Design of Distributed Randomized Orthogonal Space-Time Coding schemes for Collaborative H-ARQ

S. Savazzi and U. Spagnolini
Dip. di Elettronica e Informazione,
Politecnico di Milano, Piazza L. da Vinci 32, I-20133 Milano, Italy
e-mail: savazzi@elet.polimi.it, spagnoli@elet.polimi.it

***Abstract*— In this paper we consider a collaborative hybrid ARQ protocol where retransmissions are handled by a number of cooperating nodes employing a distributed orthogonal space time coding scheme. Outage performances are analyzed by assuming a Distributed Randomized Orthogonal Space-Time Coding scheme (DR-OSTC) to be employed by the relaying terminals during the retransmission session. The antenna selection version of the DR-OSTC requires that each cooperating node chooses *randomly* and independently to serve as one of the space-time virtual antennas. By avoiding any pre-defined terminal-to-codeword mapping, the random selection of the space-time codewords substantially reduces the needed control overhead with respect to other distributed space-time coding strategies and it simplifies the node coordination task. According to this scheme, we develop a novel analytic model to evaluate the outage probability and tackle the problem of designing the minimum number of cooperating nodes M so as to meet a specific outage probability requirement at the destination node. Finally, according to these results, we develop simple but effective design rules tailored for the collaborative hybrid ARQ protocol in practical environments.**

I. Introduction

In wireless networks channel fading is one of the main source of impairment that could be mitigated through the use of appropriate spatial redundancy also known as diversity. When the use of nodes with multiple antennas is not a viable solution due to hardware, size and costs constraints, transmitter diversity can be still achieved by exploiting cooperation among the antennas of different terminals so as to benefit from cooperative diversity [1].

In this paper we consider a cooperative wireless network where each terminal can communicate with a destination node (say a base station (BS) or an access point) with the aid of multiple relaying nodes (see figure 1). The investigated protocol, also referred to as collaborative hybrid-ARQ (H-ARQ) [1] [2], is designed so that whenever a source starts a transmission session to the BS, all the terminals covered by the same BS potentially receive the transmission intended for the destination. The BS indicates success or failure of transmission by broadcasting a single bit of feedback (ACK - NACK feedback). If a transmission failure occurs (NACK feedback), the retransmission is performed by any of the relays that overhears and decodes the earlier transmitted block employing a distributed space-time coding scheme and thus providing a spatial diversity gain [2]. As low complexity maximum likelihood (ML) decoding can be accomplished at the BS, this scheme is appropriate even when the number of transmitters is unknown, as in case of cooperative networks (see [1]).

A conventional distributed version of an orthogonal space-time coding scheme (herein referred to as D–OSTC) requires a set up phase where each cooperating node has to be informed on which space-time codeword should be transmitted. To ensure that every codeword is correctly assigned to a different active cooperating node, the coordinated D-OSTC protocol exhibits a low throughput as it requires the exact knowledge of which node is going to collaborate and therefore an high amount of control overhead that increases with the required diversity degree.

In this paper, we focus on a simple randomized transmission scheme (referred to as distributed Randomized Orthogonal Space-Time Coding scheme (DR-OSTC) or antenna selection [5]) where each cooperating node chooses *randomly* and independently the space-time codeword to be transmitted. The idea of designing and optimizing a distributed space-time coding strategy based on the simultaneous transmission of a linear combination of the space-time coding matrix was independently proposed in [5] and [6]. The original contribution of this paper is twofold: we firstly develop a novel analytic model to evaluate the outage probability for the DR-OSTC scheme, next, we tackle the problem of designing the minimum number of cooperating nodes M so as to meet a specific outage probability requirement at the destination node.

