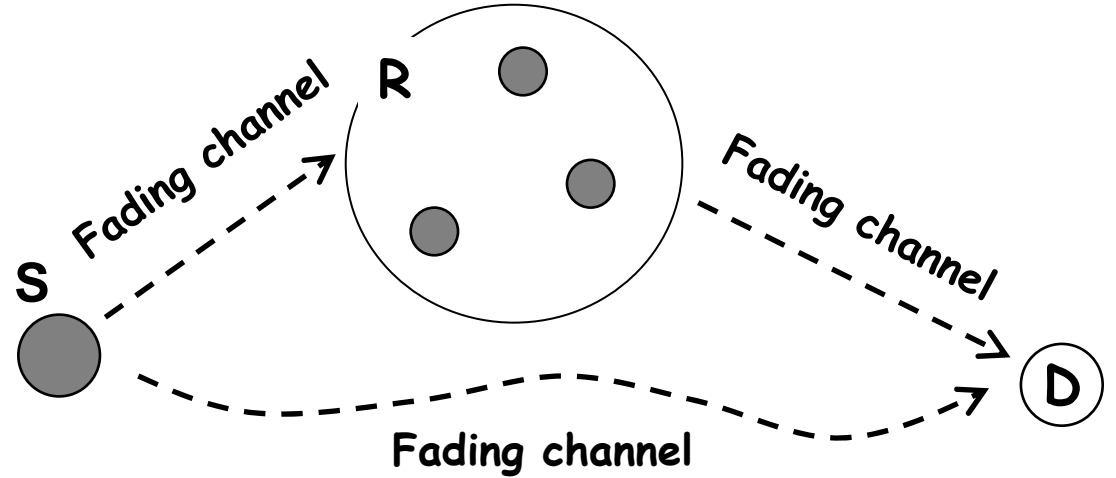
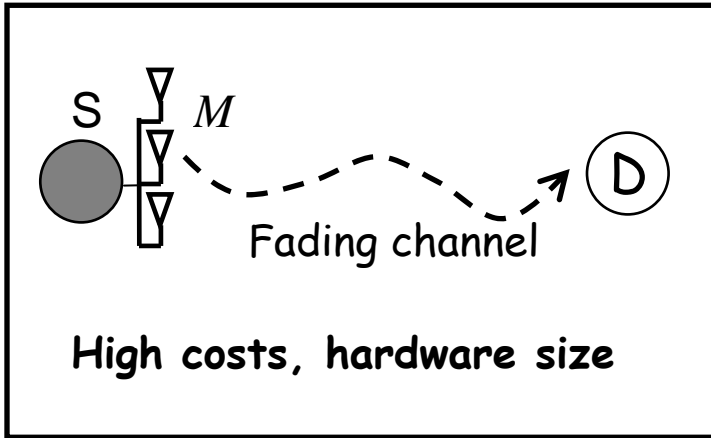
- Relay deployment design
- Virtual MIMO vs MIMO

S. Savazzi and U. Spagnolini, "Design criteria of two-hop based wireless networks with non-regenerative relays in arbitrary fading channels," to appear on IEEE Trans. On Communication, expected May 2009





Background on cooperative transmissions





Background on cooperative transmissions

Three key-research topics...

1) Distributed Synchronization

[Strogatz, Simeone, Goddard, Scaglione etc...]

2) PHY layer transmission schemes:

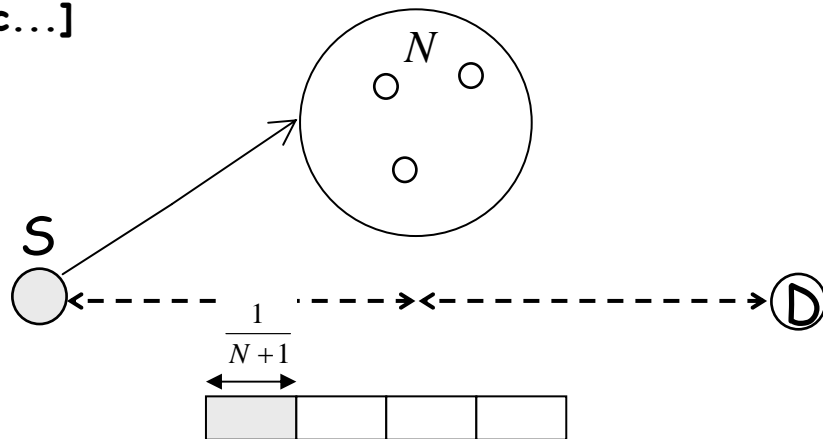
Relaying processing (DF-AF), Repetition based (space diversity..), Distributed ST coding, Coded cooperation

[Laneman, Nosratinia, Goldsmith, Poor etc...]

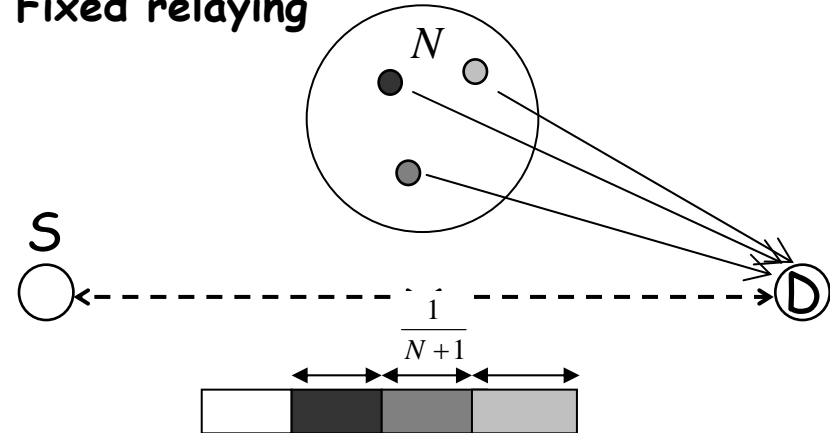
3) Medium Access Control:

fixed/adaptive relaying, relay grouping and partner selection, randomization

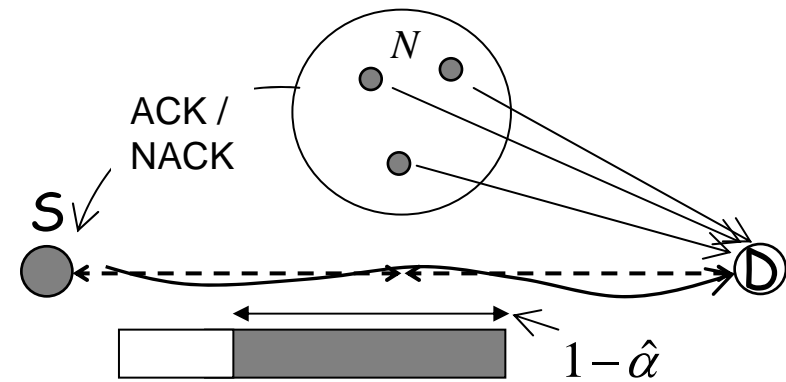
[Nosratinia, Scaglione, Valenti, Goldsmith, Kumar etc...]



Repetition based (space diversity..),
Fixed relaying

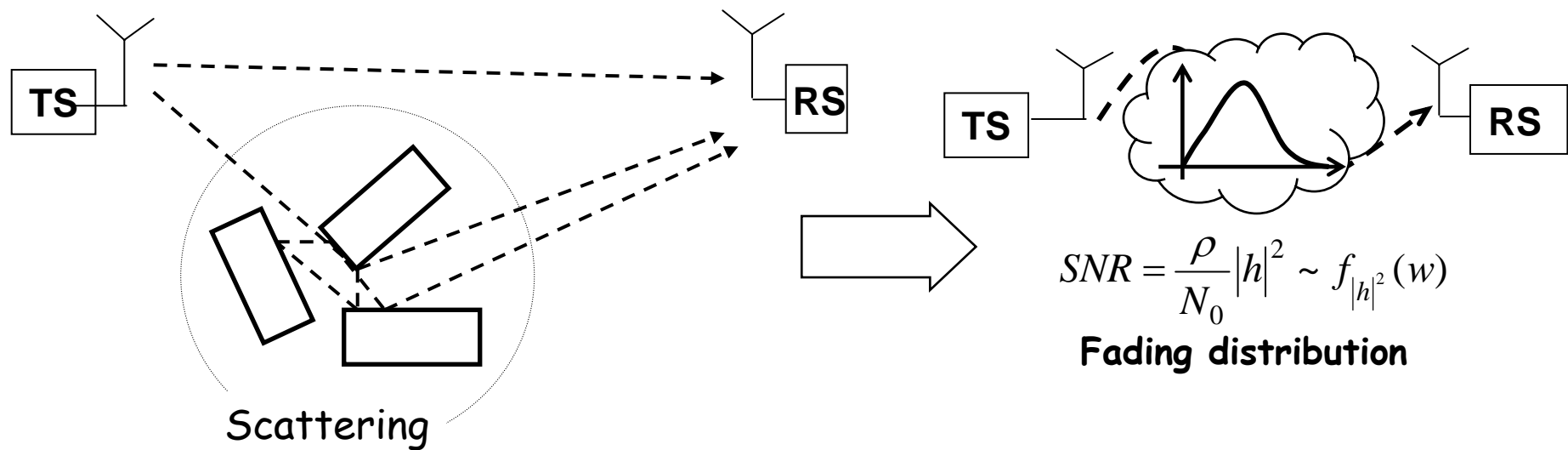


Distributed ST coding,
Adaptive relaying



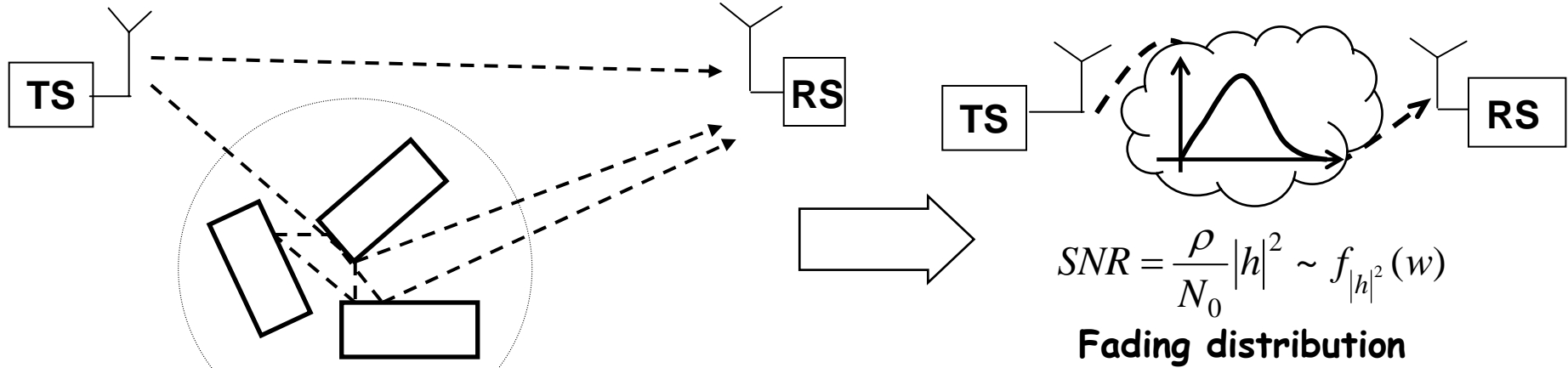


Part 1 - Outage analysis in arbitrary fading channels



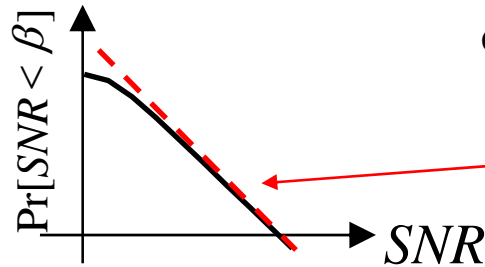


Part 1 - Outage analysis in arbitrary fading channels



Scattering

Outage probability analysis



Outage probability ruled by diversity and coding gain [Tse]:

