How Useful Is Amplitude-Versus-Offset (AVO) Analysis?

Joint SEG/EAEG Summer Research Workshop

August 9-14, 1992
Big Sky, Montana, USA

TECHNICAL PROGRAM AND ABSTRACTS

A Presentation of the Society of Exploration Geophysicists
and the
European Association of Exploration Geophysicists

Degrees of freedom and interaction of AVO and velocity information

Umberto Spagnolini

Dip. di Elettronica e Informazione - Politecnico di Milano
Piazza L. da Vinci 32, I-20133 Milano (Italy).

Abstract

The number of independent parameters necessary to describe an isolated reflection with AVO and residual velocity error is determined. A statistical analysis allows to identify an ambiguity among these parameters that cannot be solved without geological a priori information. As an example, an extreme medium totally implausible geologically but kinematically equivalent is derived.

Introduction

An accurate velocity estimation is essential to seismic data post–stack and pre–stack processing i.e. prestack depth migration.
Pre–stack analysis allows the specific estimation of elastic parameters (velocities P and S, density) which require a nonlinear elastic inversion. Although several authors (Tarantola, 1986; Kolb et al. 1986) have already examined the problem of elastic parameter estimation, there is still a need to fully understand the limits and reliability of the inversion technique. An alternative approach to elastic parameter estimation (cfr. full elastic inversion) lies in Amplitude and Phase vs. Offset (AVO and PVO) measurements. Owing to AVO dependence on physical parameters (e.g. Zoeppritz equations) the AVO inversion searches for optimal elastic parameters that fit AVO measurements. When compared to kinematic and full elastic inversions of the interface, AVO analysis and inversion can be considered as an intermediate approach. However it is to be noted that the inversion of noise free AVO measurements has more than one solution (Drufuca and Mazzotti, 1991). The need to use not only P–P reflections but also P–S, S–S and S–P reflections to improve the reliability of the interface inversion was discussed by de Haas and Berkhout (1990), Van Rijssen and Herman (1991). In fact the reliability of the inversion is greatly reduced by AVO measurement errors. Single interface AVO measurement requires a much more accurate kinematic model than does seismic imaging. However there is no guarantee that the proposed model is the real one just because the wavefield obtained from the inversion modeling matches the data.

A key role in AVO analysis for elastic parameter estimation is played by velocity analysis; even small errors can cause quite relevant inaccuracies in the estimation of the proper AVO. It was shown by Spratt (1987) and Walden (1990) that velocity residual, or Residual Normal Move Out (RNMO), reduces the reliability of AVO measurement and the inversion lacks the expected accuracy. NMO correction and stretching reduces the accuracy of AVO measurements (Herbert, 1990) so it is advisable when AVO measurements are employed to correct moveout statically.
The basic assumptions of AVO analysis are that: i) the bandwidth of the wavelet must be considered when estimating the reflection moveout; ii) an analysis is made of the isolated reflections (signal muting can reduce reflection interference).

This paper presents a statistical analysis of the interaction, due to velocity errors, between AVO and RNMO. Any elastic reflection (assumed here to have a physical AVO behavior), unknown deterministically but characterized by a known statistical distribution, is decomposed into reflections with and without AVO. In such a simple description the residual is minimized and could correspond to different media but no inversion technique is able to differentiate the media. The analysis was carried out for full bandwidth wavelets (i.e. Ricker wavelets) adding the reflection delay.
If, in a LMS sense, any elastic event can be further decomposed e.g. into two or more reflections with AVO and with distinguishable velocity parameters, then the AVO information is no longer reliable for the extraction of the elastic parameters of the interface. In fact the basic assumption for any AVO analysis and interface inversion is to have a single event with a well determined velocity. It is demonstrated here that the AVO inversion is ambiguous, not only in the physical parameter subspace (velocities P and S, and density) but also in the kinematic subspace.

Elastic Reflections Model

Consider the Common Midpoint Gather (CMG) of the elastic reflections of a horizontally layered model of earth. A few simplifications of the P-P reflections are examined for the purpose of obtaining a simple and useful relationship between AVO and velocity errors.