The paper is organized as follows. After system model definitions (Sect. II) the outage analysis of OSTC and of cooperative based schemes (D-OSTC) is reviewed in Sect. III-A. In Sect. III-B the outage probability is derived by assuming a DR-OSTC protocol to be employed at the cooperating terminals. Code design (namely in terms of degree of cooperation) with outage probability constraints is dealt with in Sect. IV, and validated by numerical simulations. Finally, Sect. IV-B sheds a light on how the

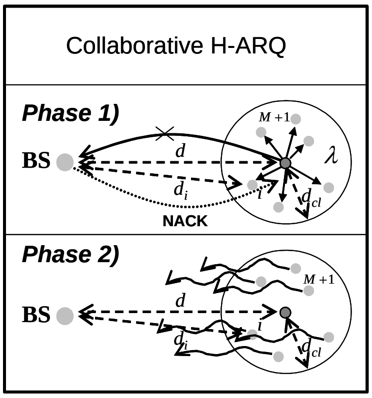

Fig. 1. Collaborative H-ARQ protocol and simulation environment

proposed analysis can help in designing a realistic planning for the collaborative H-ARQ protocol. As notation, in the following, subscripts R and D are used so as to refer to DR-OSTC and D-OSTC protocols, respectively.

II. System model

In the link model in fig. 1, each channel gain between the m-th cooperating terminal and the BS node is modelled by $h_m \sim CN(0, \Gamma_m)$, $m = 1, ..., M$ (Rayleigh fading model) and is known by the destination BS node, but it is not available by the M transmitting nodes. We assume that M nodes covered by the same BS node correctly receive and decode the source message intended for the BS node.

Let ρ be the signal to noise ratio (SNR) at the decision variable, the outage probability at the BS relative to β is defined as $\mathcal{P}_{out} = \Pr(\rho < \beta) = F(\beta)$, where $F(\beta) = \int_0^\beta f(\rho) d\rho$ is the Cumulative Density Function (CDF) of SNR from density function $f(\rho)$ and β typically specifies the minimum SNR required for acceptable performance.

In the following P_r denotes the overall power budget available at the distributed multi-antenna system composed by all the M cooperating nodes. The transmit power level at each cooperating node is set to P_r/M so as to highlight (in the comparison only) the diversity gain rather than the gain for increasing the received power. Assuming static fading over the whole codeword duration p, the received signal at the BS node $\mathbf{y}$ $(p \times 1)$ is

$$\mathbf{y}^T = \sqrt{\frac{P_r}{M}} \mathbf{v}^T \mathbf{C} + \mathbf{n}^T, \tag{1}$$

for both schemes the dimension $L \times p$ of the complex orthogonal design $\mathbf{C}$ can be decided according to the outage probability requirements $\mathcal{P}_{out}$ (and β). Being each row of $\mathbf{C}$ a linear combination of the unit power q source symbols $\{s_1, ..., s_q\}$, the space-time code rate is therefore $R_L = q/p \leq 1$. $\mathbf{n} \sim CN(0, \mathbf{I}_{p \times p})$ is the unit power additive white Gaussian noise vector with dimensions $p \times 1$. In accordance with the specific node-to-code mapping, vector $\mathbf{v} = [v_1, ..., v_L]^T$ contains a linear combination of the channel gains $\mathbf{h} = [h_1, ..., h_M]^T \sim CN(\mathbf{0}, diag(\mathbf{\Gamma}_M))$ of the links between each cooperating terminal and the BS (see Sect III). Average channel powers of each link towards the destination node are collected into the M length vector $\mathbf{\Gamma}_M = [\Gamma_1, ..., \Gamma_M]^T$, where $\Gamma_i \neq \Gamma_j$ $\forall i, j$. Due to path loss, shadowing and node distance towards the BS node, we assume each fading power $\{\Gamma_i\}_{i=1}^M$ to be a realization of i.i.d. random variable with probability density function $p_\Gamma(\Gamma)$.

To ease of notation, the average SNR at the receiving BS is herein defined as

$$\bar{\rho} = E_\Gamma[\Gamma] P_r, \tag{2}$$

and it represents a virtual retransmission as if performed by one single relay placed at an antenna that use all the power P_r with average attenuation $E_\Gamma[\Gamma] = \int x p_\Gamma(\Gamma)$.