$$\Pr[SNR < \beta] \cong \left(\frac{\beta N_0}{c \rho} \right)^d$$

Theorem

Diversity d is such that

$$d = \lim_{s \rightarrow \infty} \frac{-\log F_{|h|^2}(s)}{\log s}$$

MGF $F_{|h|^2}(s) = \int f_{|h|^2}(w) \exp(-sw) dw$

Coding gain c

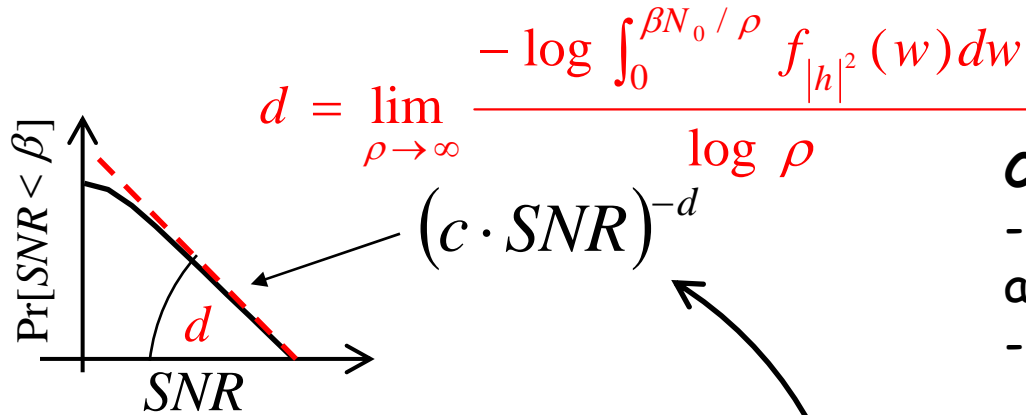
$$c = \left(\Gamma(d+1) / \lim_{s \rightarrow \infty} s^d F_{|h|^2}(s) \right)^{1/d}$$

$$\Gamma(x) = \int_0^{\infty} y^{x-1} \exp(-y) dy$$



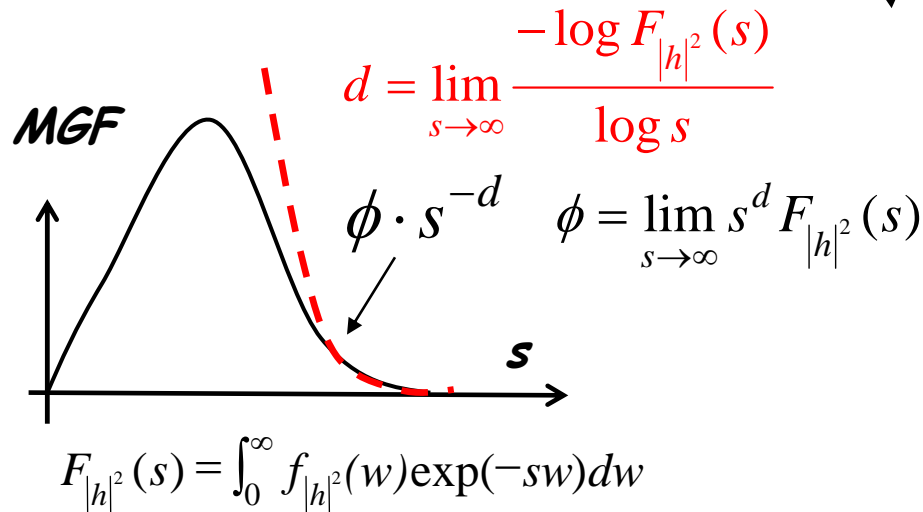
Diversity in arbitrary fading channels

Moment Generating Function based approach



Conventional approach

- requires integration over arbitrary small interval
- integration complexity is high



MGF based approach

- MGF is available for a variety of pdfs
- used for evaluating Pairwise Error Probability
- integration complexity is low (Laplace Transform)



Diversity in arbitrary fading

Integral form for the random fading power density function

$$f_{|h|^2}(w) = \int_0^{\infty} \Gamma(t+1)^{-1} \cdot D^t[f_{|h|^2}(0)] w^t dt$$

$D^t[f_{|h|^2}(w)]$ t -th order fractional derivative of random fading power pdf in $w=0$

$$f_{|h|^2}(w) = \underbrace{\int_0^{t^*} \Gamma(t+1)^{-1} \cdot D^t[f_{|h|^2}(0)] w^t dt}_{=0} + \int_{t^*}^{\infty} \Gamma(t+1)^{-1} \cdot D^t[f_{|h|^2}(0)] w^t dt = \int_{t^*}^{\infty} \Gamma(t+1)^{-1} \cdot D^t[f_{|h|^2}(0)] w^t dt$$

t^* is the order of the first non-zero fractional derivative of the random fading power pdf in zero such that

$$\int_0^{t^*} \Gamma(t+1)^{-1} \cdot D^t[f_{|h|^2}(0)] w^t dt = 0$$

The fading power density can be approximated for large SNR as

$$f_{|h|^2}(w) = \Gamma(t^*+1)^{-1} \cdot D^{t^*}[f_{|h|^2}(0)] w^{t^*} + o[w^{t^*}]$$

Diversity becomes

$$d = \lim_{\rho \rightarrow \infty} \frac{-\log \int_0^{\beta N_0 / \rho} f_{|h|^2}(w) dw}{\log \rho} = t^* + 1$$



Diversity from the MGF

Necessary and sufficient condition for t^* is

$$\lim_{s \rightarrow \infty} s^{t^*+1} F_{|h|^2}(s) > 0 \text{ and finite}$$

Condition is sufficient as

$$\forall \varepsilon > 0, D^{t^*-\varepsilon}[f_{|h|^2}(0)] = \lim_{s \rightarrow \infty} s^{t^*-\varepsilon+1} F_{|h|^2}(s) = 0$$

Condition is necessary from initial value theorem

Since

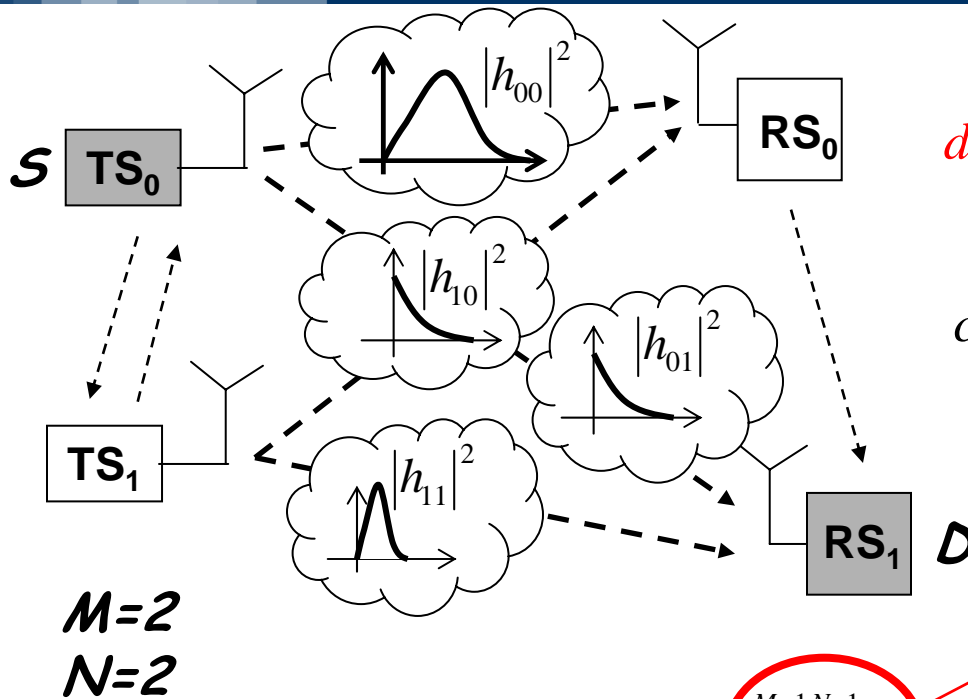
$$\lim_{s \rightarrow \infty} s^{t^*+1} F_{|h|^2}(s) > 0 \text{ and finite} \Leftrightarrow \lim_{s \rightarrow \infty} \frac{\log F_{|h|^2}(s)}{\log s^{t^*+1}} > 0 \text{ and finite}$$

then diversity is

$$d = t^* + 1 = \lim_{s \rightarrow \infty} \frac{-\log F_{|h|^2}(s)}{\log s}$$



Example - Outage performances of Distributed-MIMO in arbitrary fading channels



$$d_{ij} = \lim_{s \rightarrow \infty} \frac{-\log F_{|h_{ij}|^2}(s)}{\log s}$$

$$c_{ij} = \left(\Gamma(d_{ij} + 1) / \lim_{s \rightarrow \infty} s^{d_{ij}} F_{|h_{ij}|^2}(s) \right)^{1/d_{ij}}$$

Cooperative diversity provided by distributed MIMO transmission

$$\Pr[SNR < \beta] \cong \lambda \cdot \left(\frac{\beta N_0}{\rho} \right)^{\sum_{i=0}^{M-1} \sum_{j=0}^{N-1} d_{ij}}$$

