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Adaptive MT Time-Domain Processing

EM2.1

Umberto Spagnolini, Politecnico di Milano, Italy

SUMMARY

The spectral analysis of magnetotelluric (MT) data for impedance tensor estimation requires the stationarity of measured H-field and E-field and it is well known that noise biases the results.

A time domain technique that minimizes the residuals between the measured and the estimated E-data from H-data is more robust with respect to uncorrelated noise. The LMS tensor identification is achieved in time domain using an adaptive filter. In order to prevent strong spikes to slow down the convergence, a median filter is used in the feedback path of the adaptive identification.

An application to noisy synthetic examples will be presented.

INTRODUCTION

The MT method represents an interesting way to determine the electrical resistivity distribution in the subsurface from the measurement of electric and magnetic fields on the surface. Generally the dependency of E-field from H-field can be represented by the relation (Cantwell 1960):

$$Ex(\omega) = Zxx(\omega)\,Hx(\omega) + Zxy(\omega)\,Hy(\omega)$$

$$Ey(\omega) = Zyx(\omega)\,Hx(\omega) + Zyy(\omega)\,Hy(\omega).$$

The estimation of [Z] tensor from the measured field is carried out in the frequency domain (Swift 1967) where a short time stationarity is required to estimate the auto-power and cross-power. The noise on H-data and E-data has the effect to bias the tensor elements estimate when computed using the spectral technique (Sims et al. 1971). The effect of strong spiky-noise can be reduced if the tensor estimate is carried out in the time-domain (McMechan and Barrodale 1985). An adaptive approach that estimates the impedance impulse response using a parametric model has been previously proposed (Yee et al. 1988). The purpose of this paper is to present an adaptive time-domain technique that is robust with respect to noise and not too computer-consuming to be used directly in the field.

ADAPTIVE SYSTEM IDENTIFICATION

Consider two time series data: $\mathbf{X} = [x_1\, x_2 \dots x_N]^T$ and $\mathbf{Y} = [y_1\, y_2 \cdots y_N]^T$ (bold characters will be used to represent vector) where the vector $\mathbf{Y}$ is a noisy measurable output of the unknown system to identify and the vector $\mathbf{X}$ represents data input. The signal $\mathbf{Y}$ can be modeled as a linear combination of the input time series: $y_i = \sum f_j\, x_{i-j+1}$. It can also be written in vector notation: $y_i = \mathbf{F}^T\mathbf{X}_i$ where $\mathbf{F} = [f_1\, f_2 \dots f_M]^T$ represents the weight vector and $\mathbf{X}_i = [x_i\, x_{i-1} \dots x_{i-M+1}]^T$ is the data vector of length M extracted from the time series $\mathbf{X}$.

The single input and single output system identification resides in the estimation of the weight vector $\mathbf{F}$ that best predicts, in the LMS sense, the measured output data $\mathbf{Y}$ from input $\mathbf{X}$. The LMS solution of the system identification is the most widely used in the field of time domain signal processing. The MS error function:

$$\epsilon(\mathbf{F}) = E[(y_i - \mathbf{F}^T\mathbf{X}_i)^2] = E[y_i^2] + \mathbf{F}^T\mathbf{R}_{xx}\mathbf{F} - 2\mathbf{R}_{yx}\mathbf{F} \quad (1)$$

depends on the values of the weight vector $\mathbf{F}$. The LMS identified system is obtained minimizing the MS error (1) with respect to the weight vector $\mathbf{F}$. It can be easily recognized, in (1), that the square matrix $\mathbf{R}_{xx} = E[\mathbf{X}_i\mathbf{X}_i^T]$ is a Toeplitz matrix, the autocorrelation of $\mathbf{X}$ time serie. The vector $\mathbf{R}_{yx} = E[y_i\mathbf{X}_i^T]$ represents the cross-correlation between the $\mathbf{Y}$ and $\mathbf{X}$ time series.

The LMS solution carried out by the gradient computation of (1) is:

$$\mathbf{R}_{xx}\mathbf{F} = \mathbf{R}_{yx}. \quad (2)$$

The solution of the system identification has been developed in time domain and requires the $\mathbf{R}_{xx}$, $\mathbf{R}_{yx}$ estimation and the Toeplitz matrix $\mathbf{R}_{xx}$ inversion (the Toeplitz structure is advantageous because it requires only M element of storage and $\simeq M^2$ multiplications for inverse computation).