III. Distributed Space-time coding

In order to make the paper self-consistent, we first briefly review the outage performances of Orthogonal Space-Time Coding schemes (OSTC) that take advantage of a distributed virtual antenna system (for a detailed discussion the reader should refer to [3]). Next, we derive an analytic model for the DR-OSTC (or antenna selection) scheme.

In both cases, decoding of distributed OSTC schemes with maximum likelihood (ML) detection can be still decomposed into q scalar detection problems of the unknown symbols $s_1, ..., s_q$ (see [7]). For each symbol, the SNR ρ at the decision variable can be written (recalling that $E[|s_i|^2] = 1$):

$$\rho = \frac{P_r}{M} \|\mathbf{v}\|^2 \tag{3}$$

A. Review of D-OSTC outage performances

The istantaneous SNR ρ at the decision variable can be found by substituting in (3) $M = L$ and $\mathbf{v} = \mathbf{h}$. The probability density function of ρ differs from the conventional chi-squared distribution as for OSTC [7] in that the average channel powers $\{\Gamma_i\}_{i=1}^L$ of the links are unequal. By averaging with respect to $\mathbf{\Gamma}_L = [\Gamma_1, ..., \Gamma_L]^T$ distribution, the outage probability is (see equation (14.5.26) in [8]):

$$P_{out}^D(L) = \int p_\Gamma(\mathbf{\Gamma}_L) \sum_{i=1}^L A_i(\mathbf{\Gamma}_L) \mathcal{W}\left(\frac{\beta L}{\Gamma_i P_r}\right) d\mathbf{\Gamma}_L \tag{4}$$

where $p_\Gamma(\mathbf{\Gamma}_L) = \prod_{i=1}^L p_\Gamma(\Gamma_i)$ is the joint distribution of the average channel powers for each link, $A_i(\mathbf{\Gamma}_L) = \prod_{\ell \neq i}^L \frac{\Gamma_i}{\Gamma_i - \Gamma_\ell}$ and $\mathcal{W}(\xi) = (1 - \exp(-\xi))$.

Under large SNR $\bar{\rho} = E_\Gamma[\Gamma] P_r \gg L\beta$, the outage probability (4) can be rewritten as:

$$P_{out}^D(L) \simeq \frac{1}{L!} \left(\frac{L\beta}{\bar{\rho}\epsilon}\right)^L. \tag{5}$$

Therefore, although this scheme achieves full diversity in the number of cooperating nodes, a SNR loss $\epsilon = 1/(E_\Gamma[\Gamma] E_\Gamma[1/\Gamma])$ has to be accounted for as in [4].

B. Randomized OSTC - Antenna selection

The DR-OSTC scheme consists in that each node randomly and independently chooses to transmit one of the L space-time codewords (or codeword matrix rows). Let k_i, for $i = 1, ..., L$ be the random number of terminals that are using $i-$th coding matrix row such that $\sum_{i=1}^{L} k_i = M$, the probability of a specific terminal to space-time codeword assignment $\mathbf{k} = [k_1, ..., k_L]$ is modelled by a multinomial distribution:

$$\Pr(\mathbf{k}) = \Pr(k_1, ..., k_L) = \frac{M!}{\prod_{i=1}^{L} k_i!} \left(\frac{1}{L}\right)^M. \qquad (6)$$

Vector $\mathbf{v} = [v_1, ..., v_L]^T$ in (1) is a linear combination of entries of $\mathbf{h}$:

$$\mathbf{v} = \mathbf{G}(\mathbf{k}) \cdot \mathbf{h} \sim CN(\mathbf{0}, diag(\tilde{\mathbf{\Gamma}}_L)), \qquad (7)$$

where, for any assignment vector $\mathbf{k}$, $\mathbf{G}(\mathbf{k})$ is a random binary matrix with dimensions $L \times M$ that corresponds to a specific mapping. Being $\mathbf{G}(\mathbf{k})\mathbf{G}(\mathbf{k})^T = diag(\mathbf{k})$, $\tilde{\mathbf{\Gamma}}_L = \mathbf{G}(\mathbf{k}) \cdot \mathbf{\Gamma}_M$ or, equivalently, $\tilde{\Gamma}_i = \left[\tilde{\mathbf{\Gamma}}_L\right]_i = \sum_{m=1}^{M} g_{i,m}\Gamma_m$, for $i = 1, ..., L$.

