Virtual MIMO Transmissions: Diversity and Outage Analysis in arbitrary Fading Channels

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***Abstract*— In this paper, we evaluate the outage performances over fading channels of cooperative transmissions involving clusters of single antenna relay terminals that are collaborating for delivering the same message by engaging virtual multiple input multiple output (V-MIMO) transmissions. Diversity provided by these protocols has been widely analyzed for the Rayleigh fading case. However, ad-hoc networks and short range communications often experience propagation environments where the PDF of the impairments is largely different from Rayleigh (e.g. for cases where the line-of-sight component dominates compared to the random non line-of-sight ones). In this paper we evaluate diversity and outage performances of V-MIMO systems by considering arbitrary distributed fading over each cooperative link. We derive the conditions on the fading statistics for which the V-MIMO transmission can be regarded as a competitive option (in terms of achievable performances) compared to Space Time (ST) coded direct MIMO link. *Cooperative fading regions* collect the propagation settings that make V-MIMO preferable to MIMO transmission.**

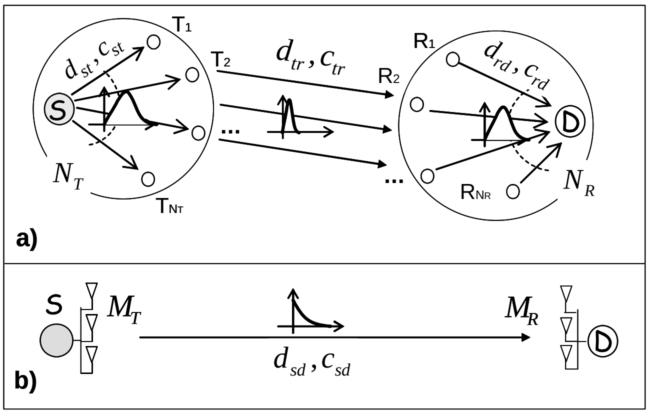

Fig. 1. Transmission setting and propagation environment for non cooperative MIMO transmission (b) and cooperative (virtual) MIMO (V-MIMO) (a) cases

I. INTRODUCTION

Wireless ad hoc networks consist of a number of terminals (or nodes) communicating with each other without the assistance of a wired or infrastructure network. The communication between nodes might take place through several intermediate nodes, creating a multi-hop network. Constructive and destructive combination of randomly delayed, reflected, scattered and diffracted signal components at the receiver result in multipath fading impairments and path loss that limit the performances of these networks.

Although the fading envelope at the receiver is typically modelled as Rayleigh distributed, ad-hoc (and sensor) networks often experience propagation environments that cause the fading impairments to exhibit largely different distributions [1]. As an example, when the signal wavelength is comparable with the size of a cluster of scatterers (that typically happens in short range and highly LOS communication), it can be shown that the PDF of the fading envelope can converge to a Gamma PDF [2]. In all cases the propagation environment has a major influence on the performances of wireless transmissions.

Multiple antenna at each terminal are known to provide spatial redundancy (or diversity order) to reduce fading impairments. When this solution is not viable due to hardware, size and cost constraints, it has been shown that cluster of nodes, with one antenna each, might form coalitions to cooperatively act as a large transmit or receive array. In Virtual MIMO (V-MIMO) systems, transmission takes place between clusters of cooperating (and coordinated) transmitters/receivers [3]-[4] so as to harness the diversity (e.g., cooperative diversity [5]) provided by a multiple input and multiple output channel.

In this paper, the performance analysis of cooperative transmissions with decode and forward relaying is dealt with by embracing *arbitrary* fading distributions for each link that is scheduled for transmission. Consider the transmission setting for the V-MIMO system depicted in figure 1: a source node S might decide to reach its destination D with the help of relayed transmissions from a cluster of N_T transmitting relays (TRs). Detection at the destination is attained with the help of a cluster of N_R receiving relays (RRs) that independently decode the received message and forward it to the destination. Since in typical settings relays (TRs or RRs) are in short range with the source (or destination), they can benefit from a marginal diffusive channel component (e.g., Nakagami fading [13]) compared to Rayleigh. The necessary conditions on the fading distribution for each collaborative link are investigated here so that the V-MIMO transmission performed by terminals with single antenna can be regarded as a competitive option (in terms of performance, bandwidth efficiency and power consumption) compared to MIMO (non-cooperative) Space Time coded systems [6]. Conditions on the fading probability density functions (PDFs) are derived in closed form for varying number of TR and RR relays (N_T, N_R) and for varying direct MIMO settings (for M_T transmit and M_R receive antennas, see figure 1-(b)).

