Velocity Analysis by Truncated Singular Value Decomposition

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SUMMARY

Standard velocity analysis uses coherency measurement along a time window to build velocity spectra. Accuracy, as well as resolution, is strictly correlated to the coherency technique adopted.
By introducing the a-priori knowledge of the wavelet amplitude spectrum (even if approximate) for matched filtering and Karhunen-Loeve decomposition of the windowed data, we show that the velocity spectra reject incoherent and coherent (e.g. interfering events) noise. It is assumed that when kinematical parameters (in this case two way traveltime and stacking velocity) are tested in a time window with one reflection, the first eigenimage obtained from Singular Value Decomposition (SVD) spans the signal subspace of the data (truncated SVD). The complex matched filter analysis (CM) of the first eigenimage shows improved noise rejection, due to SVD weighted stack, and accuracy due to matched filtering.
The techniques have been applied to data and the results are compared with velocity spectra obtained using semblance as well as other high-resolution techniques (MUSIC and wideband covariance measures proposed by Key and Smithson, 1990). The resolution improvement is obtained at the expense of moderately increasing the computational costs (less than twice the cost of semblance).

INTRODUCTION

In geophysics space coherency functionals are widely used to evaluate time delays and kinematical parameters of reflections. The velocity spectra are usually computed by coherency measurements of time corrected events using a span of velocities. Stacking, as well as semblance, gives coherency measurements with limited resolution and noise rejection capability. High resolution techniques which employ decomposition into eigenstructures of the spatial covariance matrix have been devised. Key and Smithson (1990) estimate the signal to noise ratio (SNR) and a weighting factor from the eigenvalue spread of such a matrix. This method assumes that when an event in the time window is exactly aligned, the largest eigenvalue will dominate those associated with interfering events and noise. The technique exhibits good noise rejection, but uses no wavelet model.

Parametric techniques take advantage of the properties of both eigenvalues and eigenvectors of the spatial covariance matrix. The velocities are evaluated by projecting the model of the signal onto two orthogonal subspaces: signal and noise subspaces (Multiple Signal Classification or MUSIC technique introduced by Schmidt (1986)). The signal subspace is related to spatially coherent events detected in the data and is spanned by the eigenvectors associated to the first few dominant eigenvalues. Since seismic data are wideband Biondi and Kostov (1989) have proposed the decomposition of the data into narrowband components; the ray parameter spectra are obtained by the incoherent summation of the spectra of each component. The residual NMO curvature, which departs from the data model structure, and the necessary averaging between subarrays (spatial smoothing), considerably reduce the resolution capability of this method. However MUSIC cannot resolve the arrival time of the wavefronts, as a pure direction of arrival estimation algorithm. We will show that reliable results can be obtained by correcting the data for a set of trial kinematical parameters, and calculating the MUSIC spectrum only for the null ray parameter. Moreover like all the eigenstructure methods cited so far, the MUSIC technique does not take into account any knowledge of the wavelet. As a result, when two or more wavefronts exhibit constant relative delays, they may be mistaken for a single *compound reflection*, degrading the resolution of the velocity panels.

In this paper we propose the complex matched filter analysis (CM) technique that introduces the a-priori information (even if approximate) of the wavelet amplitude spectrum in the evaluation of velocity spectra. Since complex signals analysis is used, no assumption is required on the phase (e.g. minimum or zero phase wavelet). The main advantage of CM lies in the summation of the normalized correlations along the spread. An improvement over CM is obtained by reducing the contributions resulting from interfering events and noise. In this paper we also propose the Karhunen-Loeve decomposition for the coherent weighted stack of NMO corrected data for velocity analysis. This is achieved by Singular Value Decomposition of a time window of NMO corrected data. In the enhanced complex matched filter analysis (ECM) the first eigenimage associated to the dominant singular value is analyzed using a filter matched to the reflection wavelet. The SVD allows the separation of the matched filtering of the coherent time waveshape from the Karhunen-Loeve weighting factor along the spread. In other words, the weighting acts as a multichannel filter for the rejection of interfering events. With respect to a wideband extension of narrowband parametric techniques (MUSIC), the ECM is a general wideband method for coherency measurement. Moreover, the resolution is less dependent on the misleading artifacts caused by the interaction between AVO and residual NMO (Spagnolini, 1992).