The outage probability averaged over the set $\tilde{\mathbf{\Gamma}}_L$ becomes:

$$P_{out}^R(M, L) =$$
$$= \sum_{\mathbf{k},\, k_1+...+k_L=M} \Pr(\mathbf{k}) \int p_\Gamma(\mathbf{\Gamma}_M) P_{out}^R(M, L|\tilde{\mathbf{\Gamma}}_L)) d\mathbf{\Gamma}_M. \qquad (8)$$

being $P_{out}^R(M, L|\tilde{\mathbf{\Gamma}}_L)$ the outage probability conditioned on $\tilde{\mathbf{\Gamma}}_L$ (see (4)). The summation in (8) includes all assignments $k_i \geq 0$ such that $\sum_{i=1}^{L} k_i = M$ and also the power distribution $p_\Gamma(\mathbf{\Gamma}_M) = \prod_{m=1}^{M} p_\Gamma(\Gamma_m)$.

Similarly to what shown in [5], the DR-OSTC scheme achieves a maximum diversity order of L when the average signal to noise ratio $\bar{\rho}$ is lower than a threshold $\bar{\rho}_t$ and $M \to \infty$. On the contrary, for any finite value of M, and $\bar{\rho} \to \infty$ the diversity reduces to 1 as the least likely event that all the users select the same code (i.e., $k_i = M$ and $k_j = 0$ for $\forall j \neq i$) has a dominating effect on the overall performances. In Appendix VI-A, it is derived the closed form approximation for the threshold $\bar{\rho}_t$ when $\bar{\rho} \gg L\beta$ and $M > L$ as a function of the selected code matrix spatial dimension L and of the number of cooperating terminals M:

$$\bar{\rho}_t = \beta \sqrt[L-1]{L^{M-1}}. \qquad (9)$$

Furthermore, the outage probability 8 can be approximated as:

$$P_{out}^R(M, L) \simeq \hat{P}_{out}^R(M, L)$$
$$= \frac{1}{L!} \left(\frac{\beta L}{\bar{\rho}}\right)^{L/\sqrt[M]{2}} \left(1 + \frac{\bar{\rho}}{\bar{\rho}_t}\right)^{(L/\sqrt[M]{2})-1}. \qquad (10)$$

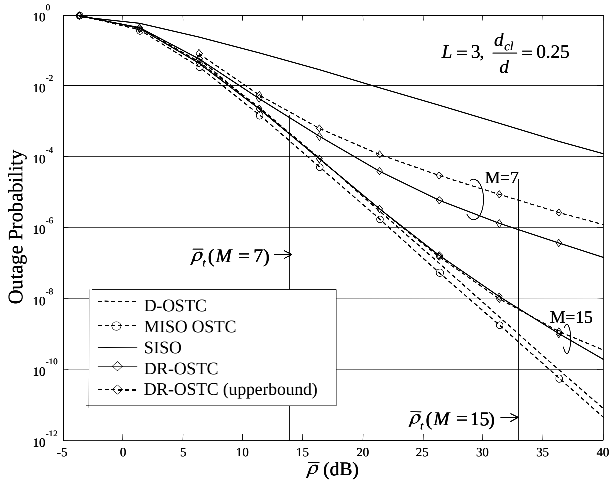

Fig. 2. Outage performances with respect to the average SNR $\bar{\rho}$ of Distributed Randomized OSTC (DR-OSTC) schemes with $M = 7$ and $M = 15$ cooperating terminals compared to D-OSTC and multiple-antenna based OSTC ($L = 3$, $d_{cl}/d = 0.25$). Outage curve approximation for DR-OSTC (10) is shown together with the SNR threshold $\bar{\rho}_t$ (9).