This paper is organized as follows. The outage probability used here as performance metric is extended to embrace arbitrary distributed fading. Fading channels are described by the inherent diversity (d) and coding gain (c) parameters [7] that are evaluated in a general way from the asymptotic behavior of the moment generating function (MGF) of the random fading power over each link as proposed in [8] (the reader might also refer to [9]). In Sect. II we evaluate the outage probability performances for the V-MIMO system. In Sect. III *cooperative fading regions* [8] are then derived as the collection of fading power distributions that make the V-MIMO transmission to provide the same diversity (and outage probability) as if the source and destination nodes would be equipped with multiple antennas. As an extension to the work proposed in [10], in Sect. IV we show how outage analysis at large SNR can be specialized for Nakagami-m fading environments. For this setting, we evaluate the conditions on the Nakagami fading figures so that V-MIMO transmissions can exhibit the same performances, bandwidth efficiency and power/energy consumption as for direct or non-cooperative MIMO systems.

A. Outage probability analysis for arbitrary fading - SISO case

We first review the basic principles of outage analysis for arbitrary fading and single antenna point-to-point transmissions [8]. As a baseline case, let E_S be the transmitted symbol energy from a single antenna source towards a destination, the transmit power is $\rho = E_S/T$ and the signal to noise ratio (SNR) referred to the transmitting side is ρ/σ^2, with $\sigma^2 = N_0/T$ the additive white Gaussian (AWGN) noise power and N_0 the single-sided noise power spectral density (to simplify, here we assume $N_0 = 1$ and $\sigma^2 = 1$ so that ρ refers equivalently to SNR or the transmit power).

The baseband complex valued channel gain is h, fading power $E[|h|^2] = g$ accounts for path loss and shadowing and the instantaneous SNR at the receiving side is $\mu = \rho\,|h|^2$. Assuming a channel with static fading for the whole transmission duration, the maximum mutual information over the link for source employing Gaussian codebook is $I = \log_2(1+\mu)$, for a target rate R [b/s/Hz], the outage probability is $\Pr[I < R] = \Pr\left[\mu < 2^R - 1\right]$.

The outage probability at high SNR ρ can be conveniently approximated as [7]

$$\Pr[I < R] = \Pr\left[\rho\,|h|^2 < 2^R - 1\right] \approx \left(\frac{2^R-1}{c \times \rho}\right)^d, \quad (1)$$

where notation $\approx$ indicates that equality holds for asymptotically high SNR ($\rho \to \infty$). For any fading with generic power distribution $|h|^2 \sim f_{|h|^2}(w)$, d is the (fractional) diversity order $d \triangleq \lim_{\rho\to\infty} \log \Pr[I < R]/\log\rho$ that is provided by the channel, while parameter c is the amount of coding gain.

For arbitrary fading channels, diversity d can be defined from the Laplace transform $\mathcal{F}_{|h|^2}(s)$ (or Moment Generating Function, MGF) of $f_{|h|^2}(w)$ as

$$d \triangleq \lim_{s\to\infty} \frac{-\log \mathcal{F}_{|h|^2}(s)}{\log s} > 0. \quad (2)$$

The main reasoning of the proof is in Appendix VI (see also [8]). Coding gain c follows as:

$$c \triangleq \left[\frac{\Gamma(d+1)}{\lim_{s\to\infty} s^d \mathcal{F}_{|h|^2}(s)}\right]^{1/d}, \quad (3)$$

where $\Gamma(x) = \int_0^\infty y^{x-1}\exp(-y)dy$ is the complete Gamma function. Notice that in Rayleigh fading it is $d = 1$, $c = g$.

B. Outage probability analysis for arbitrary fading - MIMO case

The direct ST block-coded transmission from a source node S equipped with $M_T \geq 1$ antennas and a receiver D with $M_R \geq 1$ antennas employing maximum ratio combining (MRC) represents the reference setting (figure 1) to be used for performance comparison of V-MIMO protocols. The overall transmission duration is T.

Space-time block coding (ST) is employed to combat the fading effects by harnessing the diversity of the channel without requiring channel state information at the transmitter. The transmit power of each antenna scales as ρ/M_T so that ρ is the overall energy consumption. For Gaussian codebook case, the mutual information for the direct link is $I_{sd}(\rho) = \log_2(1+\mu)$ with $\mu = \sum_{i=1}^{M_T}\sum_{j=1}^{M_R}\rho\left|h_{sd}^{(i,j)}\right|^2$ and $\left|h_{sd}^{(i,j)}\right|^2 \sim f_{|h_{sd}|^2}(w)$ are the i.i.d. (spatially independent) fading powers over each transmitter and receiver pair. The outage probability for a given power $\rho \gg 1$ and rate R after MRC at the receiver can be written as