The adaptive identification searches the minimum of the error function (1) by gradient methods. In numerical problems the error function to be searched is given while in adaptive identification the error function $\epsilon(\mathbf{F})$ is unknown and must be estimated from data. This means that gradient methods simultaneously estimate the error function and optimize the weight vector $\mathbf{F}$ towards the minimum. The minimum search by method of steepest descent to find the weight vector at k+1 iteration is expressed by the following algorithm (Widrow et al. 1976):

$$\mathbf{F}_{k+1} = \mathbf{F}_k - \mu\frac{\partial\epsilon(\mathbf{F}_k)}{\partial\mathbf{F}} = \mathbf{F}_k + 2\mu(y_k - \mathbf{F}_k^T\mathbf{X}_k)\mathbf{X}_k \quad (3)$$

in which μ is the constant that regulates the step size, the rate of convergence and the stability of adaptation. The rate of convergence depends on the eigenvalues spread of the autocorrelation matrix $\mathbf{R}_{xx}$ and, to improve the convergence, it is advisable to prewhiten the input data series before the adaptive identification. One of the most interesting properties of the adaptive identification is the ability to operate in a nonstationary environment (Widrow et al. 1976). In the MT identification the tensor is supposed to be stationary so that the weight vector should converge to a unique solution. Measured magnetic and electric data are nonstationary and noisy so that the step size should be chosen for a low adaptation (small μ) in order to reduce the influence of noise on the system identification. The gradient search method of (3) can be very sensitive to spiky-noise. An isolated and high value spike in the output data set y_k can strongly move the weight vector $\mathbf{F}_k$ even if a low adaptation has been selected. Removing spikes in the output data set requires the knowledge of the statistical properties of y_k. The identification error $(y_k - \mathbf{F}_k^T\mathbf{X}_k)$ should be uncorrelated to the input data $\mathbf{X}$ when the minimum has been reached; this means that the error can be modeled as a nonstationary gaussian process. This assumption allows to use a non-linear median filter in the adaptation loop to prevent spikes instability. The steepest descent can be modified:

$$\mathbf{F}_{k+1} = \mathbf{F}_k + 2\mu M[(y_k - \mathbf{F}_k^T \mathbf{X}_k)]\mathbf{X}_k \tag{4}$$

where $M[.]$ represents the median filter applied to the identification error that would reduce the spiky step size.

ADAPTIVE MT TENSOR IDENTIFICATION

The LMS adaptive identification has been used for MT tensor estimation. In a 1-D case where $Z_{xx}(\omega)$ and $Z_{yy}(\omega)$ tend to zero, the adaptive identification can be applied straight away because the signals can be modeled as single input single output system. The weight vectors **Zyx** and **Zxy**, when the minimum of the identification error has been reached, represent the impulse response of finite length filters that best predict, in LMS sense, the measured E-data from H-data. Formula (4) can be applied separately on two identifications and, at the end of data set or at the convergence, the weight vectors can be Fourier transformed to recover the MT tensor as a function of frequency.

For 2-D or 3-D cases two systems composed by double input and single output should be identified. The identification error, for the **Ey** data set in the time domain, can be computed as $\epsilon_{y_k} = [e_{y_k} - \mathbf{Zyx}_k^T \mathbf{Hx}_k - \mathbf{Zyy}_k^T \mathbf{Hy}_k]$. The minimum search leads to the following steepest descent algorithm:

$$\mathbf{Zyx}_{k+1} = \mathbf{Zyx}_k + 2\mu M[\epsilon_{y_k}]\mathbf{Hx}_k \tag{5}$$

$$\mathbf{Zyy}_{k+1} = \mathbf{Zyy}_k + 2\mu M[\epsilon_{y_k}]\mathbf{Hy}_k \tag{6}$$

where $M[.]$ is the error median filter. The **Zxy** and **Zxx** can be computed simply duplicating the adaptive structure. The complete block diagram is presented in Fig.1. At convergence the LMS MT tensor has been identified. An unique and stable solution can be reached if the two H-fields data sets are uncorrelated; in this case the identification evolves separately on each sub-system. Good results are achieved even when the H-field data sets show short time correlation compared to slow adaptation.

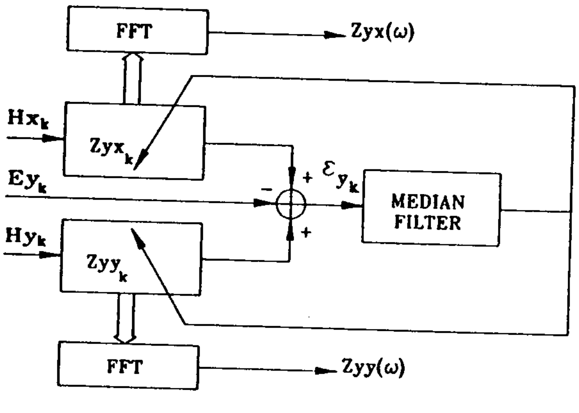

Fig.1 Block diagram of MT identification.

EXAMPLES

I have tested the adaptive identification under different conditions of noise for a synthetic model. The $Zyx(\omega)$ and $Zyy(\omega)$ that I have chosen as model are shown in Fig.2 where data sampling is 5Hz. The model has been used with real H- data so that the -20 dB/decade of reduction of telluric power spectra for this frequency range reduces the convergence rate of adaptive identification. A real H-data set has been used to test also the adaptive identification under a non-imposed correlation in the input data set. In Fig.3 the 2-D identification for a noise-free case achieves good results compared to the model chosen while in Fig.4 the results for different conditions of noise are presented ((a.): 5% gaussian noise, (b.): 1% of random spikes with gaussian amplitude and (c.): 20% gaussian noise and 1% of random spikes). The $Z_{yx}(\omega)$ and $Z_{yy}(\omega)$ have been recovered efficiently with low error after having applied the adaptive identification over 17000 data samples.