Notice that (10) does not depend on power dispersion in terms of ϵ: this latter result is further verified by numerical analysis (Sect.III-C).

C. Numerical results

As shown in figure 1, the cooperating nodes are randomly deployed within a circular area (or cluster) of radius d_{cl}, the distance between the cluster center and the BS node is d. Each average channel power $\Gamma_i = (d_i/d_{ref})^{-\alpha}$ takes into account a path loss term with exponent $\alpha = 3$, d_i is the random distance between the $i-$th cooperating node and the BS node (d_{ref} is a reference distance, say $d_{ref} = 1m$). According to these choices, here the probability density function $p_\Gamma(\Gamma)$ is evaluated numerically. For practical considerations, outage performance measurements are analyzed as a function of the ratio d_{cl}/d. For realistic values of the ratio d_{cl}/d (say $0.05 \leq d_{cl}/d \leq 0.5$), simulated curves (not displayed herein) show that the SNR degradation ϵ is a decreasing function of the ratio d_{cl}/d and $\epsilon > 1 - d_{cl}/d$ whenever $\alpha \leq 3$. SNR $\bar{\rho}$ is normalized with respect to β, or equivalently $\beta = 1$.

Figure 2 shows outage curves for the DR-OSTC scheme with $L = 3$, $d_{cl}/d = 0.25$ versus the average SNR $\bar{\rho}$ for $M = 7$ and 15 cooperating nodes. Performances of multi-antenna based OSTC and its distributed version D-OSTC are shown as reference. Outage probability approximation (dashed lines) in (10) is compared with the simulated performances (solid lines). In contrast to the D-OSTC case, the SNR degradation ϵ (or equivalently d_{cl}/d ratio) has a minor (negligible) impact on the outage performances of a DR-OSTC scheme as the fading power unbalancing is substantially reduced.

IV. Minimum number of cooperating nodes

In this section we develop design criteria for the DR-OSTC scheme given the outage constraint. Although the number of cooperating nodes M is a random variable and it has to be statistically modelled as a function of the transmit power allocation, the node and the fading distribution [3], in the following, for analytical convenience, the number of decoding nodes M is assumed to be known by the source (or the BS) node. In Sect. IV-B it is shown how the following design approach can be used in more practical scenarios where the number of cooperating nodes M is based on the probability density function of M (available at the transmitter). At first the (estimate of) average SNR $\bar{\rho}$ (2) (and of ϵ due to D-OSTC scheme) is computed at the BS node. Next, selection of the best complex orthogonal design $\mathbf{C}$ with spatial dimension L that meets a given link outage requirement $\mathcal{P}_{out}$ (at any given rate R or threshold β) is carried out by solving for $L > 1$ the inequality $P_{out}^{D}(L) \leq \mathcal{P}_{out}$.

For a given complex orthogonal design $\mathbf{C}$ with dimension L , DR-OSTC design is herein dealt with by finding the minimum number of cooperating terminals so that:

$$P_{out}^{D}(L) \geq P_{out}^{R}(M, L). \quad (11)$$

By exploiting outage approximation in (5) and (10), the inequality (11) reduces to:

$$\left(\frac{L\beta}{\bar{\rho}\epsilon}\right)^{L} \geq \left(\frac{L\beta}{\bar{\rho}}\right)^{L/\sqrt[M]{2}} \left(1+\frac{\bar{\rho}}{\bar{\rho}_t}\right)^{(L/\sqrt[M]{2})-1} \quad (12)$$