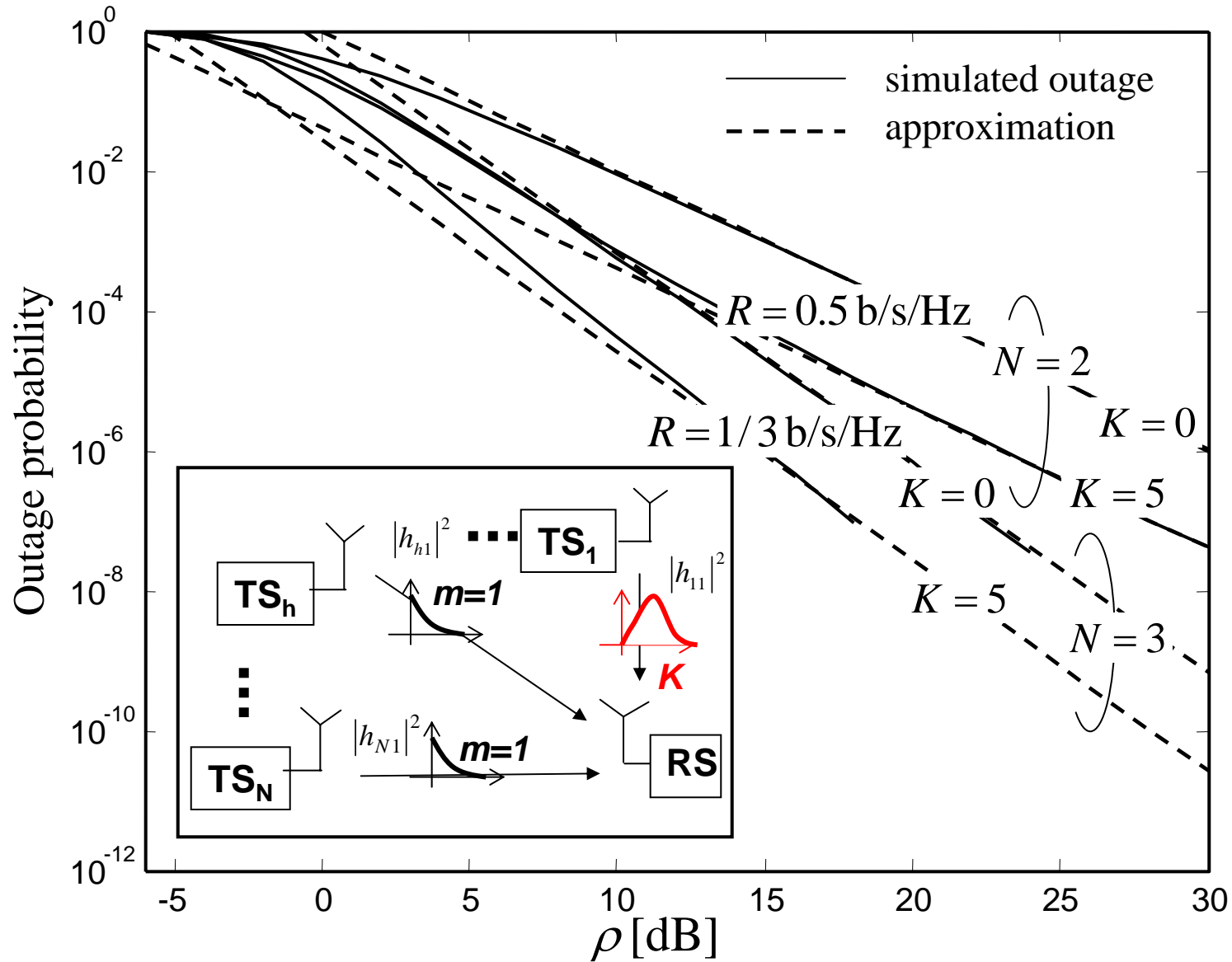
$$SNR = \frac{\rho}{N_0} \sum_{j=1}^{N-1} \sum_{i=1}^{M-1} |h_{ij}|^2$$

$$\lambda = \frac{\prod_{i=0}^{M-1} \prod_{j=0}^{N-1} \Gamma(d_{ij} + 1) / c_{ij}^{d_{ij}}}{\Gamma\left(1 + \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} d_{ij}\right)}$$

Diversity does not simply scales with the product of the number of tx antennas and rx antennas but is the sum of all diversities provided by all the links for distributed TX..

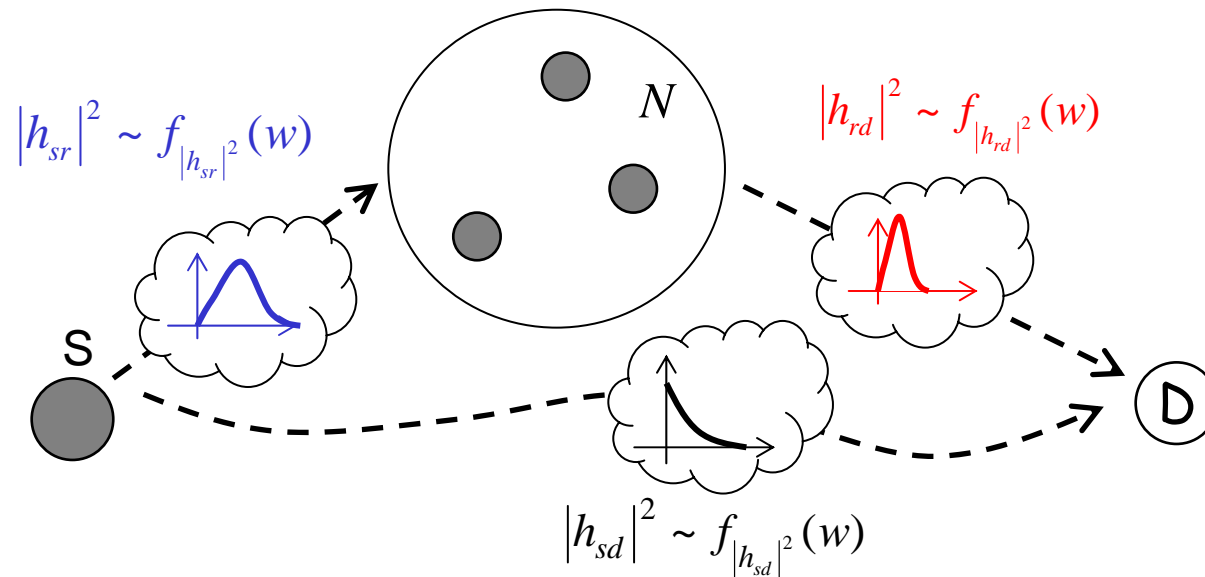


Example - Distributed ST coding in mixed Rice/Rayleigh fading



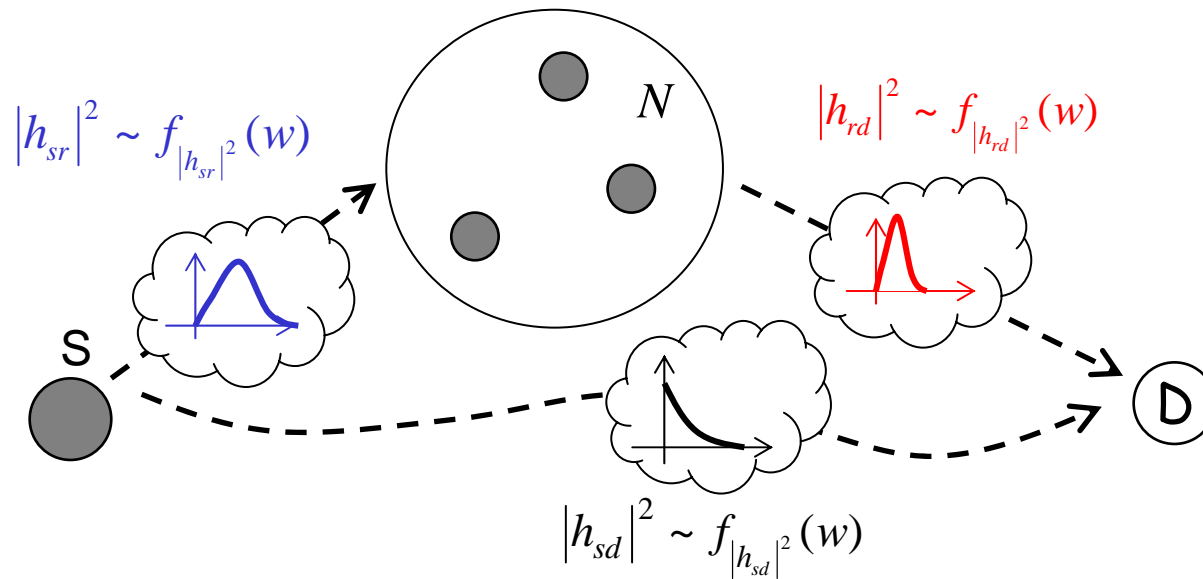


Part 2 - Cooperative fading regions





Part 2 - Cooperative fading regions: Problem definition



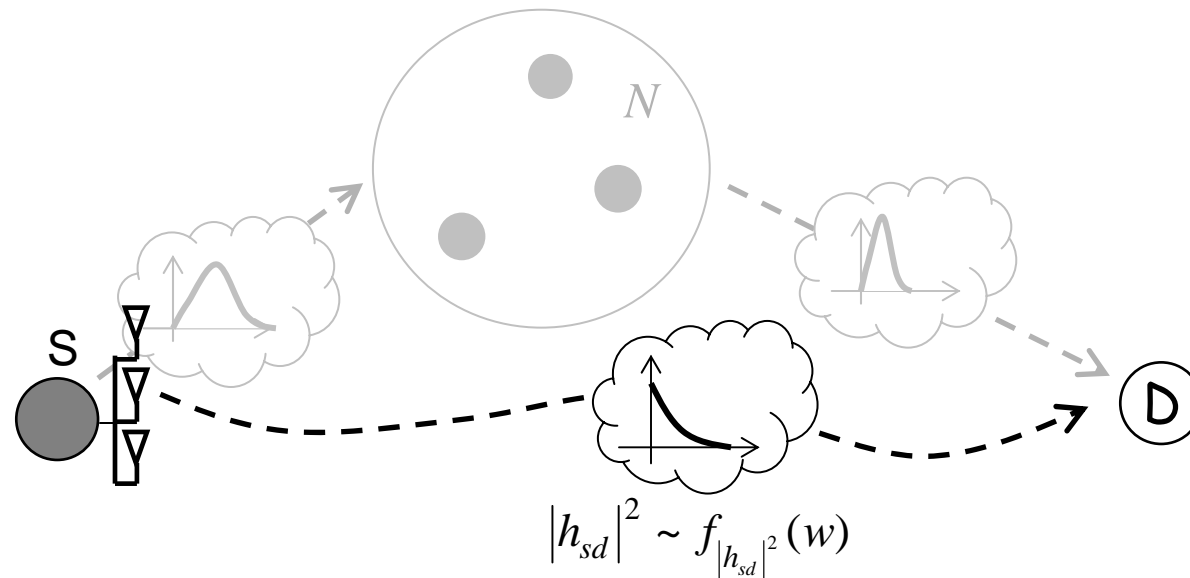
...what are the conditions on the arbitrary fading distributions

$$|h_{rd}|^2 \sim f_{|h_{rd}|^2}(w) \quad |h_{sr}|^2 \sim f_{|h_{sr}|^2}(w) \quad |h_{sd}|^2 \sim f_{|h_{sd}|^2}(w)$$

that make the use of the (overheads of the) cooperative TX to be of any advantage