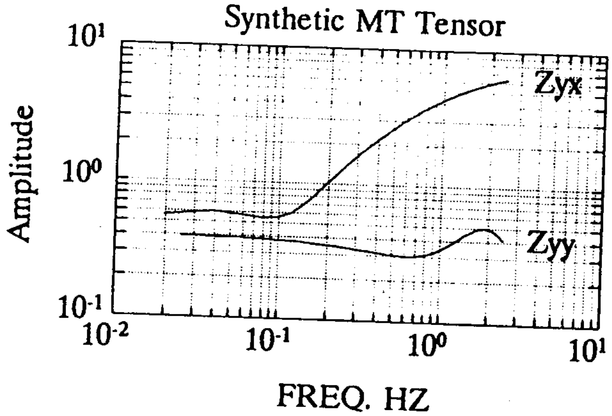

Fig.2 Synthetic MT tensor.

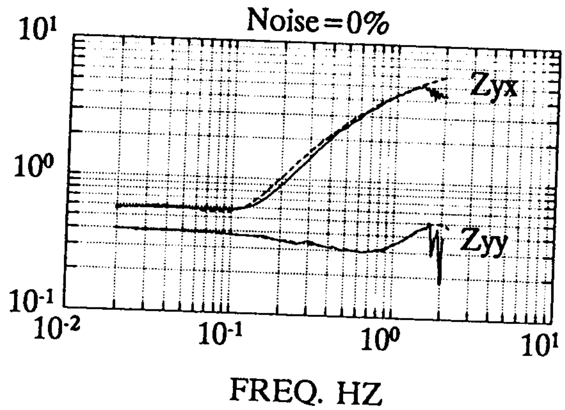

Fig.3 Adaptive noise-free identification (solid) vs. synthetic (dashed).

DISCUSSION

Adaptive MT tensor identification has been presented. This method, because it performs a time domain processing of MT data sets using a median filter, allows a less noise sensitive identification of the impedance tensor than spectral techniques. On the other hand a prewhitening filter is required in the frequency range where H-data autocorrelation matrix presents high eigenvalues spread. A high correlation between the two H-time series can lead to a wrong tensor identification. An approximate orthogonalization of the H-data set could improve the results even under these severe circumstances.

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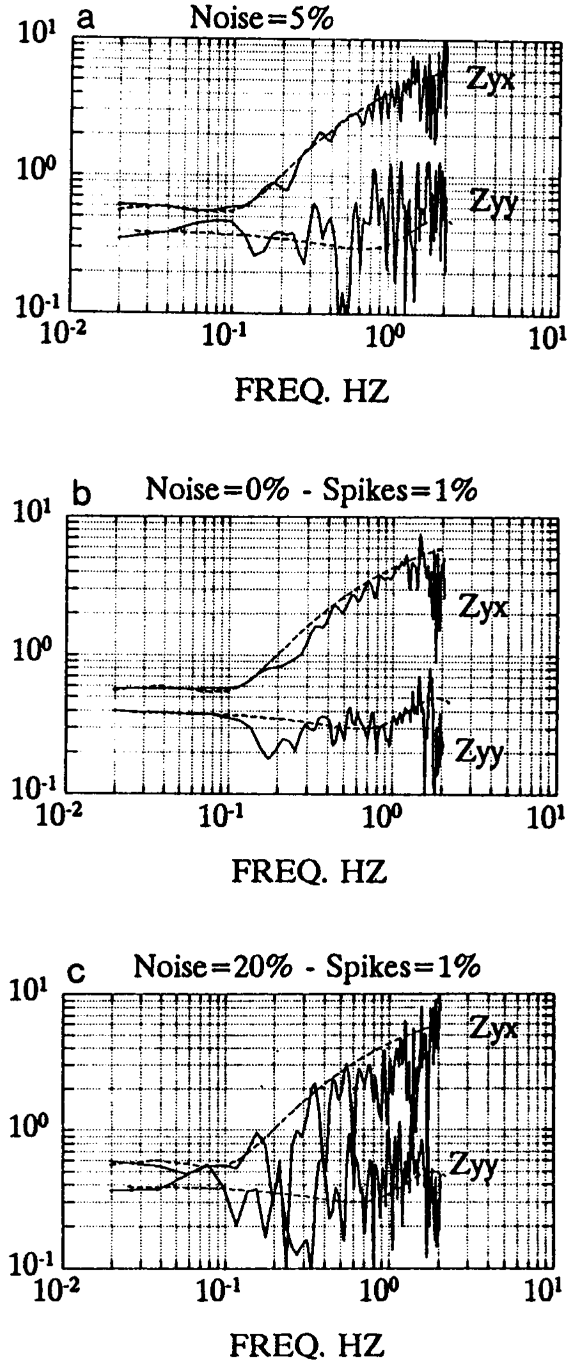

Fig.4 Adaptive identification for different conditions of noise (solid) vs. synthetic (dashed).