After straightforward algebraic computations, M for DR-OSTC can be designed as:

$$M \geq M_{\min}(L,\epsilon,\bar{\rho}) = \begin{cases} M_{\min}^{\bar{\rho}<\bar{\rho}_t} \text{ when } \epsilon > \sqrt[L]{1/L} \\ M_{\min}^{\bar{\rho}\geq\bar{\rho}_t} \text{when } \epsilon < \sqrt[L]{1/L} \end{cases} \quad (13)$$

where $M_{\min}^{\bar{\rho}<\bar{\rho}_t} == \max\left\{\left[\log_2\left(\log_{\frac{\bar{\rho}\epsilon}{L\beta}}\left(\frac{\bar{\rho}}{L\beta}\right)\right)\right]^{-1},\ \mathcal{G}\left(\frac{\bar{\rho}}{\beta}\right)\right\}$ and $M_{\min}^{\bar{\rho}\geq\bar{\rho}_t} = \mathcal{G}\left(\frac{\epsilon\bar{\rho}}{\beta}\right) + \log_L(\epsilon)$, $\mathcal{G}(\psi) = 1 + (L-1)\log_L(\psi)$ (details around the computation are given in [9]).

A. Numerical examples

By numerical evaluation of (12) and (13), the minimum number of cooperating nodes, $M_{\min}(L,\epsilon,\bar{\rho})$, is herein computed for different scenarios and compared to simulated values.

At first, we evaluate the minimum required number of cooperating nodes $M_{\min}(L,\epsilon,\bar{\rho})$ of a DR-OSTC scheme as in (13) to achieve the same outage performances of a D-OSTC with a given set of parameters: d_{cl}/d (and thus ϵ), $\bar{\rho}$ and $L = 3$. For any d_{cl}/d ratio ranging between 0.2 and 0.42, the minimum number of cooperating nodes $M_{\min}(L,\epsilon,\bar{\rho})$ is computed in figure 3 for SNR $\bar{\rho} = 16dB$ and $\bar{\rho} = 24dB$ (solid lines). The resulting settings are validated by extensively simulating the outage performances from (4) and (8) (cross markers). As the d_{cl}/d ratio decreases (or equivalently if increasing ϵ), the number of required cooperating nodes for the DR-OSTC increases with $1/(1-\epsilon)$. On the contrary, when the propagation environment is such that (from (13)) $\epsilon < \sqrt[L]{1/L} \simeq 0.69$ (or for cluster of large size as $d_{cl}/d \simeq 0.36$), requirements on M become less stringent.

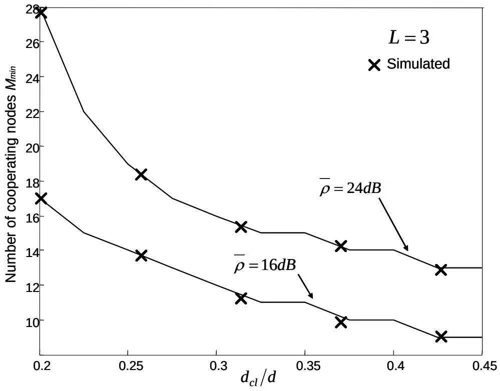

Fig. 3. Minimum required number of cooperating nodes, $M_{\min}(L,\epsilon,\bar{\rho})$, versus the ratio d_{cl}/d, for $L = 3$, $\bar{\rho} = 16dB$ and $\bar{\rho} = 24dB$: approximated results obtained through the proposed design strategy (Sect. IV) are validated by extensive numerical simulations (cross markers).

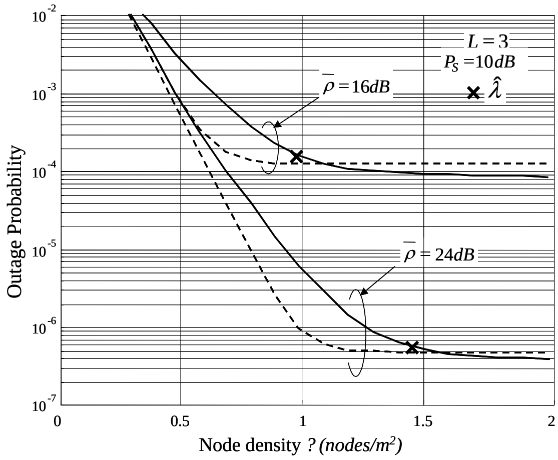

Fig. 4. Outage probability at the destination BS for the DR-OSTC scheme ($L = 3$) with respect to the local node density λ for the collaborative H-ARQ protocol. Neighbor nodes within the transmission cluster are distributed according to a Poisson random point process. Dashed lines refer to the outage performances for the coordinated D-OSTC case. Lowerbound $\hat{\lambda}$ in (16) is indicated by cross markers.