Cooperative fading regions: Problem definition



...what are the conditions on the arbitrary fading distributions

$$|h_{rd}|^2 \sim f_{|h_{rd}|^2}(w) \quad |h_{sr}|^2 \sim f_{|h_{sr}|^2}(w) \quad |h_{sd}|^2 \sim f_{|h_{sd}|^2}(w)$$

that make the use of the (overheads of the) cooperative TX to be of any advantage compared to multiantenna transmission



Cooperative fading regions and outage

Define the diversity and the coding gain provided by each cooperative channel:

$$d_{sr} = \lim_{s \rightarrow \infty} \log F_{|h_{sr}|^2}(s) / \log s$$

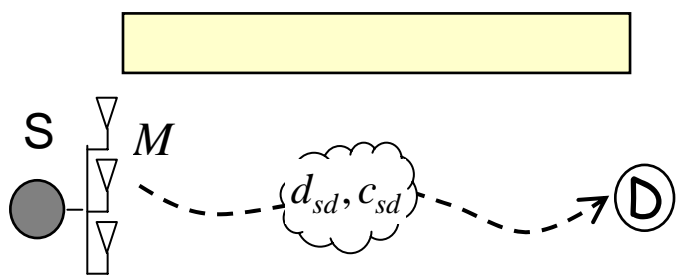
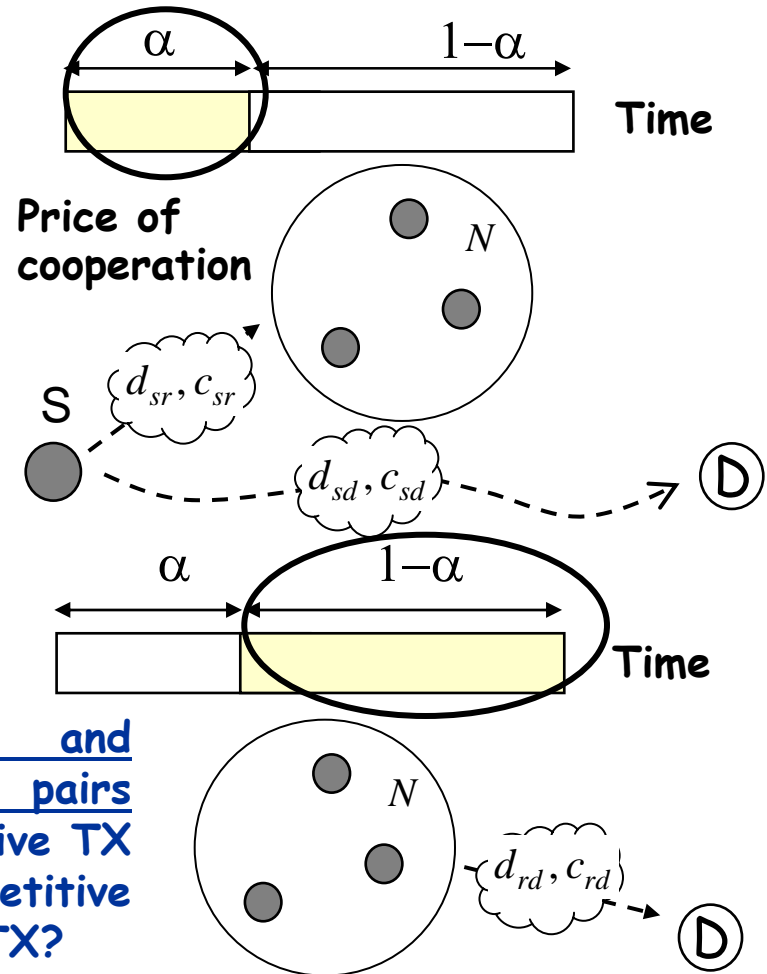
$$d_{rd} = \lim_{s \rightarrow \infty} \log F_{|h_{rd}|^2}(s) / \log s$$

$$d_{sd} = \lim_{s \rightarrow \infty} \log F_{|h_{sd}|^2}(s) / \log s$$

$$c_{sr} = \left(\Gamma(d_{sr} + 1) / \lim_{s \rightarrow \infty} s^{d_{sr}} F_{|h_{sr}|^2}(s) \right)^{1/d_{sr}}$$

$$c_{rd} = \left(\Gamma(d_{rd} + 1) / \lim_{s \rightarrow \infty} s^{d_{rd}} F_{|h_{rd}|^2}(s) \right)^{1/d_{rd}}$$

$$c_{sd} = \left(\Gamma(d_{sd} + 1) / \lim_{s \rightarrow \infty} s^{d_{sd}} F_{|h_{sd}|^2}(s) \right)^{1/d_{sd}}$$



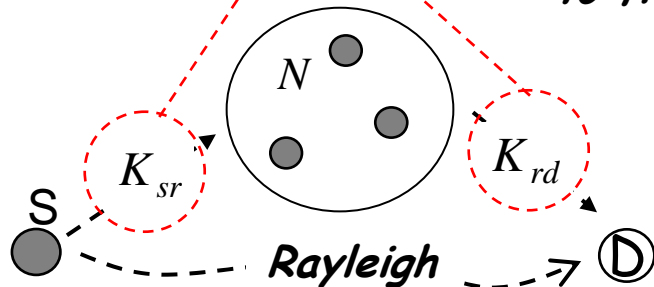
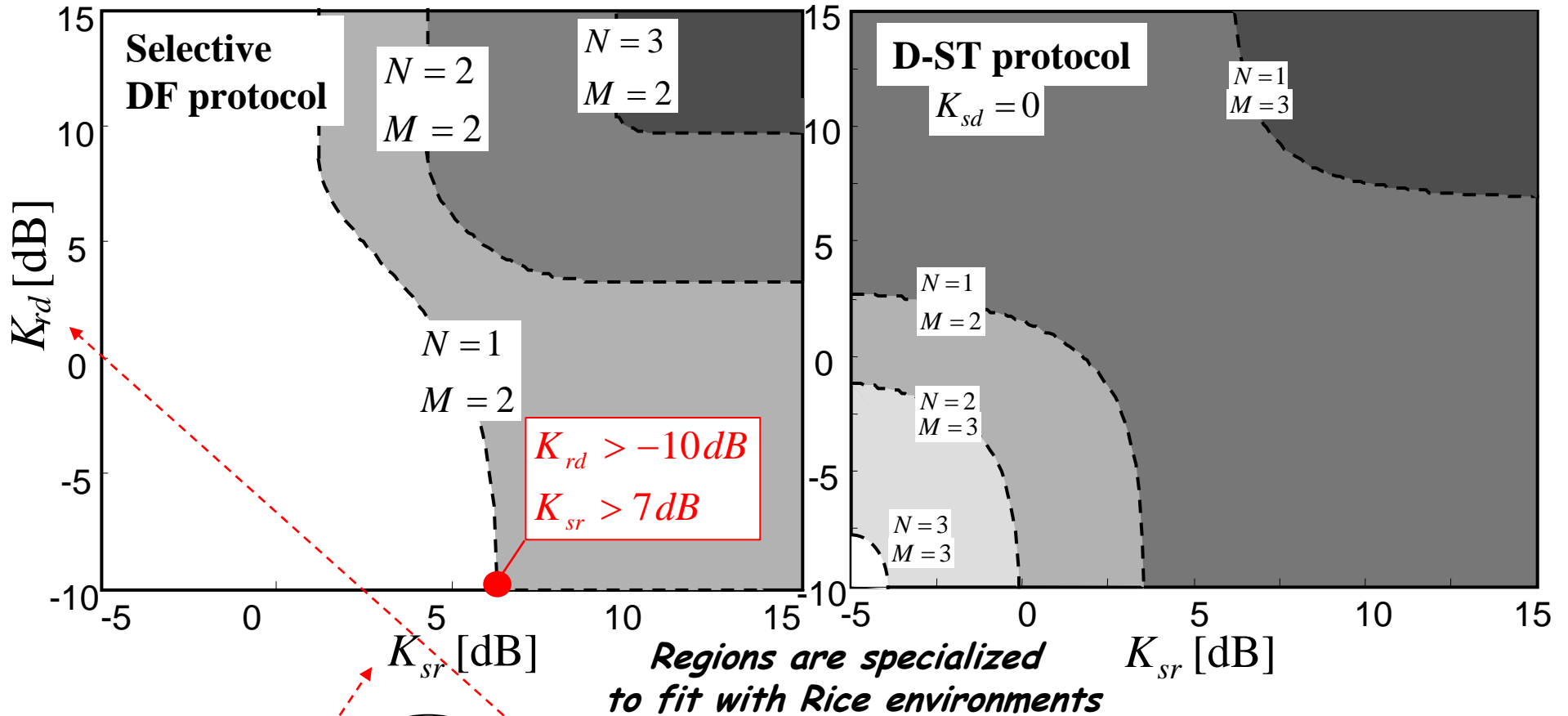
Non-cooperative (direct tx)

...diversity and coding gain pairs for cooperative TX to be competitive with direct TX?

Cooperative tx
(e.g., protocols DF, AF D-ST...)