The Complex Matched filter analysis (CM) coherency functional

Let us consider the Common Midpoint gather analytical data *data*(x,t) obtained by Hilbert transform, the time windowing and the correction of the relative time delay; this allows the windowed data matrix to be defined as:

$$\mathbf{D}(\tau_i, v_i) = [\mathbf{d}(x_1), \mathbf{d}(x_2), \ldots, \mathbf{d}(x_m)]; \tag{1}$$

vector $\mathbf{d}(x)$ represents, for an assigned offset x, the NMO corrected data using hyperbolic time dependency vs. offset (kinematical parameters):

$$\mathbf{d}(x) = [data(x, t_1 - t_i(x)), \ldots, data(x, t_n - t_i(x))]^T;$$
$$t_i(x) = \sqrt{\tau_i^2 + \frac{x^2}{v_i^2}} \tag{2}$$

herein, τ_i indicates the two way traveltime and v_i the stacking velocity. If an exactly NMO corrected reflection is present in the time window it can be described as $\mathbf{wv}^*$, where $\mathbf{w} = [w(1),w(2),..,w(n)]^T$ is the vector of the sampled analytic wavelet and $v=[v(1),v(2),..,v(m)]^T$ gives the amplitude (AVO) and phase (PVO) variations across the spread (T indicates matrix transposition while * indicates conjugate transposition). The $n \times m$ matrix $\mathbf{D}(\tau_i,v_i)$ depends on the kinematical parameters and can thus be written as $\mathbf{D}(\tau_i,v_i) = \mathbf{wv}^* + \mathbf{N}(\tau_i,v_i)$; $\mathbf{N}(\tau_i,v_i)$ accounts for interfering events with residual moveout and correlated and/or uncorrelated noise within the time window. Supposing that the vector wavelet $\mathbf{w}$ is known (e.g. it can be assigned a-priori or estimated from data), the least squares (LS) estimation of $\mathbf{v}$ yields

$$\hat{\mathbf{v}}(\tau_i,v_i) = \frac{\mathbf{w}^*\mathbf{D}(\tau_i,v_i)}{\|\mathbf{w}\|} \tag{3}$$

where $\|\bullet\|$ indicates the Euclidean norm. Without taking into consideration any normalization factor, (3) is equivalent to the inner product between the wavelet $\mathbf{w}$ and all the traces of $\mathbf{D}(\tau_i,v_i)$. This is the optimum evaluation, in the LS sense, of the AVO and PVO parameters using a complex matched filter. Given the single trace normalized correlation

$$\rho(x_j;\tau_i,v_i) = \frac{\mathbf{w}^*\mathbf{d}(x_j)}{\|\mathbf{w}\|\|\mathbf{d}(x_j)\|} = \frac{\hat{\mathbf{v}}(x_j)}{\|\mathbf{d}(x_j)\|} \tag{4}$$

the coherency using CM is defined as the weighted stack using a single trace complex correlation $\rho(x_j)$:

$$CM(\tau_i,v_i) = \frac{\left|\sum_{j=1}^{m}\rho(x_j;\tau_i,v_i)\right|}{\sum_{j=1}^{m}\left|\rho(x_j;\tau_i,v_i)\right|} \tag{5}$$

As the data is normalized on a trace by trace basis CM will not be affected by amplitude change vs. offset (AVO). However it will still be affected by PVO, which is, in any case, usually negligible below critical angles. Even if the wavelet is chosen with a degree of arbitrariness, giving only an approximate amplitude spectrum, the applications, as well as routine use, indicate that the CM functional performs better than traditional attributes (e.g. Semblance) on synthetic as well as on real data.

The generalization of semblance using complex signals has led to a slight improvement of resolution, as described by Sguazzero and Vesnaver (1987).

Indeed, an analogy can be seen between the CM and the Semblance (Neidell and Taner, 1971), as the latter is easily seen as the unweighted stack of the dot products between each trace and the average of all the traces across the spread. This corresponds to implementing a filter matched to the "average" event of the NMO corrected data.

SVD as a coherent weighted stack: Enhanced Complex Matched filter analysis (ECM)

An improvement over simple CM application could be obtained by removing contributions from interfering events (i.e. not horizontally aligned) and noise for each time corrected window (1). The weighted stack for interfering events rejection could be covered using Wiener-Hopf multichannel filtering but this requires a model of coherent noise (Sengbush and Foster, 1968).