- The hyperbolic moveout of each reflection depends on the normal incidence traveltime τ_0 and on the velocity v_0. Only short offsets are considered so that the parabolic moveout approximation holds true

$$t(x) \simeq \tau_0 + \frac{x^2}{2\tau_0 v_0^2} = \tau_0 + p_0 x^2. \qquad (1)$$

The residual normal moveout (RNMO), rather than the moveout of a CMG after correction for constant velocity, is approximated by the parabolic moveout. The reflection times can be represented by a Taylor series of the form $t^2(x) = \tau_0^2 + (1/v_0^2)x^2 + c_2x^4 + c_3x^6 + ...$, where $c_2, c_3, ...$ are coefficients. Obviously, a simplification of the reflection time polynomial is considered here.

- The amplitude of the reflections vs. offset x (AVO) are approximated by the relationship $A + Bx^2 = A(1 + qx^2)$. Assuming that only small incidence angles are considered ($< 30^0$) the P–P reflection coefficient ($R_{pp}(\theta)$) depends on the P–P and S–S normal incidence reflectivity (Spratt, 1987) $R_{pp}(\theta) \simeq R_{pp}(0) + 2[R_{pp}(0) - R_{ss}(0)]\sin^2\theta$. The zero offset amplitude A depends on the acoustic impedance contrast whereas the S–S normal incidence reflectivity can be estimated from the AVO parameter q. The kinematic inversion of V_P, obtained from the moveout measurements (i.e. velocity analysis), provides only an approximate estimate of P–P normal incidence (apart from density ρ).
  Therefore, pre–stack AVO analysis allows the estimation of interface elastic parameters (velocities P and S, density); the normal incidence or the short offset reflections are mainly useful for subsurface imaging (post–stack or P–P kinematic section) or image processing (i.e. migration).
- The seismic wavelet $w(t)$ considered here has no phase shift vs. offset (PVO). The reflection model chosen deals with narrow, limited and full bandwidth wavelets.

The CMG seismic signal $s(x,t)$ can be obtained as a linear combination of elastic reflections:

$$s(x,t) = \sum_i A_i(1+q_ix^2)w(t-\tau_i-p_ix^2)+\epsilon(x,t), \quad (2)$$

The residual $\int |\epsilon(x,t)|^2\, dt\, dx$ is used in the estimation of the parameters of the equivalent model of the i-th reflection:

$(\tau,p)_i$ are used in the estimation of the equivalent kinematic model, i.e. V_p velocity and layer thickness;

$(A,q)_i$ depends on the interface elastic parameters (the kinematic model, density ρ and V_s velocity).

The interaction between RNMO and AVO of NMO corrected elastic reflection in the (x^2,t) domain can be analyzed in the Fourier transformed domain (k_{x^2},ω). The spectrum of an isolated reflection with AVO (Fig.1 (a)) is ambiguous and could be considered as the superimposition of two or more events, each at a

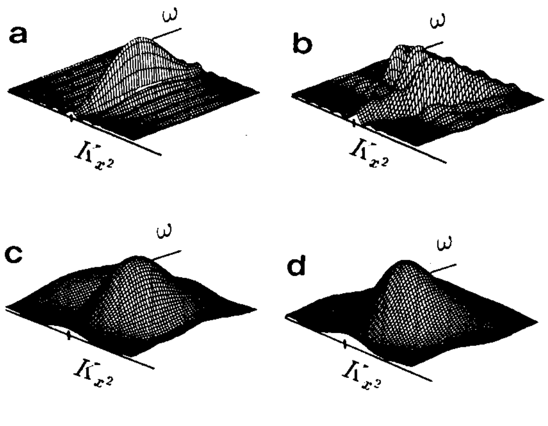

Figure 1: 2D Fourier transform of AVO and RNMO

constant value of the wavenumber k_{x^2} (lateral bands). The spectrum of two or more reflections with different RNMO, and without any amplitude behavior vs. offset, contains the spectra of each reflection on separate slopes, each slope depending on the reflection RNMO parameter p_i (Fig.1 (b)). The limited time resolution depends on the wavelet bandwidth whereas the aperture limits the RNMO resolution as shown in Fig.1 (c)-(d), this leads to ambiguity in the spectra. Owing to this spectra ambiguity, a strong interaction occurs between the AVO and RNMO. In other words the spectrum of an isolated reflection with AVO can be interpreted as a superimposition of two or more reflections without AVO. Moreover the velocity errors are an additional cause of further ambiguity.