$$P_{out}^{(NC)}(\rho, R) = \Pr(I_{sd}(\rho) < R) \approx$$
$$\approx \left(\frac{\Gamma(d_{sd}+1)^{M_T M_R}}{\Gamma\left(M_T M_R d_{sd}+1\right)}\right)\left(\frac{M_T\left(2^R-1\right)}{\rho c_{sd}}\right)^{M_T M_R d_{sd}}, \quad (4)$$

where diversity d_{sd} is evaluated in (2) by setting $\mathcal{F}_{|h_{sd}|^2}(s) \equiv \mathcal{F}_{|h|^2}(s)$ and, similarly, the coding gain c_{sd} is in (3) with $d = d_{sd}$. The overall diversity provided by the MIMO transmission is

$$d_{MIMO} = -\lim_{\rho\to\infty} \frac{\log\left[P_{out}^{(NC)}(\rho, R)\right]}{\log(\rho)} = M_T M_R \times d_{sd}, \quad (5)$$

it scales with the product of the number of transmit and receive antennas $M_T \times M_R$ and the diversity d_{sd} provided by the fading statistics.

II. VIRTUAL MIMO TRANSMISSION: PROTOCOL DESCRIPTION AND OUTAGE ANALYSIS

Analysis is now focused on different V-MIMO protocols (see figure 1-(a) and 2) consisting of a cooperative transmission that involves a cluster of N_T TRs (and the source S) and a cooperative reception that is handled by a cluster of N_R RRs (and the destination D). In this paper we assume that the source node is not joining collaborative transmission: this is done to minimize the energy consumption and the transmission duration. Moreover the destination can receive from its neighboring RRs only. As shown in figure 2 V-MIMO transmission operation is organized into three phases:

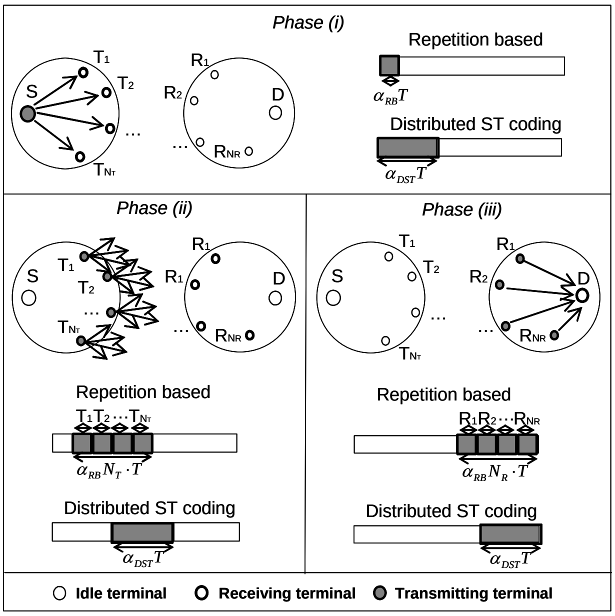

Fig. 2. V-MIMO protocol description with repetition based relaying (RB) and distributed ST coding (DST)

i) at first the source node S broadcasts the message to the N_T active TRs (for *intra-cluster* communication) so that they can set-up the collaborative transmission; *ii)* the relays that successfully decoded the message engage virtual multi-antenna transmission (realizing *inter-cluster* transmissions) by simply repeating the message using time division[1] (repetition based coding (RB), distributed ST coding (D-ST) can be also employed during this phase); *iii)* the N_R (single antenna) RRs decode and forward the received message to the destination D (phase *iii* thus involve a final *intra-cluster* transmission) by repeating the message using time division (D-ST coding is another possible option). The destination (and RRs) combines by MRC all the signal replicas from the RRs (or TRs) to gain from diversity.

We now derive the outage performances of the V-MIMO protocol for the cases where transmitting and receiving relays are employing RB coding (similar analysis cane be dealt with for the D-ST coding case). We assume that each TR node might elect itself as a collaborating node if decoding is accomplished [11]. Since performance is based on outage probability, we assume that a relay node fails in decoding whenever an outage event occurs, therefore outage is for $\alpha I < R$ [12] where I is the mutual information over the link and α is the fraction of the available time T that is reserved for transmission. Notice that the V-MIMO transmission duration is T as for the direct scheme to constrain the same bandwidth efficiency.