The Karhunen-Loeve (K-L) decomposition is proposed for the coherent weighted stack of NMO corrected data for velocity analysis. The weighted stack is obtained by performing the singular value decomposition (SVD) of the windowed data matrix $\mathbf{D}(\tau_i,v_i)$. Ulrych et al. (1988) have discussed the relationship between the SVD and the Karhunen-Loeve decomposition. The SVD of the NMO corrected and time windowed analytical signal is

$$\mathbf{D}(\tau_i,v_i) = \sum_{k=1}^{r}\sigma_k\mathbf{u}_k\mathbf{v}_k^* = \sum_{k=1}^{r}\sigma_k\mathbf{E}_k(\tau_i,v_i),\quad r = \mathrm{rank}\{\,\mathbf{D}(\tau_i,v_i)\,\}, \tag{6}$$

σ_k are the singular values in descending order and $\mathbf{E}_k(\tau_i,v_i)$ are called eigenimages of $\mathbf{D}(\tau_i,v_i)$. In other words, the original data matrix is split into the sum of several rank-one matrices, weighted by their singular values σ_k. Any horizontally aligned event can be described by a rank-one matrix. The basic assumption of K-L decomposition is that $\mathbf{E}_1(\tau_i,v_i)$ spans the signal subspace of the data $\mathbf{D}(\tau_i,v_i)$ given by the signal time shape ($\mathbf{u}_1$) and its spatial variation ($\mathbf{v}_1$). Noise and interfering events will largely be confined to other eigenimages, achieving a high degree of noise rejection (Freire and Ulrych 1988). When the kinematical parameters have been correctly estimated, $\mathbf{u}_1$ represents the wavelet (i.e. $\mathbf{u}_1=\mathbf{w}$) or a spatially coherent compound reflection and $\mathbf{v}_1$ is mostly constant. However, $\mathbf{u}_1$ will not resemble the correct wavelet and $\mathbf{v}_1$ will vary randomly both in phase and amplitude when no coherent event is present in $\mathbf{D}(\tau_i,v_i)$.

The Enhanced Complex Matched filter analysis (ECM) technique basically consists in applying the CM concept to the truncated SVD $\mathbf{E}_1(\tau_i,v_i)$ instead of $\mathbf{D}(\tau_i,v_i)$, taking advantage of its structure which allows the separation of the time and space factors. Not considering any normalization factor, the CM of the first eigenimage can be factored into the matched filter of the time factor $\mathbf{u}_1$ only, while $\mathbf{v}_1$ becomes the weight factor in the stack:

$$ECM(\tau_i,v_i) = \frac{\mathbf{u}_1^*\mathbf{w}}{\|\mathbf{u}_1\|\|\mathbf{w}\|}\cdot\frac{\mathbf{v}_1^*[1\,1\,1..1]}{\sqrt{m}\|\mathbf{v}_1\|}. \tag{7}$$

Since $\mathbf{E}_1(\tau_i,v_i)$ will essentially extract any horizontally coherent events, $\mathbf{u}_1$ should highly correlate with $\mathbf{w}$, and the coherence along the space axis would reflect in the regularity of $\mathbf{v}_1$, both in amplitude and phase. However, low amplitude in the elements of the weight vector $\mathbf{v}_1$ rejects interfering events while privileging spatially coherent NMO corrected events in the stack.

Through the use of SVD, ECM exploits the eigenstructure of both the time and space correlation matrices, separating the data onto two orthogonal bases for the aligned and interfering events, rather than calculating the weight of the first eigenimage relative to the others, as done by Key and Smithson (1990). Unlike the proposed use of

MUSIC for the search of horizontally aligned events, regardless of their time shape. SVD naturally works on wideband signals without imposing a data model of reflections. Furthermore knowledge about the wavelet facilitates ECM's resolving of temporal ambiguities.