Eigenstructure Decomposition

In seismic data the elastic reflections of narrow bandwidth were considered so as to obtain the interaction of AVO and RNMO; furthermore the number of the degrees of freedom of uncertain AVO and RNMO for elastic reflections was evaluated.

After NMO correction the time windowed CMG narrow band elastic reflection is characterized by normalized RNMO (p_i) and AVO (q_i) at frequency ω_0; sampling along the offset square x^2, gives the data vector

$$\underline{a}(p_i,q_i) = A_i[(1+q_ix_1^2)e^{j\omega_0 p_i x_1^2},\ ...,\ (1+q_ix_n^2)e^{j\omega_0 p_i x_n^2}]'. \quad (3)$$

The offset is normalized to the cable length ($x_{max}^2 = 1$) and the maximum considered RNMO has a phase difference of $\omega_0 p_{max} x_{max}^2 = \pi$ over the cable length so that the normalized RNMO parameter in now limited $|p_i| \leq 1$.

Each elastic reflection in the time window is characterized by a RNMO and an AVO behavior that is not deterministically known a priori; it can be modeled by its statistical distribution. In other words,

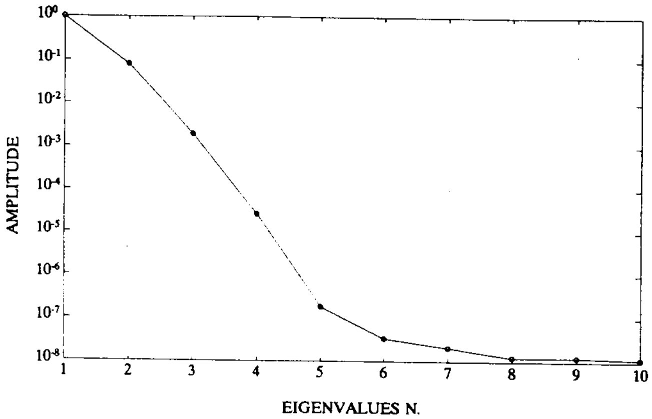

Figure 2: Eigenvalues of the covariance matrix for the uniform distribution of AVO and RNMO. From the not negligible eigenvalues, the number of degrees of freedom for this model is N=5.

let us suppose the probability densities of AVO $g(q)$ and RNMO $f(p)$ are known. The covariance matrix for independent events (the normal incidence reflection coefficients are taken as statistically independent variables i.e. $\overline{A_i} = 0$; $\overline{A_i A_j^*} = 0$ if $i \neq j$; $\overline{|A_i|^2} = 1$) is

$$\underline{\mathbf{R}} = \int\int \underline{\mathbf{a}}\,\underline{\mathbf{a}}^* \, f(p)\, g(q)\, dp\, dq = \mathrm{E}[\underline{\mathbf{a}}\,\underline{\mathbf{a}}^*] \qquad (4)$$

where E is the expectation. The number of not negligible eigenvalues of $\underline{\mathbf{R}}$ indicates the number of degrees of freedom of the data i.e. the number of independent parameters that can be used to model the data. In fact the eigenvectors (or *eigenstructures*) $\underline{\mathbf{v}}_i$, $i = 1, ..., N$ of the covariance matrix $\underline{\mathbf{R}}$ completely span the data subspace, provided that the eigenvalues are $\lambda_1 > \lambda_2 > .... > \lambda_N > \lambda_{N+1} \simeq \lambda_{N+2} \simeq .... \simeq 0$ ($Rank\{\underline{\mathbf{R}}\} = N$). Each event considered in the modeled distribution of AVO and RNMO can be obtained as a linear combination of the eigenstructures spanning the data subspace:

$$\underline{\mathbf{a}}(\overline{p}, \overline{q}) = \sum_{i=1}^{N} \beta_i \underline{\mathbf{v}}_i + \underline{\varepsilon}(\overline{p}, \overline{q}) \qquad (5)$$

so that the residual $\| \underline{\varepsilon}(\overline{p}, \overline{q}) \|^2 \simeq 0$.