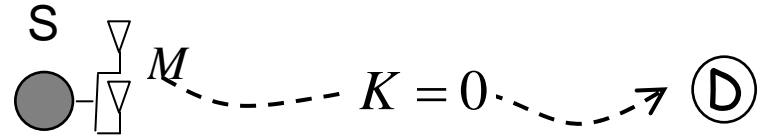
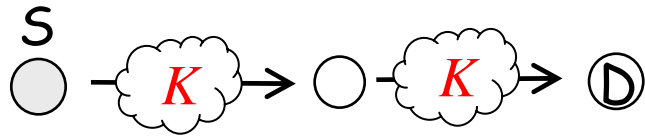


Example - Cooperative fading regions for DF and DST in Rice fading

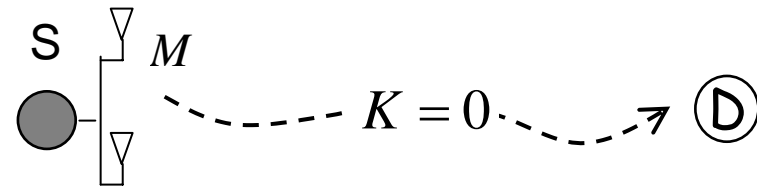
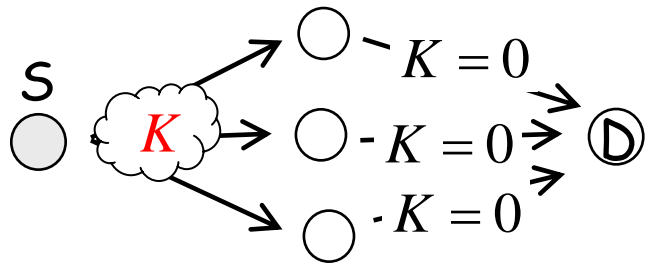




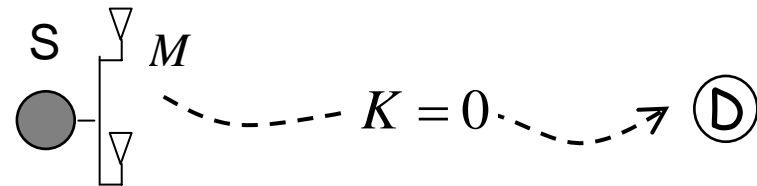
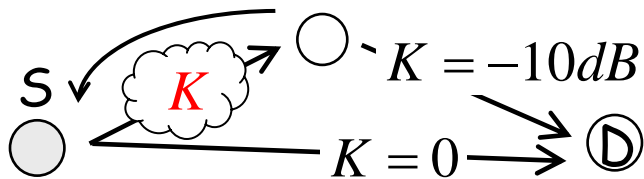
Summary of main results (from extensive simulations...)



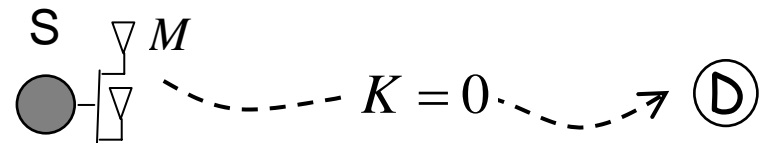
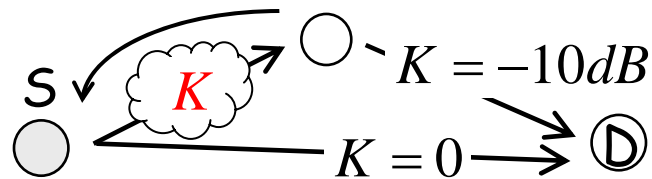
only for $K > 9\text{dB}$...



for $K > 6-7\text{dB}$ (fixed decode and forward)



for $K > 6-7\text{dB}$ (selective decode and forward)

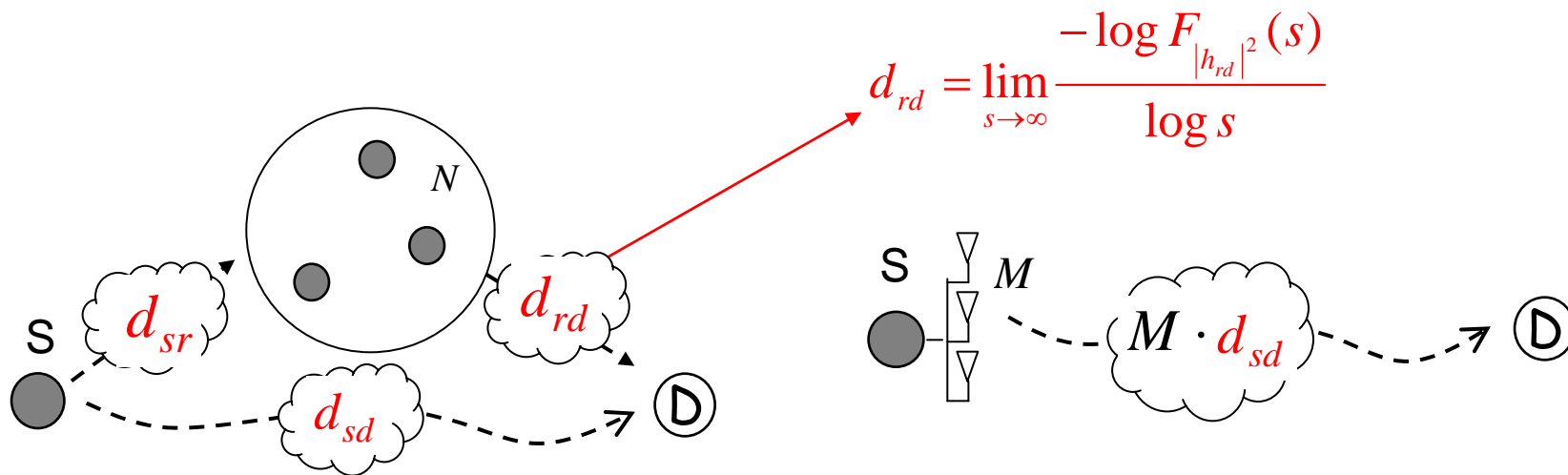


for $K > 4\text{dB}$ (distributed ST coding)



(Asymptotic) Cooperative fading regions

For asymptotically high SNR the performances of cooperative systems are ruled by the diversity only (or cooperative diversity)

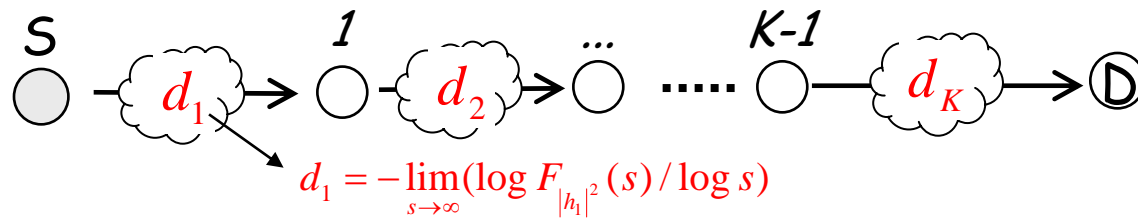


...what are the required "inherent" diversities provided by each cooperative link that make cooperative diversity to be larger than diversity provided by multiple antenna TX?

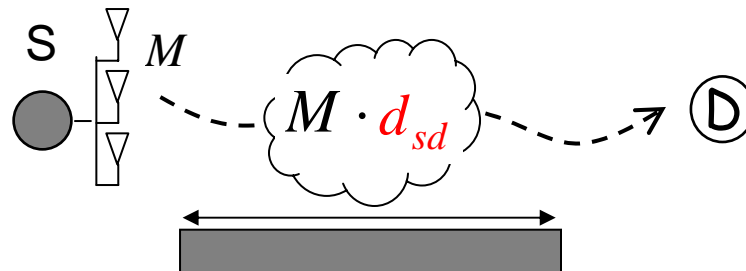


Example - Multi-hopping vs. direct multiple antenna transmission

Multi-hopping or Single path relaying ($K+1$ single antenna terminals)



Multiple antenna transmission (M-antenna terminal)



Asymptotic Cooperative Fading Regions

$$\mathcal{R}_{MH}^{\infty} = \{ \min[d_1, \dots, d_K] > M d_{sd} \}$$

Performance is ruled by the channel that provides the lowest diversity

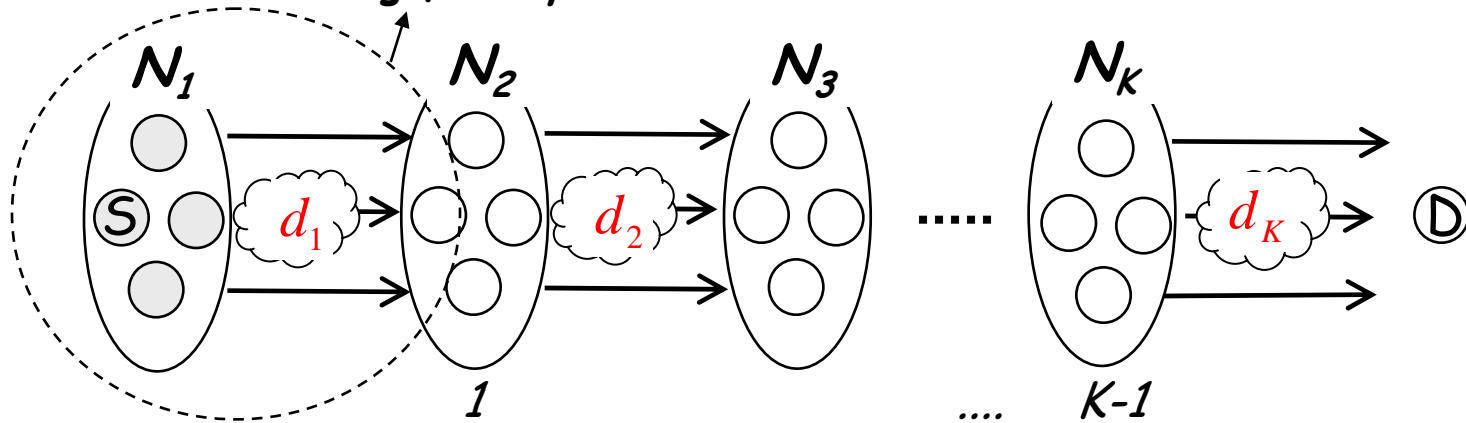
(E.g. for $M=1$, Multi-hopping vs direct single antenna transmission) $\mathcal{R}_{MH}^{\infty} = \{ \min[d_1, \dots, d_K] > d_{sd} \}$



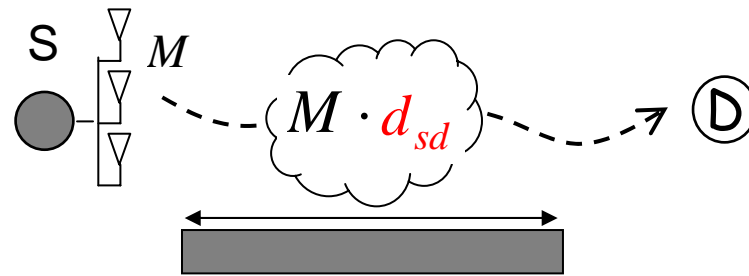
Example - Multi-hop cooperative transmission vs. direct

Multipath relaying ($K+1$ clusters)

e.g., DST protocol



Multiple antenna transmission (M-antenna terminal)