The outage probability for the V-MIMO system depicted in figure 2 can be written in a general form as

$$P_{out}(\rho, R) = \textstyle\sum_{j=0}^{N_R} \Pr[\alpha I_{rd}(j,\rho) < R] \times \Pr(j; RR), \quad (6)$$

where $\Pr[\alpha I_{rd}(j,\rho) < R]$ with $I_{rd}(\rho) = \log_2(1+\sum_{k=1}^{j}\mu_{rd,k})$ (subscript rd indicates the RRs-to destination link) is the outage probability at the destination, given that $j = 0, ..., N_R$ RRs are decoding; $\mu_{rd,k} = \rho\,|h_{rd,k}|^2$ is the SNR over each RR-to-destination link pair. Notice that $\Pr[\alpha I_{rd}(0,\rho) < R] = 1$, revealing that an outage event occurs if none of the RRs is able to decode. Term $\Pr(j; RR)$ accounts for the probability that j RRs are decoding and collaborating

$$\Pr(j; RR) = \textstyle\sum_{i=0}^{N_T} \Pr(i; TR) \times \Pr(j; RR | i; TR) \quad (7)$$

and it is the weighted sum over the probabilities $\Pr(j; RR|i; TR)$ that j RRs decode given that $i = 0, ..., N_T$ TRs are collaborating. Probability $\Pr(i; TR)$ is

$$\begin{aligned}\Pr(i; TR) &= \tbinom{N_T}{i}\,[1 - p(\alpha)]^i\, p(\alpha)^{N_T - i} = \\ &= \mathcal{B}(N_T, p(\alpha), i).\end{aligned} \quad (8)$$

Term $p(\alpha) = \Pr[\alpha I_{st}(\rho) < R]$ is the outage probability at one TR during the source broadcast of duration α, $I_{st}(\rho) = \log_2(1+\mu_{st})$ and $\mu_{st} = \rho\,|h_{st}|^2$ (subscript st refers to source-to-TRs links). The probability term $\Pr(j; RR|i; TR)$ has the same binomial structure as in (8)

$$\Pr(j; RR|i; TR) = \mathcal{B}(N_R, q(\alpha|i), j) \quad (9)$$

with $q(\alpha|i) = \Pr[\alpha I_{tr}(i,\rho) < R]$ the outage probability at one RR provided that i TRs are collaborating and transmitting. Total outage probability (6) can be written now in a compact form as

$$P_{out}(\rho, R) = p(\alpha)^{N_T} + \mathbf{b}_{st}^T(\alpha)\mathbf{B}_{tr}(\alpha)\mathbf{b}_{rd}(\alpha) \quad (10)$$

where the i-th element ($i = 1, ..., N_T$) of the N_T length vector $\mathbf{b}_{st}$ is $[\mathbf{b}_{st}]_i = \Pr(i; TR) = \mathcal{B}(N_T, p(\alpha), i)$, $N_T \times (N_R + 1)$ matrix $\mathbf{B}_{tr}$ is

$$\mathbf{B}_{tr} = \begin{bmatrix} \mathcal{B}(N_R, q(\alpha|1), 0) & ... & \mathcal{B}(N_R, q(\alpha|1), N_R) \\ ... & ... & ... \\ \mathcal{B}(N_R, q(\alpha|N_T), 0) & ... & \mathcal{B}(N_R, q(\alpha|N_T), N_R) \end{bmatrix} \quad (11)$$

with element $\Pr(j; RR|i; TR) = \mathcal{B}(N_R, q(\alpha|i), j)$ being the probability that j RRs can decode provided that $i \geq 1$ TRs are collaborating. Vector $\mathbf{b}_{rd} = \{\Pr[\alpha I_{rd}(j,\rho) < R]\}_{j=0}^{N_R}$ collects the outage probabilities at the destination D given j decoding RRs.

Outage performances (10) are limited by the probability $\mathcal{B}(N_T, p(\alpha), 0) = p(\alpha)^{N_T}$ that none of the TRs can decode the source message and by the corresponding probability at the receiving cluster. These observations will be helpful for evaluating the cooperative diversity performances of the V-MIMO system.

The propagation environment under study is depicted in figure 1. The source node is activating N_T TRs using N_T links with the same fading statistics (independent and identically distributed, i.i.d.) These are described by the inherent diversity d_{st} (2) and the coding gain c_{st} (3) that are related to the specific fading power distribution, say $|h_{st}|^2 \sim f_{|h_{st}|^2}(\cdot)$ and, more specifically, to the asymptotic behavior of the MGF $\mathcal{F}_{|h_{st}|^2}(s)$. Similarly, the inter-cluster links suffer from i.i.d. fading with power $|h_{tr}|^2 \sim f_{|h_{tr}|^2}(w)$. Fading power statistic is different from Rayleigh and it is characterized by the diversity/coding gain pair d_{tr} and c_{tr} derived from the

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[1] Each relay has exclusive access to the wireless medium.

asymptotic behavior of the MGF $\mathcal{F}_{|h_{tr}|^2}(s)$. The N_R relayed transmissions from the receiving cluster to the destination are performed over channels with diversity d_{rd} and coding gain c_{rd}, related to the MGF $\mathcal{F}_{|h_{rd}|^2}(s)$ of the fading power distribution $|h_{rd}|^2 \sim f_{|h_{rd}|^2}(\cdot)$.