Considering a uniform (i.e. equally likely) distribution of AVO and RNMO between the two extreme values ($| q_{max} | \leq 1$ and $| p_{max} | \leq 1$), the number of degrees of freedom is N=5 as shown in Fig.2 where the eigenvalues' ratio is represented. Increasing either the maximum offset or the RNMO p_{max} in the a priori distribution, increases the number of degrees of freedom. The linear slope of N vs. the maximum offset or RNMO ($\frac{dN}{dx^2_{max}} = \frac{dN}{dp_{max}} \simeq 1$) is evident from the number of not negligible eigenvalues of the covariance matrices corresponding to increasing RNMO spread (Fig.3(a)). Obviously the number of detectable events, and their resolution, increases with the cable length. The number of degrees of freedom N is more or less independent of the a priori q distribution. This has been shown in Fig.3(b) where the eigenvalues of the covariance matrices have been represented for increasing q_{max}. This suggests that AVO information can be mistaken for the RNMO. Eigenstructures are an orthogonal basis of events but they have little physical meaning. The importance of each eigenstructure can be appreciated from, for instance, the evaluation of the MS error

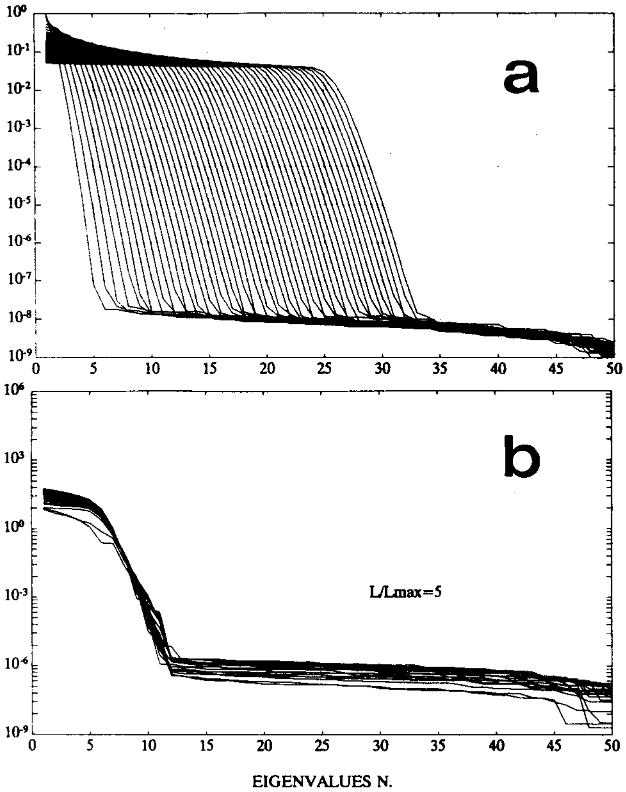

Figure 3: Eigenvalues of covariance matrix $\underline{\mathbf{R}}$: (a) for a uniform distribution of AVO and increasing RNMO (p_{max}) or equivalently maximum offset (normalized RNMO step is 0.5); (b) for a uniform distribution of RNMO ($| p_{max} | \leq 1$) and increasing AVO ($-1 \leq q \leq 1 + \delta q$; δq =0, 1, 2, ...). From the eigenvalue analysis it is evident that the number of degrees of freedom $N(p_{max})$ increases almost linearly with RNMO (a), but is almost independent of the AVO (b).

$$\| \underline{\varepsilon}(p, q) \|^2 = \| \underline{\mathbf{a}}(p, q) - \sum_{i=1}^{N} \beta_i \underline{\mathbf{v}}_i \|^2, \qquad (6)$$

with respect to the decomposition N and the parameters (p, q).

Narrowband Elastic Reflections Decomposition

Eigenstructures span the narrowband data subspace provided their AVO and RNMO have been modeled; therefore, eigenstructures are unusable for seismic inversion due to their poor physical meaning. An alternative and more physical approach is the decomposition of the elastic reflections previously modeled into a nonorthogonal basis of elastic reflections. The narrowband data subspace could be spanned on the basis of reflections corresponding to events characterized by a RNMO and an AVO or by reflections with no AVO, thus having RNMO only.