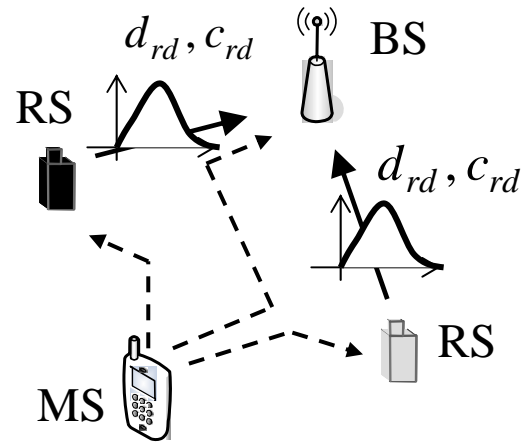


Asymptotic Cooperative Fading Regions

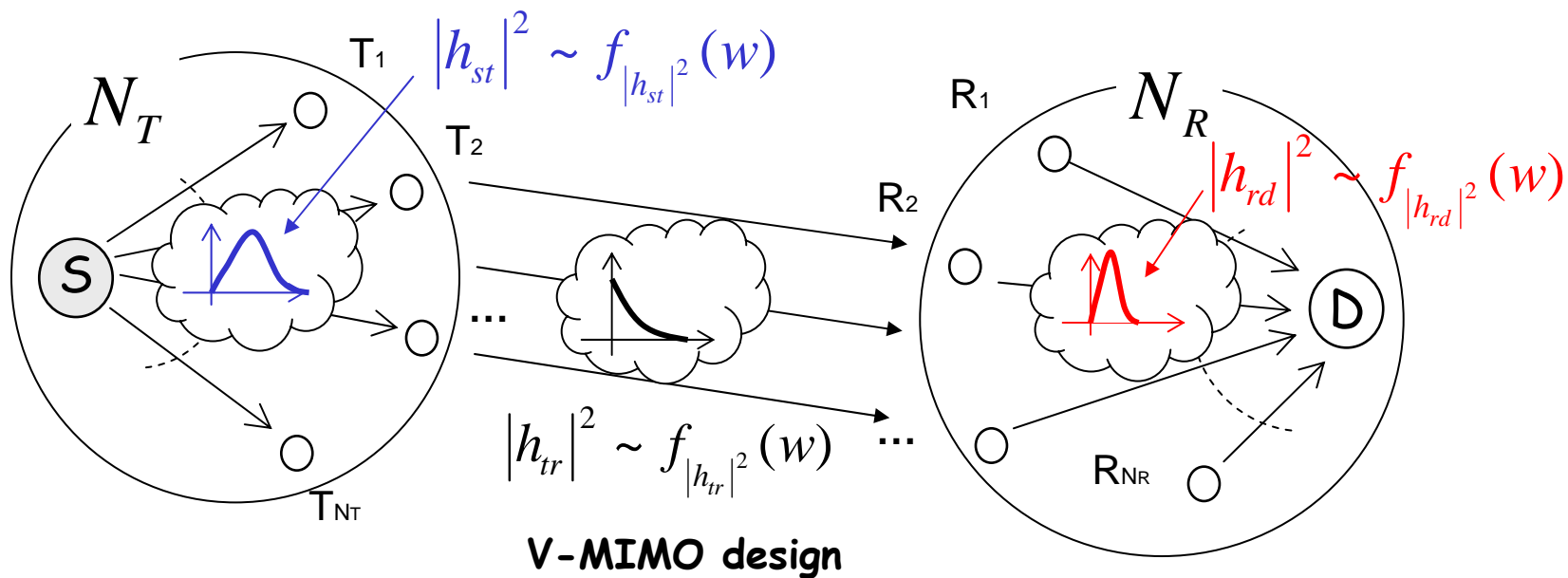
$$\mathcal{R}_{MH}^{\infty} = \{ \min[N_1 d_1, \dots, N_K d_K] > M d_{sd} \}$$



Part 3 - Cooperative network design



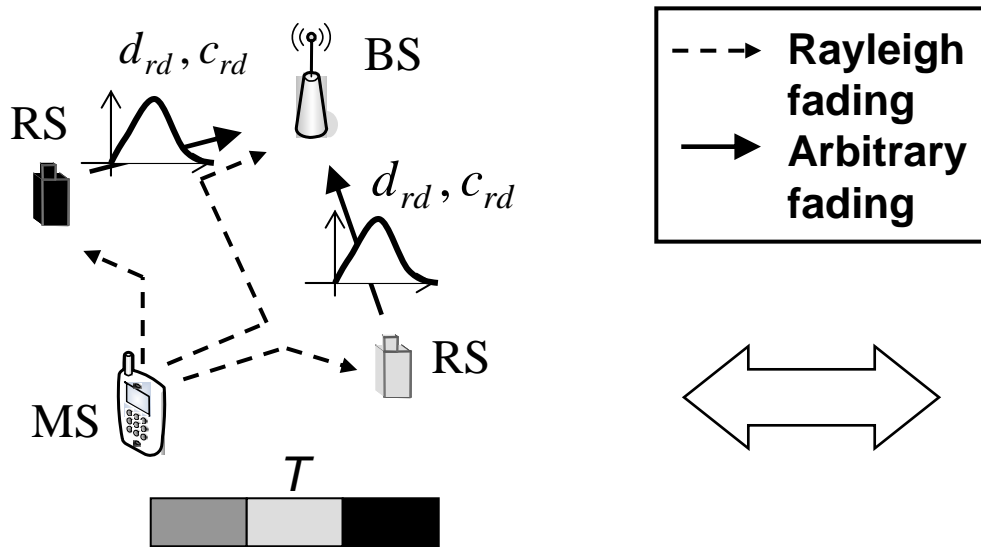
Deployment design of fixed relay networks



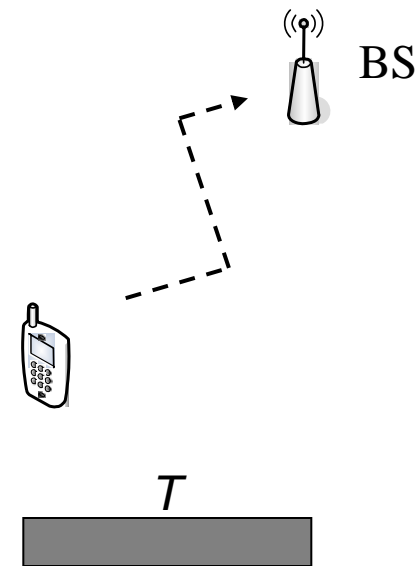


Deployment design of fixed non-regenerative relays

N=2 FIXED single antenna Relay Stations (RS) Mobile Station (MS) is single antenna



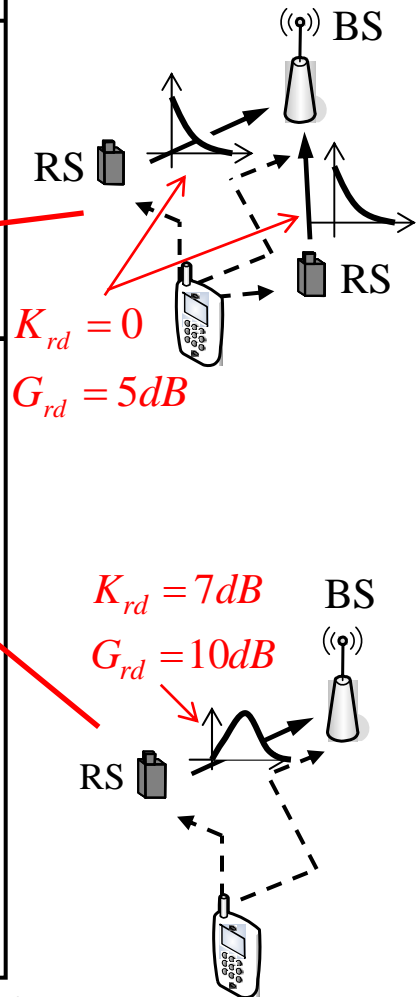
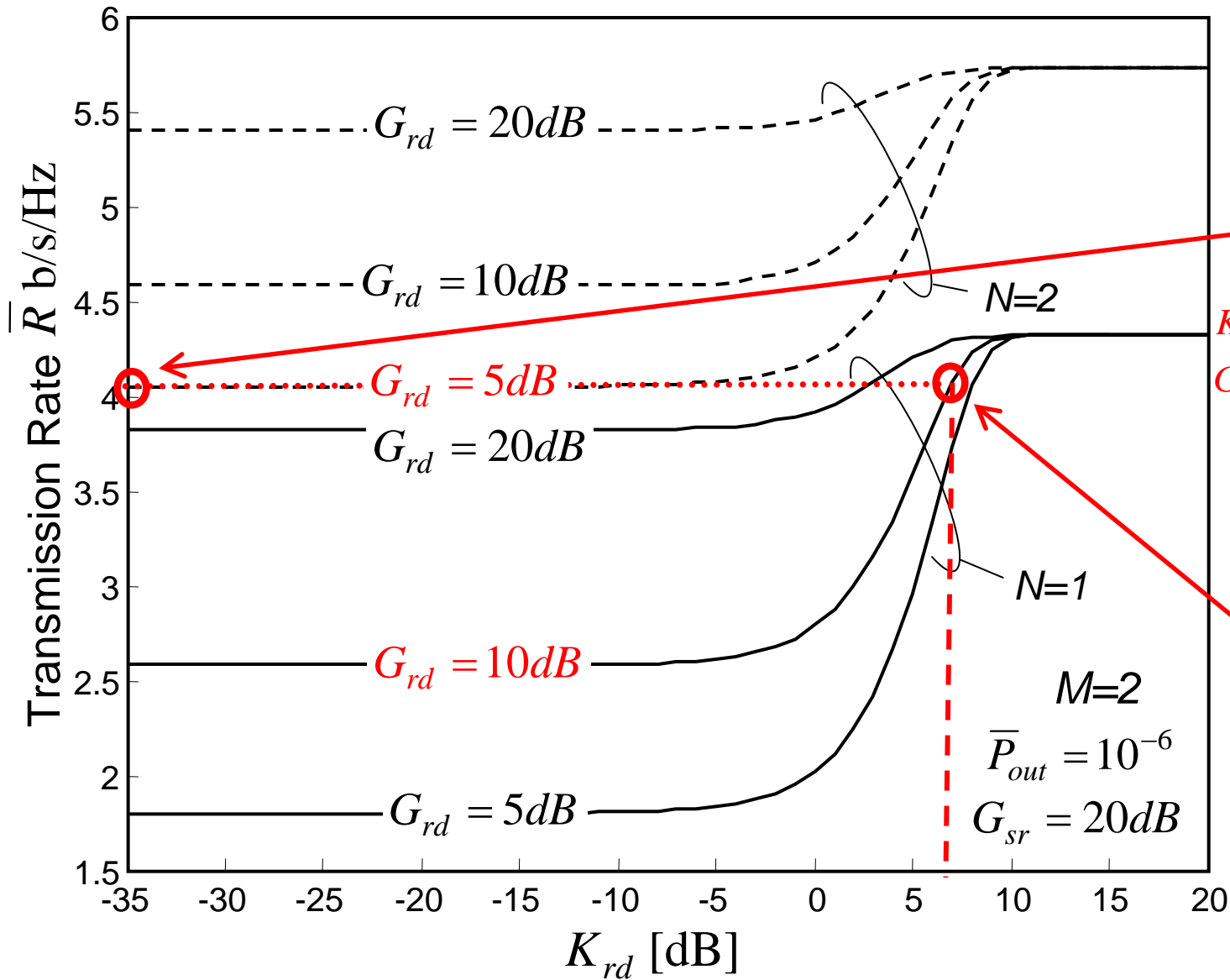
M=2 antennas at the Mobile Station (MS)



...what are the requirements on the fading distribution for the fixed RS-to-MS link that make the use of fixed AF relay stations to be of any advantage compared to multiantenna transmission?