In what follows the outage probability is specialized for RB coding by evaluating the outage terms at high SNR. Similar results can be obtained for DST coding with some minor changes on the analytical part (not shown here for lack of space)

Repetition based coding (RB)

Medium access control (MAC) operation (for Time Division Duplex (TDD) systems) is subdivided into $N_T + N_R + 1$ slots for a total transmission duration of T. Source and relays are therefore allowed to use the bandwidth for a fraction $\alpha = \alpha_{RB} = (N_T + N_R + 1)^{-1}$ of the available duration T. Since each TR (and RR) is simply repeating the same codeword of the source, each TR (and RR) is allowed to use the channel for the same time fraction α_{RB} (i.e., each terminal has exclusive access to the wireless medium based on time division policy). Since the V-MIMO scheme is constrained to have the same bandwidth efficiency (end-to-end rate R) as for the direct MIMO case, each cooperative link must support an increased rate R/α_{RB}[2]. In Rayleigh fading this scheme does not provide the full diversity in the number of transmit and receive relays and it exhibits sub-optimum performances compared to D-ST coding. However, it allows nodes equipped with single antenna to exploit a signal redundancy similar to that of co-located multiantenna systems while leveraging the MAC layer complexity, the overhead[3] and the hardware and battery resources (e.g. the per-node energy consumption). The outage probability $P_{out}^{(RB)}(\rho, R)$ for RB coding is in (10) with $\alpha = \alpha_{RB}$. For high SNR the outage probability performances are ruled by the diversity and coding gain of each link so that outage terms become

$$p(\alpha_{RB}) \approx \frac{(2^{R/\alpha_{RB}} - 1)^{d_{st}}}{\Gamma(d_{st}+1)(\rho \times c_{st})^{d_{st}}} \tag{12}$$

$$q(\alpha|i) \approx \frac{\Gamma(d_{tr}+1)^i}{\Gamma(id_{tr}+1)} \left(\frac{2^{R/\alpha_{RB}} - 1}{\rho \times c_{tr}} \right)^{id_{tr}} \tag{13}$$

$$\Pr[\alpha_{RB} I_{rd}(j,\rho) < R] \approx \frac{\Gamma(d_{rd}+1)^j}{\Gamma(jd_{rd}+1)} \left(\frac{2^{R/\alpha_{RB}} - 1}{\rho \times c_{rd}} \right)^{jd_{rd}} \tag{14}$$

III. DIVERSITY ANALYSIS AND COOPERATIVE FADING REGIONS

Cooperative diversity for V-MIMO transmission is derived here for arbitrary distributed fading over each cooperative link. Next, we analyze the conditions on the fading statistics where cooperation is beneficial compared to direct transmission case in that it provides the same (or higher) diversity.

A. Cooperative diversity and asymptotic cooperative fading regions

The achievable diversity for the V-MIMO cooperative scheme can be easily evaluated as

$$d_{VMIMO} = \lim_{\rho \to \infty} \frac{-\log[P_{out}(\rho, R)]}{\log(\rho)} = \\ = \min\{N_T d_{st},\ N_T N_R d_{tr},\ N_R d_{rd}\}. \tag{15}$$

In Rayleigh fading (for $d_{st} = d_{tr} = d_{rd} = 1$) diversity reduces to $d_{VMIMO} = \min\{N_T, N_R\}$. Cooperative diversity provided by the V-MIMO protocol scales with the product of the number of TRs and RRs only if $d_{st} > N_R d_{tr}$ and $d_{rd} > N_T d_{tr}$. These are the necessary conditions for the fading distributions (therefore the diversities d_{st} and d_{rd} provided by the propagating channels) that constrain the error probability of intra-cluster connections to be marginal compared to the inter-cluster ones. By comparing (15) with (5), it can be easily shown that cooperation is beneficial in providing higher diversity with respect to the non-cooperative case only if $d_{VMIMO} > d_{MIMO}$, therefore when $(d_{st}, d_{tr}, d_{rd}, d_{sd}, N_T, N_R, M_T, M_R) \in \mathcal{R}^\infty$ and where

$$\mathcal{R}^\infty = \{d_{st} > \frac{M_T M_R}{N_T} \times d_{sd}, \\ d_{tr} > \frac{M_T M_R}{N_T N_R} \times d_{sd},\, d_{rd} > \frac{M_T M_R}{N_R} \times d_{sd}\}. \tag{16}$$

collects the conditions that establish the (asymptotic) *cooperative fading region* for the V-MIMO system under study.