Considering the basis with both RNMO and AVO, each vector $\underline{\mathbf{a}}_i$ depends only on p_i and q_i (i.e. $\underline{\mathbf{a}}_i = \underline{\mathbf{a}}(p_i, q_i)$), thus the basis becomes

$$\underline{\mathbf{T}} = [\underline{\mathbf{a}}_1, \underline{\mathbf{a}}_2, ..., \underline{\mathbf{a}}_k]. \quad (7)$$

Any narrowband elastic reflection $\underline{\mathbf{a}}(p,q)$ modeled from the relationship (3) can be decomposed into the nonorthogonal basis

$$\underline{\mathbf{a}}(p,q) \simeq \sum_{i=1}^{k} A_i(p,q)\underline{\mathbf{a}}_i = \underline{\mathbf{T}}\,\underline{\alpha}(p,q). \quad (8)$$

The MS error

$$\| \underline{\varepsilon}(p,q,\underline{\mathbf{T}}) \|^2 = (\underline{\mathbf{T}}\,\underline{\alpha}(p,q) - \underline{\mathbf{a}}(p,q))^* (\underline{\mathbf{T}}\,\underline{\alpha}(p,q) - \underline{\mathbf{a}}(p,q)) \quad (9)$$

depends on the choice of the weight vector $\underline{\alpha}(p,q)$. The optimum choice is obtained by minimizing the MS error (9) with respect to the weight vector so that the LMS error for each event is obtained. From the gradient computation:

$$\frac{\partial \| \underline{\varepsilon}(p,q,\underline{\mathbf{T}}) \|^2}{\partial \underline{\alpha}(p,q)} = 0 \quad (10)$$

the LMS error becomes

$$\| \underline{\varepsilon}(p,q,\underline{\mathbf{T}}) \|^2_{min} = \underline{\mathbf{a}}^*(p,q)(\underline{\mathbf{I}} - \underline{\mathbf{P}})\underline{\mathbf{a}}(p,q). \quad (11)$$

The matrix $\underline{\mathbf{P}} = \underline{\mathbf{T}}(\underline{\mathbf{T}}^*\underline{\mathbf{T}})^{-1}\underline{\mathbf{T}}^*$ corresponds to the projection of $\underline{\mathbf{a}}(p,q)$ onto the subspace spanned by the nonorthogonal basis $\underline{\mathbf{a}}_i$. The MS error for an optimal choice is better expressed as:

$$\| \underline{\varepsilon}(p,q,\underline{\mathbf{T}}) \|^2_{min} = \mathrm{Tr}[(\underline{\mathbf{I}} - \underline{\mathbf{P}})\underline{\mathbf{a}}(p,q)\underline{\mathbf{a}}^*(p,q)(\underline{\mathbf{I}} - \underline{\mathbf{P}})]. \quad (12)$$

Once (p,q) have been defined, the MS error (12) depends on the event and on the basis $\underline{\mathbf{T}}$. To evaluate the MS error for an a priori statistical distribution the previously considered uniform distribution of AVO and RNMO was used; the MS error becomes:

$$\| \underline{\varepsilon}(\underline{\mathbf{T}}) \|^2 = \mathrm{Tr}[(\underline{\mathbf{I}} - \underline{\mathbf{P}})\underline{\mathbf{R}}(\underline{\mathbf{I}} - \underline{\mathbf{P}})]. \quad (13)$$

The residual depends on the covariance matrix $\underline{\mathbf{R}}$ for a known distribution once the nonorthogonal basis with RNMO and AVO has been chosen.

The next step implies the use of the a priori information in order to define the covariance matrix. This was to find the optimal combination of the basis events $\underline{\mathbf{a}}_i$ that minimize the MS residual (13) that depends on the covariance matrix. From (13), the minimization is obtained from the gradient computation:

$$\frac{\partial \| \underline{\varepsilon}(\underline{\mathbf{T}}) \|^2}{\partial(p_1, p_2, ..., p_k)} = 0. \quad (14)$$

N. Events	p value NO-AVO	p value AVO
1 Event	0	0
2 Events	±0.5100	±0.6460
3 Events	0; ±0.7621	0; ±0.8210
4 Events	±0.3706; ±0.8875	±0.3280; ±0.8375

Table 1: Normalized velocity sampling of nonorthogonal narrowband basis with and without AVO that optimizes the MS error for the assigned a priori distribution (equally likely elastic reflections).