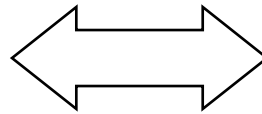
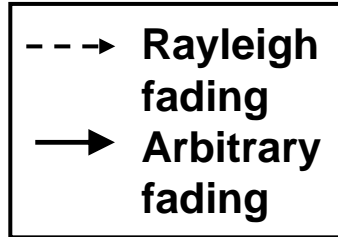
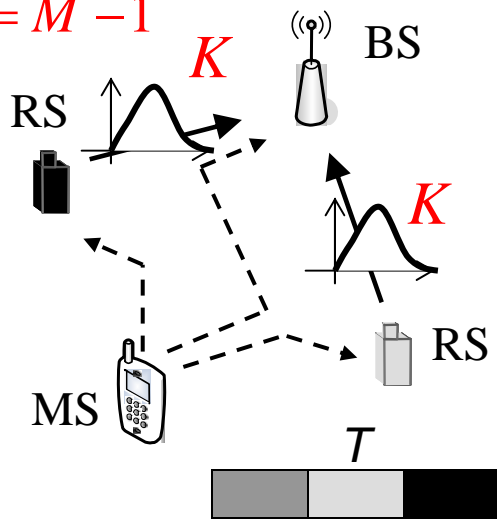
Example - Deployment design of fixed AF relay networks in Rice fading



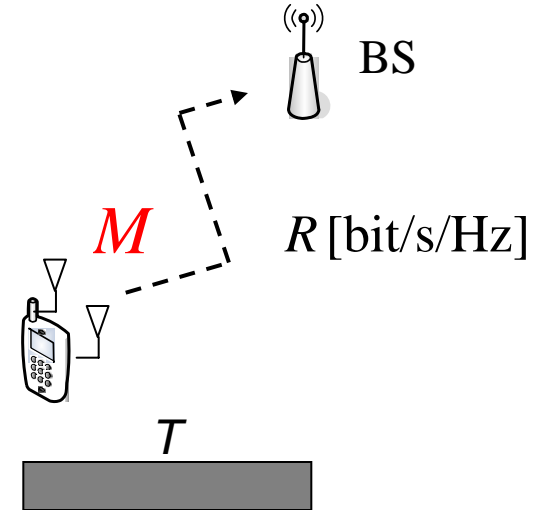


Some design rules

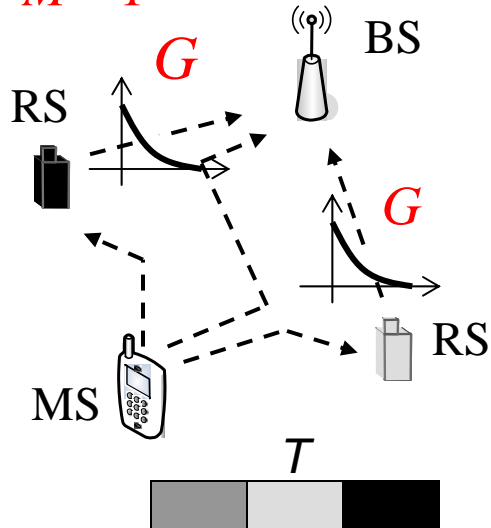
$$N = M - 1$$



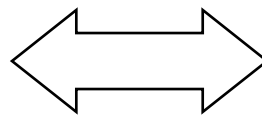
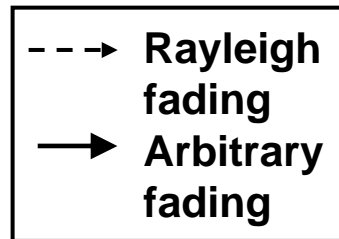
$$K \approx M \times R$$



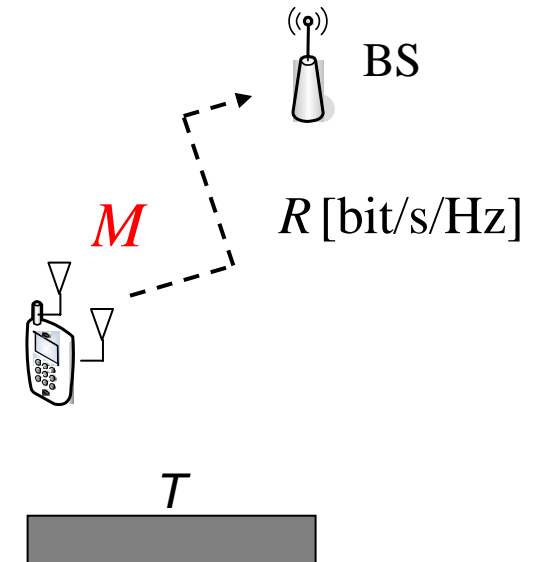
$$N = M - 1$$



G: average fading power gain compared to direct MS-to-BS link

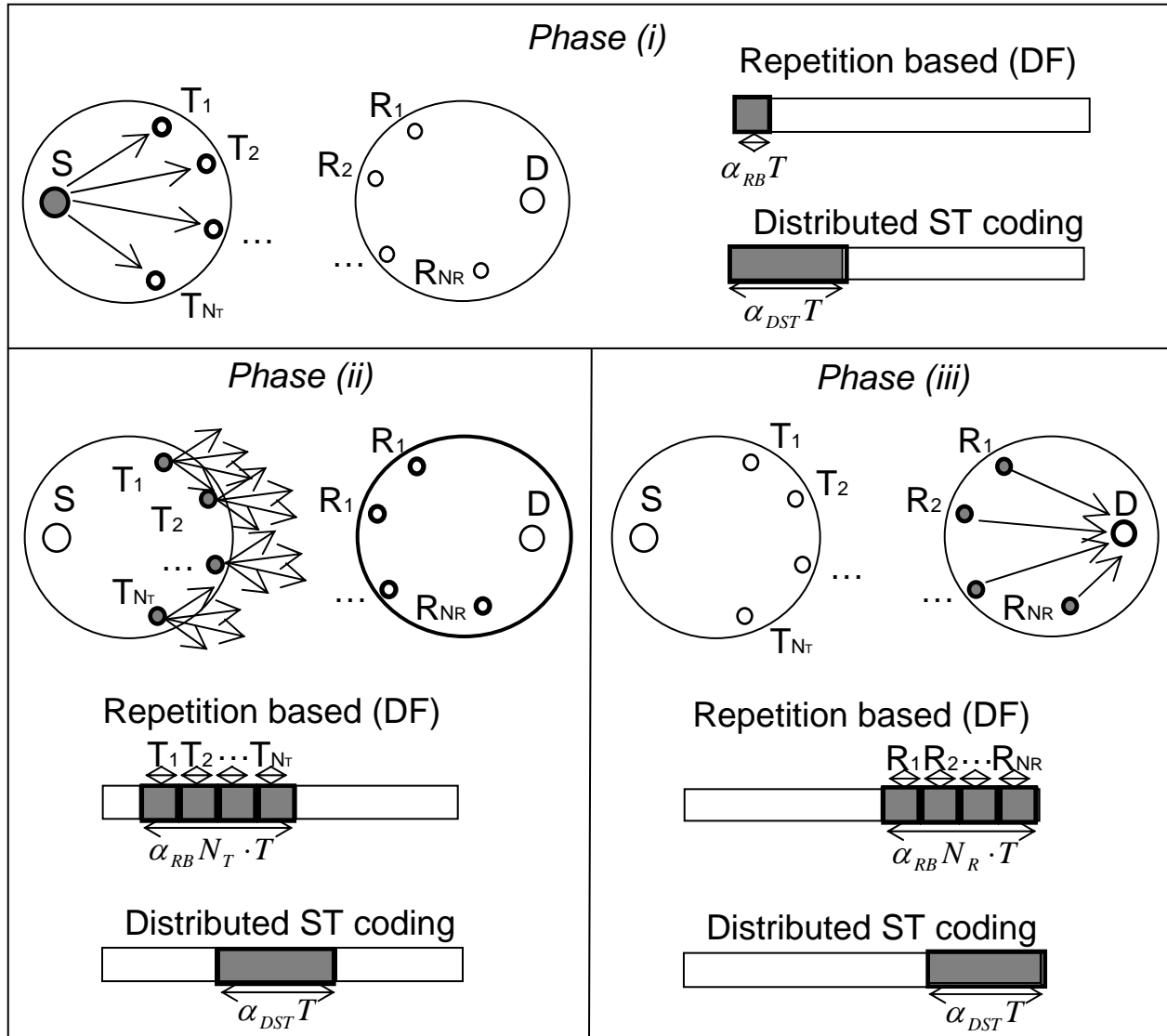


$$G \approx \exp(M \times R)$$





V-MIMO transmission for decode and forward relaying



- Idle terminal
- Receiving terminal
- Transmitting terminal

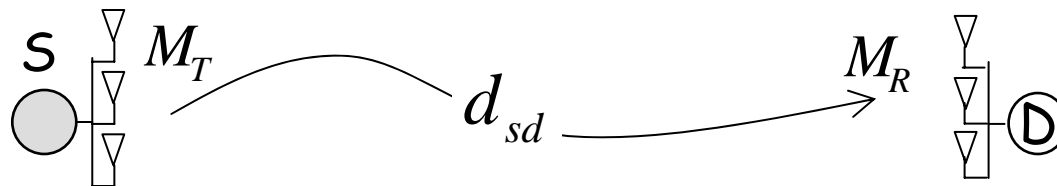
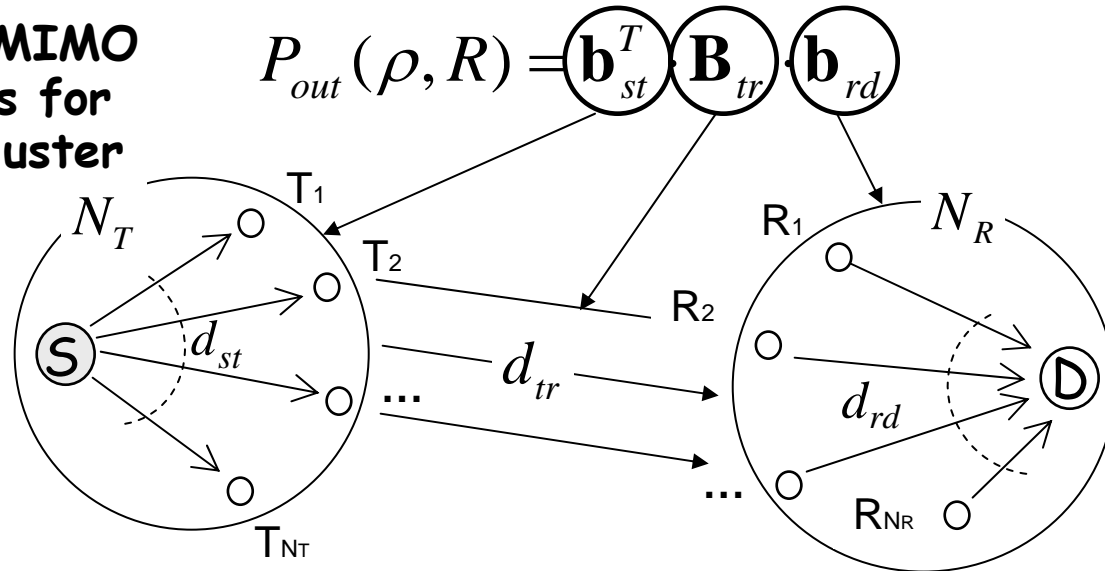


Asymptotic cooperative fading regions for V-MIMO

Outage probability of V-MIMO collects the outage events for intra-cluster and inter-cluster communications.