Applications

The CM and ECM techniques have been compared with others high resolution techniques as well as with semblance.
In order to appreciate the resolution capability as well as noise rejection of the techniques, the comparison is carried out on the velocity spectra basis. As previously discussed, the velocity spectra of the MUSIC technique have been obtained by using the null ray parameter in every investigated window.
Figure 1 shows a synthetic CMP gather and the comparison of the velocity spectra using semblance, CM, ECM, Key and Smithson (K&S) and MUSIC. The SNR is approximately 3dB and a zero-phase 60Hz Ricker wavelet has been used for both synthetic signal and a-priori wavelets (kinematic parameters are: $v1$=2000m/s, $\tau 1$=1s; $v2$=1990m/s, $\tau 2$=1.02s). In the time-offset the two reflections are approximately parallel so that one compound reflection is shown. Whenever no wavelet parametrization has been introduced in the coherency model (i.e. MUSIC and K&S), $v_i^2\tau_i$=constant describes the kinematical parameter uncertainty of the compound event. Owing to the a-priori model of the wavelet, ECM allows the reduction of the uncertainty given by the locus of maxima in the velocity spectra.
The application to real data is shown in Figure 2. The velocity spectra of ECM is compared to the semblance and the resolution of the reflections is considerably enhanced (here, 45Hz Ricker wavelet roughly resembles the amplitude spectra).
Table 1 shows that the computational costs of ECM is approximately twice the cost of semblance, but considerable time could be saved by calculating only the first eigenimage instead of the complete SVD, as suggested in Ulrych and Freire (1988).

CONCLUSIONS

In this paper we have discussed and compared coherency measurement techniques for velocity analysis. The introduction of the analytic seismic wavelet, though approximate, and Complex Matched filter analysis (CM) has improved resolution with respect to semblance without cost increase. An application of the MUSIC algorithm to curved wavefronts has been proposed. It searches only for exactly corrected events, as this is the only case in which the MUSIC parametric model holds.
The CM concept has then been applied to the first eigenimage of the data matrix obtained using SVD (truncated SVD) as a spatial coherency filter, yielding interesting results on both synthetic and real data. The corresponding functional shows good resolution, and uncorrelated or correlated noise immunity. Comparison with the high-resolution algorithms here examined (MUSIC and the covariance measure proposed by Key and Smithson (1990)) shows that 1) the resolution can be further improved by the a-priori knowledge of the wavelet; 2) SVD could be used as a weighting stack in wideband coherency measurement. The favorable correlation between high-resolution technique computer cost and the high resolution, as well as the noise rejection capability, recommends their routine use in standard processing.

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Method	**Cost**
Semblance	1.0
CM	1.2 (using FFT)
MUSIC (modified)	1.9
K&S	1.1
ECM	2.0 (standard SVD package)

Table 1: Comparison of computational costs of velocity analysis algorithms. The routine use of CM is justified by low expense while ECM cost can be further reduced by calculating , as suggested in Ulrych and Freire (1988), the first eigenimage instead of the complete SVD.