In order to take into account the effect of a limited signal to noise ratio, the eigenvalue spread of the $\underline{\mathbf{T}}$ matrix is limited to 30 dB; this corresponds to the eigenvectors used for the evaluation of $\underline{\mathbf{P}}$ in (14) [1]. In this way the optimum sampling for AVO and no AVO in the RNMO parameter p is found by minimizing the MS error. The RNMO sampling in Table 1, obtained for events with parabolic amplitude behavior, can be analogously obtained for events without AVO (events with $q = 0$). In the same way Fig.4 shows the LMS error for the optimum velocity sampling as a function of the number of events with AVO that would span the data subspace corresponding to the previously considered distribution of velocities and AVO. The same figure shows the LMS error for the events without AVO. It has been shown that any distribution of events pertaining to the data set characterized by $\underline{\mathbf{R}}$ could also be described with: either 1 event with AVO or with 2 events without AVO, with 2 events with AVO or with 4 events without AVO. These descriptions correspond to different media but no inversion technique can differentiate them without further information.

Seismic Data Decomposition

An analysis of the narrowband is a valuable guideline for a more physical decomposition of full bandwidth CMG. If one elastic reflection at normal incidence traveltime τ_0 and moveout p_0 is considered the traveltimes of the basis of N elastic reflections are

$$t_i(x) = \tau_0 + \delta\tau_i + (p_0 + \delta p_i)x^2 = \tau_0 + p_0 x^2 + \delta t_i(x). \quad (15)$$

The basis of full bandwidth elastic reflections contains the time dependency that was neglected in the narrowband elastic decomposition. For the sake of simplicity it is convenient to define the basis vector as

$$\underline{\mathbf{T}}(x, t, \underline{\delta\tau}, \underline{\delta p}) = \{w(t - \delta t_1(x)), ..., w(t - \delta t_N(x)),$$

[1] The computation of the projection matrix $\underline{\mathbf{P}}$ is obtained from the SVD of the matrix $\underline{\mathbf{T}}$: $\underline{\mathbf{U}}^*\underline{\mathbf{T}}\underline{\mathbf{V}} = \underline{\mathbf{\Lambda}}$ and the singular values spread of 30dB defines the singular values matrix $\underline{\mathbf{\Lambda}}_S$ that corresponds approximately to that signal to noise ratio. The factorization of $\underline{\mathbf{T}}$ becomes:

$$\underline{\mathbf{T}} = \underline{\mathbf{U}}_S \underline{\mathbf{\Lambda}}_S \underline{\mathbf{V}}_S^*$$

and the projection matrix, from SVD is

$$\underline{\mathbf{P}} = \underline{\mathbf{U}}_S \underline{\mathbf{U}}_S^*.$$

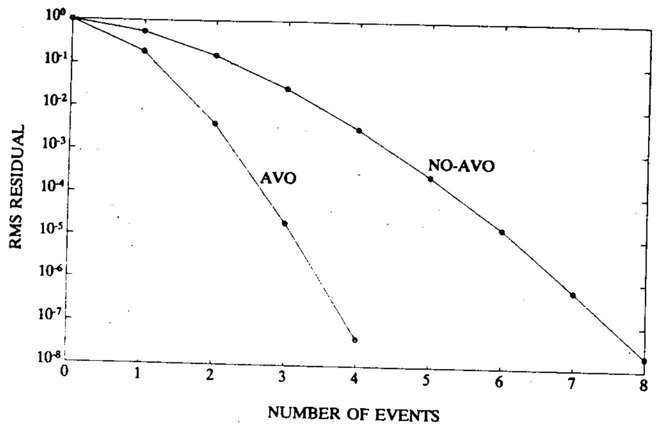

Figure 4: MS residual as a function of the number of events representing the data set. The a priori distribution can be described by 2 events with AVO, or with 4 events without AVO, obtaining a comparable negligible residual.