B. Asymptotic fading regions in Nakagami-m fading

Experimental data show that Nakagami-m is a general parametric fading distribution that can be adjusted to fit a variety of empirical measurements [13].

Propagation setting is now specialized as shown in down left corner of figure 3: intra-cluster links are modeled as Nakagami-m distributed with fading figure m_{st} (for $m_{st} = m_{rd}$), inter-cluster links are characterized by fading figure m_{tr}. Diversities $d_{st} = m_{st}$, $d_{tr} = m_{tr}$, $d_{rd} = m_{rd}$ and $d_{sd} = m_{sd}$ provided by the fading that impairs each link are found from the asymptotic behavior of the MGF (2) as $\mathcal{F}_{|h|^2}(s) = (1 + sg/m)^{-m}$ and $d = \lim_{s\to\infty} -\log \mathcal{F}_{|h|^2}(s)/\log s = m$. For Nakagami-m fading the asymptotic cooperative fading region (16) is the collection of fading figures m_{st}, m_{tr} and m_{rd} for each collaborative link such that the cooperative diversity provided by the V-MIMO transmission is larger than the diversity provided by the non-cooperative MIMO scheme

$$\mathcal{R}^\infty = \{m_{st} > \frac{M_T M_R}{N_T} \times m_{sd}, \\ m_{tr} > \frac{M_T M_R}{N_T N_R} \times m_{sd},\, m_{rd} > \frac{M_T M_R}{N_R} \times m_{sd}\}. \tag{17}$$

IV. OUTAGE PROBABILITY ANALYSIS IN NAKAGAMI-M FADING

Cooperative fading regions are now specialized by assuming the outage probability as performance metric and using the outage results for large SNR under RB coding in Sect. II. In

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[2] For non-Gaussian codebook case, it constrains the use of a modulation/coding pair with higher spectral efficiency.

[3] In contrast to D-ST coding syncronization at symbol-level is not mandatory.

this section performance analysis is also dealt with for D-ST coding (without providing the extensive proof). Orthogonal space–time codes can provide a diversity that scales with the random number of collaborating relays. Two complex orthogonal designs are used during the TR and RR transmissions with spatial dimensions $N_T > 1$ and $N_R > 1$, respectively. Even if one cooperating terminal is not able to decode, orthogonal ST coding can still provide residual diversity benefits [5]; recall that decoding requires symbol-level synchronization among the relays. Compared to repetition based coding, the cooperative link consists in this case of separate radios *simultaneously* encoding and transmitting the same message in coordination: each relay (either TRs or RRs) are serving as a different virtual antenna for ST encoding during a fraction $\alpha_{DST} = 1/3$ of the available transmission duration T (see also figure 2). Outage probability $P_{out}^{(DST)}(\rho, R)$ has same expression as (10) with $\alpha = \alpha_{DST} = 1/3$. Transmit power during simultaneous transmissions is scaled to constrain the same energy consumption (compared to RB coding and direct MIMO transmission). Large SNR outage analysis can be dealt with in a similar way as done in Sect. II.

We now derive the conditions on the fading distributions of the cooperative links for which the V-MIMO system with repetition based coding (RB) or distributed ST coding (DST) can exhibit comparable or improved *outage* performances compared to direct MIMO transmission. Both systems are constrained to support an end-to-end rate of R with transmit power (or energy) ρ. The *cooperative fading regions* for repetition based (RB) $\mathcal{R}_{RB}$ and distributed ST coding (DST) $\mathcal{R}_{DST}$ V-MIMO protocols, are now the regions in terms of the number of relays (N_T, N_R), number of antennas for the reference direct MIMO transmission (M_T, M_R) and fading figures for which

$$\mathcal{R}_{RB}(\rho, R) = \{d_{st}, c_{st}, d_{rd}, c_{rd}, d_{tr}, c_{tr}, N_T, N_R, M_T, M_R : P_{out}^{(RB)}(\rho, R) < P_{out}^{(NC)}(\rho, R)\ \} \tag{18}$$

and

$$\mathcal{R}_{DST}(\rho, R) = \{d_{st}, c_{st}, d_{rd}, c_{rd}, d_{tr}, c_{tr}, N_T, N_R, M_T, M_R : P_{out}^{(DST)}(\rho, R) < P_{out}^{(NC)}(\rho, R)\} \tag{19}$$

Notice that

$$\lim_{\rho \to \infty} \mathcal{R}_{DST}(\rho, R) = \lim_{\rho \to \infty} \mathcal{R}_{RB}(\rho, R) = \mathcal{R}^{\infty} \tag{20}$$

as expected.