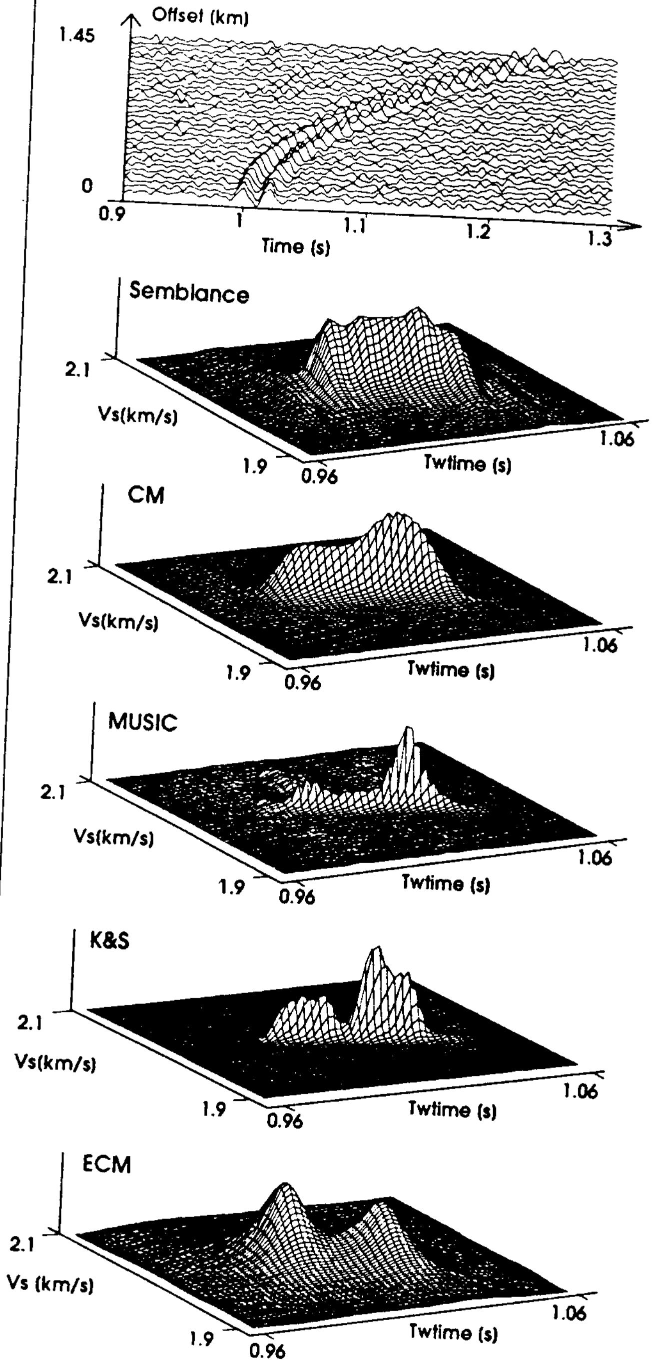

Figure 1 Comparison of velocity spectra. The normalized coherency (e.g. Semblance, CM and ECM) have been represented as a ratio c/(1-c) for display purpose.

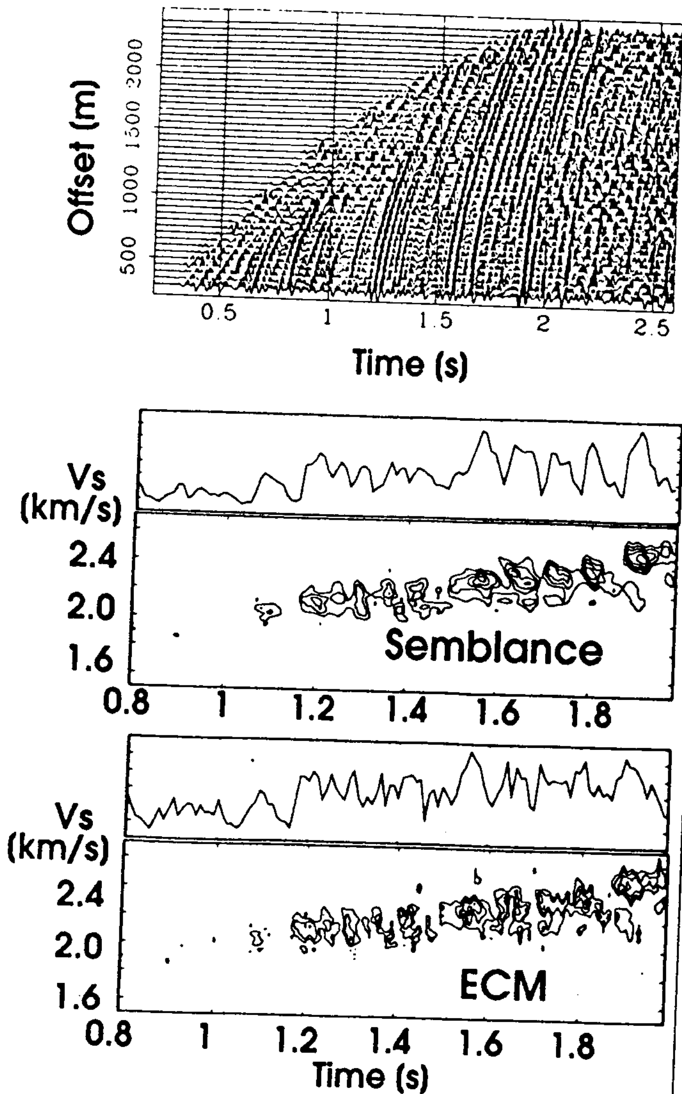

Figure 2 Application of ECM to a CMP gather field data. The comparison with semblance shows an improved resolution of compound reflections (thin layers).