$$x^2 w(t - \delta t_1(x)), ..., x^2 w(t - \delta t_N(x))\}'. \quad (16)$$

The MS error depends on the basis time reference $\underline{\delta\tau} = \{\delta\tau_1, \delta\tau_2, ..., \delta\tau_N\}'$ and $\underline{\delta p} = \{\delta p_1, \delta p_2, ..., \delta p_N\}'$ and on the amplitude coefficients $\underline{\alpha} = \{A_1, ..., A_N, B_1, ..., B_N\}'$ (acoustic reflection coefficient A_i and AVO B_i):

$$\| \epsilon(\underline{\delta\tau}, \underline{\delta p}) \|^2 = E_s + \underline{\alpha}' \, \underline{\Phi}(\underline{\delta\tau}, \underline{\delta p}) \, \underline{\alpha} - 2\underline{\alpha}' \, \underline{\Psi}(\underline{\delta\tau}, \underline{\delta p}), \quad (17)$$

where $E_s = \int\int | s(x,t) |^2 \, dt \, dx$ represents the signal energy. The symmetric positive definite matrix, i.e. basis correlation matrix, $\underline{\Phi}(\underline{\delta\tau}, \underline{\delta p}) = \int\int \underline{\mathbf{T}}'\underline{\mathbf{T}} \, dt dx$ (the basis vector (x,t) dependence is understood) depends on the basis time sampling as well as on the crosscorrelation vector between the basis and the data $\underline{\Psi}(\underline{\delta\tau}, \underline{\delta p}) = \int\int s(x,t)\underline{\mathbf{T}} \, dt dx$. The optimal weight vector can be estimated from a linear inversion; nevertheless, the optimal time sampling of the basis should be evaluated using an optimization stage. The optimal velocity sampling, estimated for equally likely reflections in the narrowband analysis, can be extrapolated to the limited bandwidth reflections. Obviously the analytical signals should be considered taking into consideration the Hilbert transform of limited bandwidth data.

Consider the synthetic CMG elastic reflections of center frequency $f_0 = 30Hz$ and raised cosine bandwidth $\Delta f = 20Hz$: the time window is centered on $\tau_0 = 512ms$, the average stacking velocity is $v_0 = 4.0Km/s$ and the receiver sampling is $dx = 50m$ while the maximum offset is $x_{max} = 2450m$. Any limited bandwidth elastic reflection, or any combination, is decomposed into a nonorthogonal basis. The residual $\int\int | \varepsilon(x,t) |^2 \, dt \, dx$ of the decomposition into reflections without AVO of isolated narrow bandwidth elastic reflection is reported in Fig.5. By increasing the number of events the basis spans the data subspace more accurately.

A synthetic elastic model (dashed) is shown in Fig. 6; using a 60Hz Ricker wavelet each reflection has been decomposed into three elastic reflections with parabolic AVO. The equivalent kinematic model

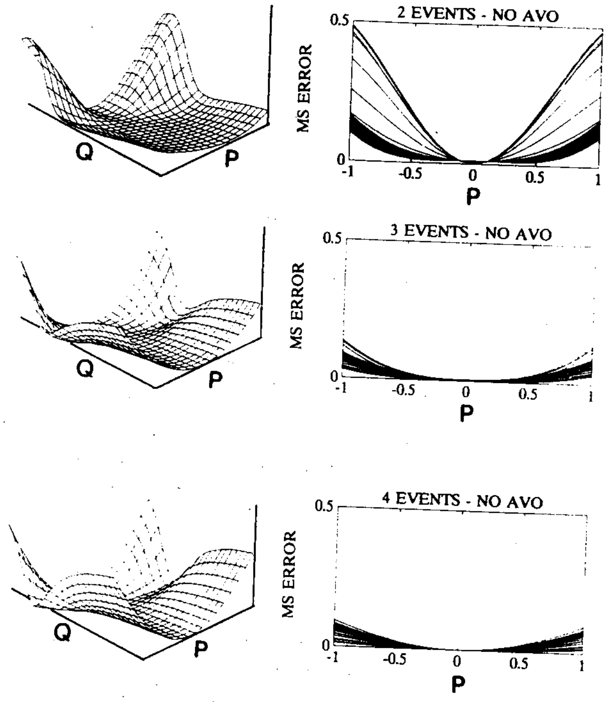

Figure 5: Nonorthogonal decomposition of elastic reflections using limited bandwidth reflections without AVO. Residuals vs. 2, 3, 4 events decomposition of elastic reflections with AVO (normalized q) and RNMO (normalized p) (right: residual projection on the RNMO parameter). Increasing the nonorthogonal basis, the residuals decrease as predicted for narrow band example in Fig.4.