Diversity provided by V-MIMO:

$$\lim_{\rho \rightarrow \infty} \frac{-\log P_{out}(\rho, R)}{\log \rho} = \min[N_T d_{st}, N_T N_R d_{tr}, N_R d_{rd}]$$

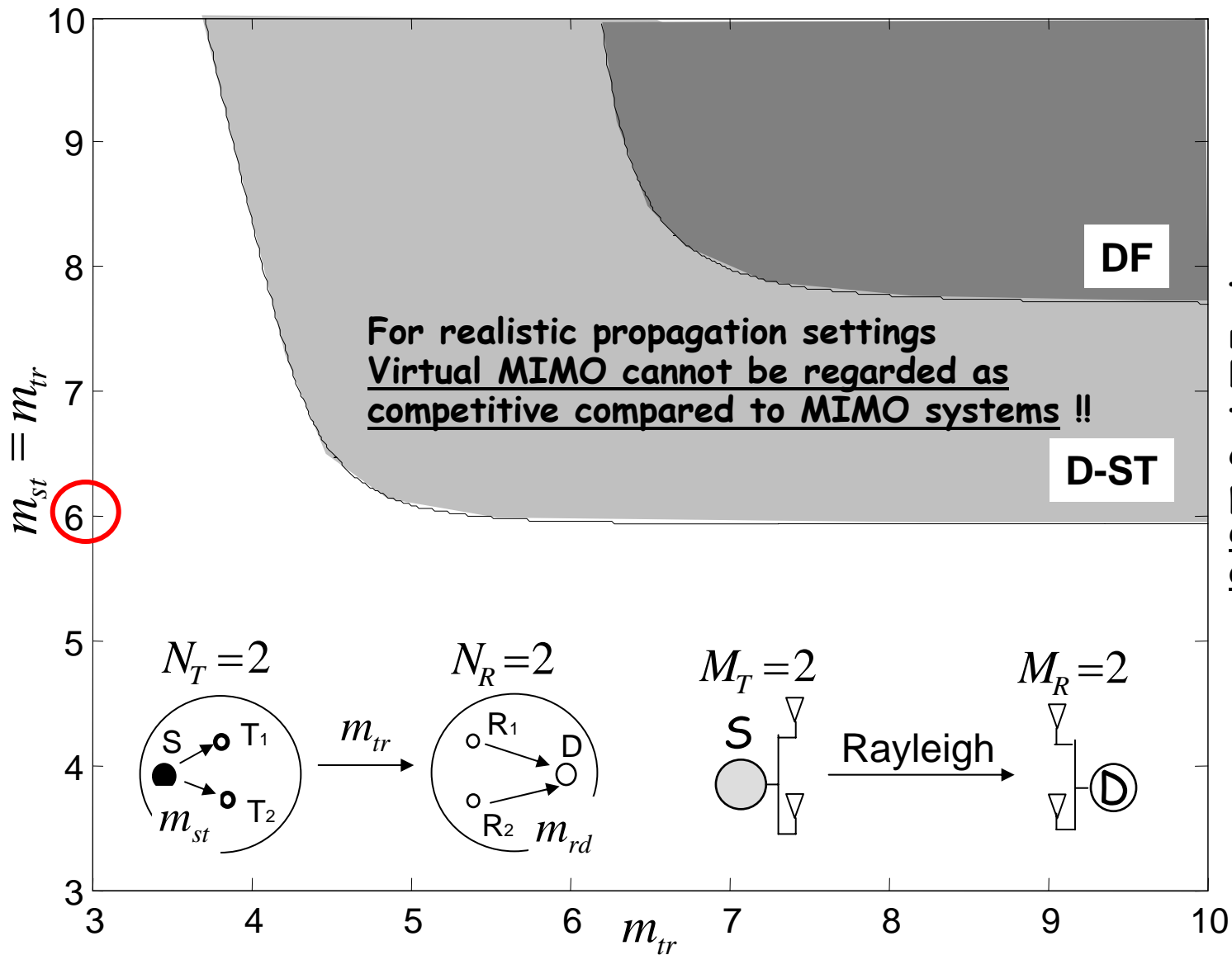


$$\mathcal{R}_{VMIMO}^{\infty} = \{ \min[N_T d_{st}, N_T N_R d_{tr}, N_R d_{rd}] > M_T M_R d_{sd} \}$$

Asymptotic Cooperative Fading Regions



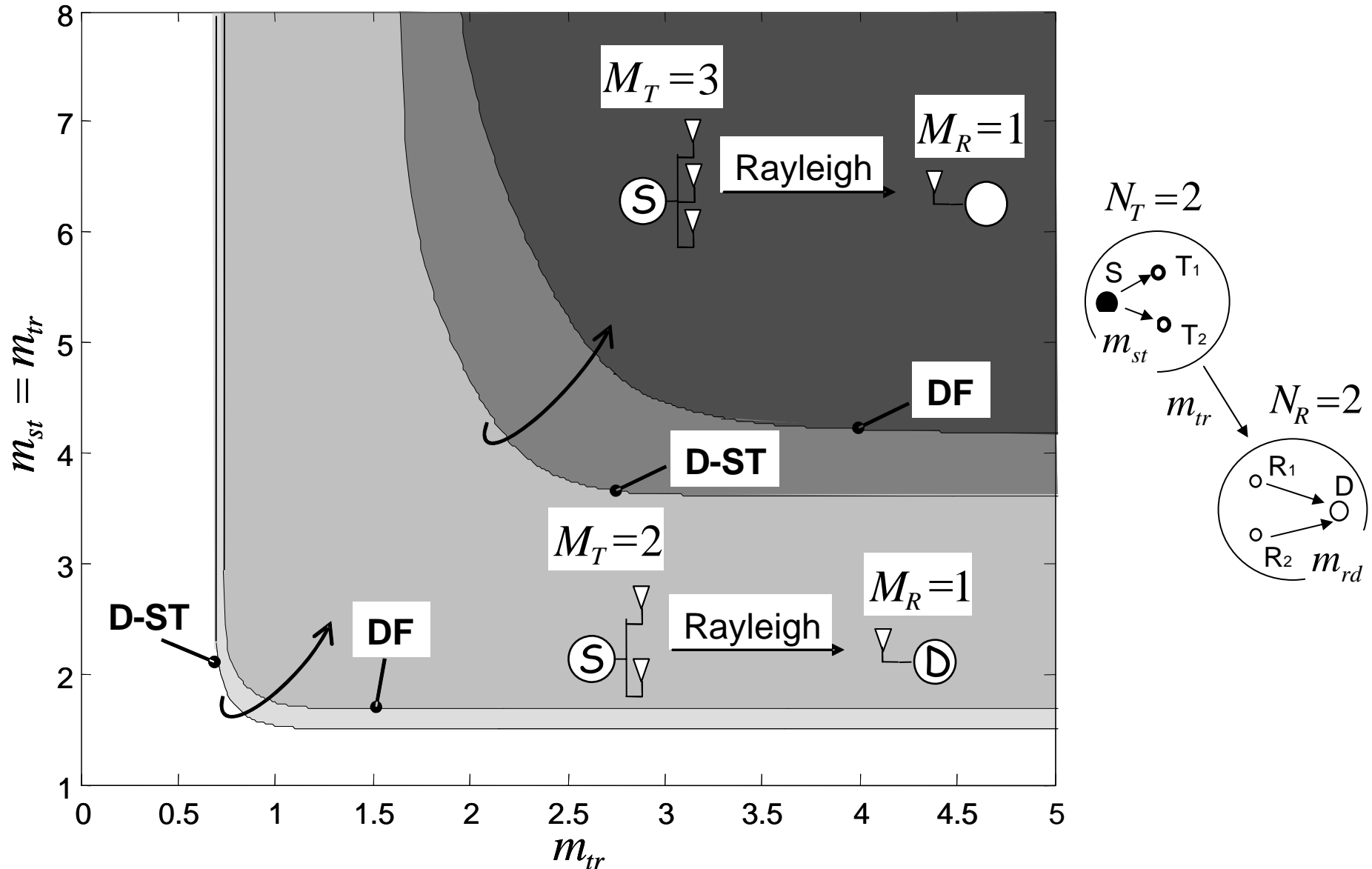
Example - Cooperative fading regions in Nakagami-m fading: V-MIMO v.s. MIMO



Transmission resources should be allocated to enable collaborative processing either at the TX or at the RX



Example - Cooperative fading regions in Nakagami- m fading: V-MIMO v.s. MISO





Concluding remarks and future directions...

- ✓ **MGF based approach to performance analysis of (distributed) wireless communications** in arbitrary fading is proposed. Outage probability analysis is carried out using information theoretic tools
- ✓ **Advantages of cooperative transmissions (for any protocol) are closely related to the propagation environment** and to the channel statistics of the links involved in collaboration that limit the price of cooperation
- ✓ **The price of cooperation for cooperative wireless systems compared to direct multi-antenna transmission is quantified in terms of outage probability and for arbitrary fading.**
- ... **Analysis shall include the effect of mobility in time-varying fading and channel out-dating due to imperfect channel estimation**
- ... **Energy consumption should be properly modeled to include the impact of the increased energy expenditure of cooperating devices at the receiving front-end**
- ... **Effective channel measurements are needed to translate the proposed theoretical analysis into a practical model for network design**



Thank you
for your attention

Any question?