The regions $\mathcal{R}_{RB}(\rho, R)$ (18) and $\mathcal{R}_{DST}(\rho, R)$ (19) are specialized in figures 3 and 4 for the statistical parameters of Nakagami-m fading distributions. Diversities d and coding gains c are now computed from the asymptotic behavior of the Nakagami-m fading power MGF as $d = m$ and $c = (g/m) \times [\Gamma(m+1)]^{1/m}$. Outage performances can be similarly derived for other fading distributions (not detailed in this paper but it is just a straightforward extension).

Figures 3 and 4 we study the impact of the Nakagami fading factor for intra-cluster ($m_{st} = m_{rd}$) and for the inter-cluster (m_{tr}) links on the performances of V-MIMO system, by evaluating the conditions on the fading figures for which the V-MIMO system provide (outage) performance improvements compared to multiple-antenna (MIMO) transmission, for varying number of relays N_T, N_R and number of antennas for the reference direct ($M_T \times M_R$) MIMO transmission used as reference.

The cooperative fading regions are analyzed in figure 3 and 4 by comparing the performances of the V-MIMO protocols considered with respect to direct link with M_T and M_R transmit and receive antennas. Cooperative regions are illustrated as shaded areas in all figures to highlight the propagation settings (here evaluated in terms of the fading figures m of Nakagami-m PDFs) for relayed links where cooperation is beneficial in providing enhanced performances with respect to the multi-antenna case in figure 1-(b), used as reference. For both figures, the following assumptions have been made for the ease of exposition: *i)* the number of relay terminals is fixed $N_T = N_R = 2$; *ii)* direct (non-cooperative) MIMO transmission is performed over Rayleigh fading channels, $m_{sd} = 1$. Figure 3 compares the V-MIMO protocol with an Alamouti ST coded MIMO system ($M_T = M_R = 2$), while figure 4 shows the necessary condition for fading figures so that the V-MIMO protocol has same outage performances as if a ST coded MISO (single output) system would be employed with $M_T = 2, 3$ (and clearly $M_R = 1$).

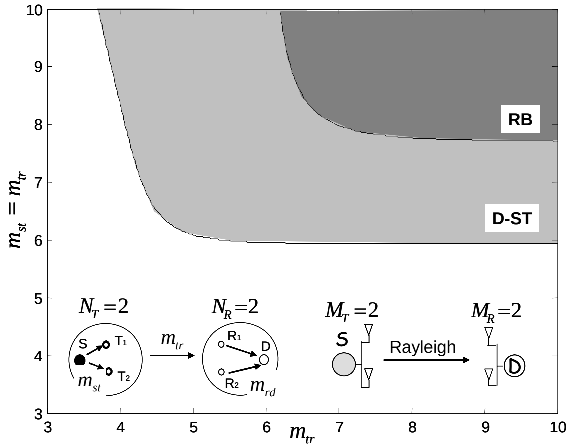

Fig. 3. Cooperative fading regions $\mathcal{R}_{RB}$ and $\mathcal{R}_{DST}$ for V-MIMO transmission ($P_{out} = 10^{-6}$, $R = 0.5$ [b/s/Hz], $N_T = N_R = 2$) for RB and DST coding with varying Nakagami factors for each link: $m_{st} = m_{rd}$ and m_{tr}. Region boundaries are in solid lines. The reference direct MIMO transmission ($m_{sd} = 1$) with $M_T = M_R = 2$ is shown below.

Assume that $m_{st} = m_{rd} = 6$ (that models short range communication [1]), figure 3 shows that a V-MIMO protocol that uses DST coding requires a fading figure $m_{tr} \geq 5$ for the inter-cluster link. Instead, the inefficiencies of the repetition based coding (RB) make the requirements for the fading statistics over each collaborative link more stringent ($m_{st} = m_{rd} \geq 8$ and $m_{tr} \geq 7$). Figure 4 shows that the conditions on the fading figures for each link become less stringent when outage performances are compared to those provided by non-cooperative MISO ST coded transmission. Still by assuming $m_{st} = m_{rd} = 6$, it is shown that a V-MIMO transmission with $N_T = N_R = 2$ can be regarded as a competitive option compared to a non-cooperative MISO system with $M_T = 3$ antennas when $m_{tr} \geq 1.5$ (for DST relaying) and $m_{tr} \geq 2$ (for RB coding). V-MIMO system can provide same outage performances as Alamouti ST coded transmission ($M_T = 2$) with single antenna receiver when $m_{tr} \geq 0.75$ (fading can be even more severe than Rayleigh).