B. Collaborative hybrid-ARQ design

For completeness, starting from the previous results, we now assess the performance of the DR-OSTC scheme when applied to the collaborative H-ARQ protocol outlined in Sect. I. The random number of decoding (or cooperating) nodes in short range wireless applications is modelled according to the "disk model" for the links between the transmitter and the relays [10]. A successful transmission occurs as long as the received SNR exceeds the threshold β: $P_S d_r^{-\alpha} d_{ref}^{\alpha} > \beta$, where d_r is the random distance between the source and the relay and P_S is the source transmit power (in general $P_S \neq P_r$). The node range r_g for successful transmission can be therefore defined as $r_g = d_{ref} \sqrt[\alpha]{P_S/\beta}$.

For the sake of simplicity, we consider the probability of M successfully decoding nodes in an area $\mathcal{A} = \pi r_g^2$ be given by the Poisson distribution:

$$P_{\mathcal{A}}(M, \lambda) = \exp(-\lambda \pi r_g^2) \frac{\left(\lambda \pi r_g^2\right)^M}{M!} \tag{14}$$

with average number of decoding nodes $E_{P_{\mathcal{A}}}[M] = \lambda \pi r_g^2 = \lambda \pi d_{ref}^2 \sqrt[\alpha]{(P_S/\beta)^2}$, λ is the local node density (with respect to the source node).

When considering the collaborative H-ARQ protocol, the source power is fixed so as to guarantee a given link reliability towards the BS. Here, the design is focused on the required local node density λ to guarantee an outage probability at the BS, $\mathcal{P}_{out}$, *after* the retransmission (perfect feedback channel from BS to the node cluster is assumed). The latter is found by solving for λ:

$$\mathcal{P}_{out} \simeq \sum_{M=0}^{\infty} \hat{P}_{out}^{R}(M, L) P_{\mathcal{A}}(M, \lambda\ ). \tag{15}$$

From the Jensen inequality and the results in Sect. IV, a lower bound for λ is:

$$\lambda \geq \hat{\lambda} = \frac{M_{\min}(L, K(\mathcal{P}_{out}, L, \bar{\rho}), \bar{\rho})}{\pi d_{ref}^2} \sqrt[\alpha]{\left(\frac{\beta}{P_S}\right)^2}. \tag{16}$$

where $K(\mathcal{P}_{out}, L, \bar{\rho}) = \min\left\{\frac{L\beta}{\bar{\rho}\sqrt[L]{L!\mathcal{P}_{out}}}, 1\right\}$ is derived by solving the equation $\mathcal{P}_{out} = \hat{P}_{out}^{D}(L, \epsilon)$, see (5), for ϵ. By evaluating $\hat{\lambda}$, the source node can decide whether or not the node density is enough for a cooperative retransmission to be effective given the outage probability constraint.

In figure 4 we assess the tightness of the proposed lower bound. We show the outage probability versus the local node density at the destination BS node for a Poisson random network obtained from the summation in (15). Results are shown for various $\bar{\rho}$ values ($d_{cl}/d = 0.25$), cross markers refer to the node density $\hat{\lambda}$. Dashed lines refer to the outage performances for the D-OSTC case with $L = 3$. Although the DR-OSTC scheme requires an higher demand of network resources with respect to D-OSTC (in terms of an increased node density), we show that the bound $\lambda = \hat{\lambda}$, is tight as compared to the optimal solution of (15).

V. Conclusion

In this paper we considered an H-ARQ protocol where retransmission of corrupted symbols is handled by a number of cooperating nodes employing a distributed randomized orthogonal space time coding scheme (DR-OSTC or antenna selection). An analytic model to evaluate the outage probability has been derived. We compared performance results with those obtained from distributed space-time coding schemes (D-OSTC) and from conventional multiple antenna based OSTC. Next, simple and accurate design rules have been investigated for the required minimum number of cooperating nodes $M_{\min}$ (Sect. IV) so as to meet a specific outage probability requirement at the BS. The randomization rule simplifies the node coordination task at the price of an increased number of cooperating nodes with respect to the D-OSTC protocol, $M \geq M_{\min} > L$ (13), that constrains the minimum sensor density (as shown for the collaborative H-ARQ protocol) within a given deployment area.