(solid) is able to describe the muted CMG within 2–3% of residual. The accepted velocity error of each reflection is approximately 0.4%. The equivalent signal model (i.e. made by reflections used to decompose the isolated reflection with RNMO) corresponds to a combination of three reflections with basis traveltimes (15), such a combination depending on the effective Ricker wavelet bandwidth. This extreme equivalent kinematic media could be geologically meaningless, due to impossible density contrast, but no inversion technique could tell the difference between them if the MS error was the only discriminating factor.

Conclusions

The unreliability of the MS solution of AVO analysis and inversion was explored and the importance of geological information revalued. It was found that seismic elastic reflections can be replaced by reflections

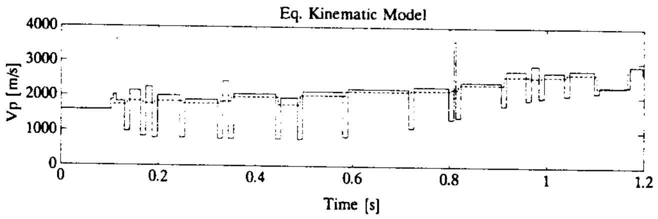

Figure 6: Full bandwidth (60Hz Ricker wavelet) synthetic example of the elastic model (dashed) where each reflection has been decomposed into 3 kinematically equivalent elastic reflections (solid). Even if the equivalent model could be geologically meaningless, the inversion is unable to differentiate between the solutions in the kinematic subspace without any a priori information.

without AVO and this makes the reliability of AVO inversion questionable. Furthermore it was shown that AVO does not generally increase the number of degrees of freedom of the data. It was demonstrated that any combination of narrowband elastic reflections can be decomposed into either K AVO reflections or 2K reflections without AVO, the residuals being comparable. The decomposition of the narrowband into reflections with and without AVO has been extrapolated to a limited bandwidth that can be compared with the bandwidth of practical seismic data. The more realistic Ricker wavelet was also examined in a synthetic example, this led to a kinematic (but perhaps not geologically) equivalent medium.
In the kinematic subspace the equivalent medium which can be obtained is not unique; in fact, only the geological a priori information, or other geophysical measurements (e.g. well logs), can identify the relevant solution in the ambiguous subspace.

Acknowledgements

The author is most grateful to Fabio Rocca for his encouragement, guidance and many suggestions, all of which were indispensable to the development and completion of this work.
The work was partially funded by AGIP SpA (Italy) within the framework of the Project Geoscience sponsored by the CEC, DG XII, Contract JOUF 0037.

Bibliography

Drufuca,G. and Mazzotti,A., 1991, Null spaces in AVO inversion of single interfaces, 53rd E.A.E.G. Meeting, Florence, Expanded Abstracts.

de Haas,J.C., and Berkhout,A.J., 1990, On the information content of P-P, P-SV, SV-SV and SV-P reflections, 58th Ann. Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 1190-1194.

Herbert,W.S., 1990, Amplitude versus Offset analysis in a finely layered media: 58th Ann. Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 1195-1198.

Kolb,P., Collino,F., and Lailly, P., 1986, Prestack inversion of a 1-D medium, Proceeding of IEEE, v.74, n.3, 498-508.

Spratt,S., 1987, Effect of Normal Moveout errors on amplitude versus offset-derived shear reflectivity: 57th Ann. Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 634-637.

Tarantola,A., 1986, A strategy for nonlinear elastic inversion of seismic reflection data, Geophysics **51**, 1893-1903.

Van Rijssen,E,P.F., Herman,G.C., 1991, Resolution analysis of band-limited and offset-limited seismic data for plane-layered subsurface models, Geophysical Prospecting **39**, 61-76.