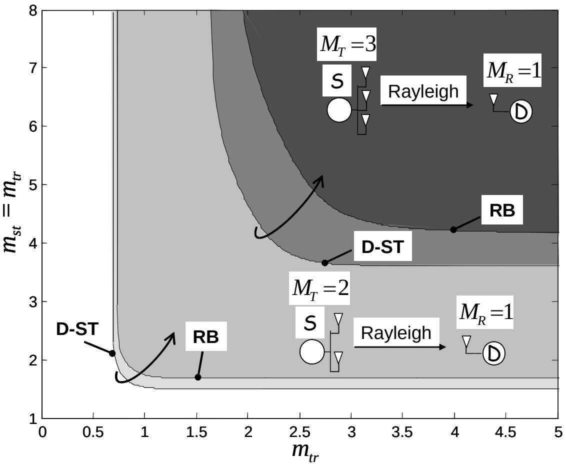

Fig. 4. Cooperative fading regions $\mathcal{R}_{RB}$ and $\mathcal{R}_{DST}$ for V-MIMO transmissions (for similar transmission parameters as in fig. 3). The reference non-cooperative transmission is now a MISO system ($m_{sd} = 1$) with $M_T = 2$ and $M_T = 3$ antennas.

V. CONCLUDING REMARKS

In this paper the transmission protocols are analyzed based on the collaboration of a number of relaying nodes that engage virtual MIMO transmissions employing repetition based (RB) coding or distributed space-time coding (DST). Since ad-hoc and sensor networks often experience propagation environments where the fading envelope distribution is largely different from Rayleigh PDF, benefits of cooperative transmission in terms of provided diversity and outage are analyzed for arbitrary distributed fading by evaluating the *cooperative fading regions*. These regions define the statistical propagation settings so that the V-MIMO transmission can be regarded as a competitive option compared to multi-antenna non-cooperative (MIMO) transmission. By limiting the analysis to simple and low-complexity transmission policies, it has been shown that relevant propagation settings exist where the V-MIMO protocol can exhibit comparable diversity/outage performances as for direct MIMO (and MISO) transmissions.

VI. APPENDIX

Outage performances in fading channels are primarily limited by the (outage) events that cause the SNR $\mu = \rho |h|^2$ to be small with probability that depends on the terms $f_{|h|^2}(w)|_{w\to 0^+}$. The probability density function $f_{|h|^2}(w)$ can be written through the integral expansion in [8] so that the probability density function can be written for w small enough as

$$f_{|h|^2}(w) = \Gamma(t^*+1)^{-1}\mathcal{D}^{t^*}\left[f_{|h|^2}(0)\right] w^{t^*} + o(w^{t^*}), \quad (21)$$

where t^* is the order of the first non-zero fractional derivative of $f_{|h|^2}(w)$ in $w \to 0^+$, $\mathcal{D}^{t^*}\left[f_{|h|^2}(0)\right]$, that satisfies $\lim_{\varepsilon\to 0^+}\int_0^{t^*-\varepsilon}\Gamma(t+1)^{-1}\mathcal{D}^t\left[f_{|h|^2}(0)\right] w^t dt = 0$. The outage probability is a function of t^* and scales with the power ρ as

$$\Pr[I(\rho) < R] \approx \left(2^R - 1\right)^d \times (c\rho \times \mathcal{A})^{-d}, \quad (22)$$

with diversity $d = t^* + 1$ and coding gain $c = \left(\mathcal{D}^{t^*}\left[f_{|h|^2}(0)\right]/\Gamma(t^*+2)\right)^{-1/(t^*+1)}$. We used the notation $\approx$ to indicate that the equality holds for $\rho \to \infty$.

For right-continuous distributions that can be adjusted to fit empirical measurements a *necessary and sufficient* condition for calculating t^* is

$$\phi = \lim_{s\to\infty} s^{t^*+1} F_{|h|^2}(s) > 0 \quad (23)$$

and finite (Proof can be inferred from the initial value theorem). The value of t^* satisfies (23) iff

$$d = t^* + 1 = -\lim_{s\to\infty}\left[\log \mathcal{F}_{|h|^2}(s)/\log s\right] > 0 \quad (24)$$

and finite. Diversity d is thus related to the order t^* of the first non-zero fractional derivative. The coding gain from (22) and (24) is $c = (\phi/\Gamma(d+1))^{-1/d}$ and ϕ is in (23). Extended results for distributed multi-antenna transmissions can be found in [8].

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