VI. Appendix

A. DR-OSTC

We define $\Pr(i) = \mathcal{Q}(i) - \mathcal{Q}(i-1)$ as the probability that i ($i \leq L$) rows of code matrix $\mathbf{C}$ are selected by the M cooperating nodes, $\mathcal{Q}(i) = \binom{L}{L-i}\left(\frac{i}{L}\right)^M$. Assuming $\bar{\rho} \gg L\beta$ and $M > L$, we derive a lower bound on the outage probability:

$$P_{out}^{R}(M, L) > \sum_{i=1}^{L} \Pr(i) \left(\frac{i\beta}{\bar{\rho}}\right)^i \frac{1}{i!} \tag{17}$$

details around the computation are shown in [9].

When M is large such that $\Pr(1) \leq \left(\frac{\beta}{\bar{\rho}}\right)^{L-1}$ or $\bar{\rho} < \bar{\rho}_t = \beta \sqrt[L-1]{L^{M-1}}$ then, using the result in (17) and the Jensen upperbound, the diversity performance at finite $\bar{\rho}$ [8] (but still $\bar{\rho} \gg L\beta$) can be upperbounded as:

$$\frac{-\log(P_{out}^{R}(M, L))}{\log(\bar{\rho})} < \sum_{i=1}^{L} i \Pr(i) < \frac{L}{\sqrt[M]{2}}. \tag{18}$$

Therefore the approximated outage curve versus $\bar{\rho}$ ($\bar{\rho} \gg L\beta$) results in (10).

References

[1] J. N. Laneman, G. W. Wornell, "Distributed Space-Time-Coded Protocols for Exploiting Cooperative Diversity in Wireless Networks," *IEEE Trans. Inform. Theory*, vol.49, no. 10 pp.2415-2425, Oct. 2003.

[2] B. Zhao, M. C. Valenti, "Practical Relay Networks: a Generalization of Hybrid-ARQ," *IEEE Journal on Sel. Areas in Comm.* vol. 23, no. 1, pp. 7-18, Jan. 2005.

[3] T. E. Hunter, S. Sanayei, A. Nosratinia, "Outage analysis of Coded Cooperation," *IEEE Trans. on Inform. Theory*, vol. 52, no. 2, pp.375-391, Feb. 2006.

[4] J. Mietzner, R. Thobaben, P. A. Hoeler, "Analysis of the Expected Error Performance of Cooperative Wireless Networks employing distributed Space-Time Codes," *IEEE Globecom* 2004.

[5] B. Sirkeci, A. Scaglione, "Randomized Space-Time Coding for Distributed Cooperative Communication," *IEEE ICC* 2006, June 2006.

[6] S.Yiu, R. Schober and L. Lampe, "Distributed Space-Time Block Coding," *IEEE Globecom.*, vol. 3, pp. 1592-1597, Dec. 2005 also in *IEEE Trans. on Comm.* vol. 54, no. 7, pp. 1195 - 1206, July 2006.

[7] V. Tarokh, H. Jafarkhani, A. R. Calderbank, "Space-Time Block Codes from Orthogonal Designs," *IEEE Trans. on Inform. Theory*, vol. 45, no. 5, pp. 1456-1466, July 1999.

[8] J. Proakis, *Digital Communications*, 4th Ed., McGraw Hill, 2001.

[9] S.Savazzi, U.Spagnolini "Outage constrained design of Orthogonal Space-Time coding schemes for randomized cooperative networks," submitted to *IEEE Trans. on Wireless Comm.*

[10] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Trans. Inf. Theory*, vol. 46, no. 2, pp. 388–404, Mar. 2000.