Collision Model for Bit Error Rate analysis of Time Hopping Impulse Radio in Multipath Nakagami-m Channels

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***Abstract*—In presence of multiple access interference, the performance of impulse radio system is affected by collisions with other users. In this paper we evaluate the bit error probability by employing a collision model for describing the non-stationary interference caused by simultaneous transmissions in time hopping systems. The collision model provides a simple approximation for the error probability even in multipath channels since the decision variable can be conditioned to the collision events. Numerical validations of the error probability for several propagation settings (e.g., over Nakagami-m fading channels or realistic scenarios) employing the collision model show that the proposed method is simple and accurate for most practical applications.**

I. Introduction

Time hopping (TH) impulse radio transmission is typically used in wireless systems in order to mitigate the effect of multi-access interference (MAI) especially in absence of coordinated usage of the medium. Ultra Wide Band (UWB) Impulse Radio (IR) is currently the most important example of technology based on TH pulse position modulation. This emerging technology, suitable for short range wireless communications [1], [4], is based on the modulation of the amplitudes and positions of trains of narrow pulses (Sec. II).

In this work, performance estimation is derived by using the statistics of the mutual delay between pulses of interfering users due to TH generated by pseudo-noise (PN) level sequences, to propagation delays and also to asynchronous transmission. The analysis takes into account the effect of multi-access interference and multipaths providing the average bit error rate (BER) of a conventional single user RAKE receiver.

In the literature dedicated to UWB systems, considerable effort has been done for modelling accurately the multiple access interference and also for removing the Gaussian assumption [5], [6], [7], [8], [9], [4]. The novelty of this work, that extends an analytical approach based on the statistical study of collisions among users, is the derivation of closed form solutions for the average BER in a general class of multipath channels, characterized by Nakagami fading and exponential power intensity profile. However, numerical analysis shows that the collision model has turned out to be satisfactory also in some more sophisticated channel models like those considered in Sec. IV from IEEE 802.15 [11], [10].

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Of course, TH transmission is associated to several modulation formats of information (e.g. pulse position or pulse amplitude modulations) and the corresponding demodulation techniques can admit coherent or non-coherent implementations. In addition, the use of random TH guarantees the coexistence of more channels, improves interference rejection (from external sources) and matches the spectral requirements of the signal. The study presented here addresses the performance evaluation of binary pulse amplitude modulation (PAM) and pulse position modulation (PPM) in uncoded systems.

The overall organization is as follows: Section II describes the system model and the channel assumptions; Sec. III presents the analytical evaluations of performance for the single user receiver in the case of interest. Finally, Sec. IV is dedicated to the numerical results, obtained by simulations and analysis.

II. Problem definition

In this paper we consider an asynchronous UWB-IR scenario [4] where a terminal decodes the information corresponding to the related link (from now on the link $i = 0$) and the remaining N_U terminals make a non-coordinated usage of the same bandwidth. In this scenario the overall performance is dominated by the MAI and a TH sequence is assigned to each user in order to reduce the overall interference that arises from collision with other users' transmission. The coded pulse train can be arranged to span several time frames for decreasing the impact of collisions. Even if, in principle, the assigned TH sequences can be deterministically optimized for minimizing MAI, the propagation delays and multipath make the effective delays from each interfering link in each frame be better described by a random TH model.

Let us consider the transmitted signal from the i-th terminal for PAM modulation or PPM modulation

$$s_i(t) = \sum_k \sum_{m=0}^{M-1} a_i[k] w\left(t - (kM+m)T_f - c_{i,m}T - b_i[k]\frac{\Delta}{2}\right) \tag{1}$$

where $w(t)$ is the pulse waveform, T is the chip time, T_f is the frame interval ($T_f = N \cdot T$ with N chips per frame) and M is the total number of frames used for transmitting a binary symbol; $a_i[k]$ or $b_i[k] \in \{-1, +1\}$ at k-th time represent the PAM or PPM data modulation (the PPM has an offset equal to

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$\Delta/2$) and $c_{i,m} \in \{0, 1, \cdots, N-1\}$ is the TH code associated to the i-th user. Let the signal received from the reference user ($i = 0$) and the MAI contributions ($i = 1, ..., N_U$) be:

$$r(t) = \sum_{i=0}^{N_U} s_i(t) * g_i(t) + n(t), \qquad (2)$$

where each signal $s_i(t)$ experiences a different multipath channel $g_i(t)$ that accounts for fading and delays from $i-th$ user to the reference one. The additive white Gaussian noise (AWGN) $n(t)$ has double-sided noise spectral density σ^2. The convolution between the pulse $w(t)$ and the channel $g_i(t)$ will be modelled by a causal equivalent channel $h_i(t)$ over the causal support LT (i.e., $h_i(t) = 0$ for $t \notin [0, (L-1)T]$) and affected by a random time shift t_i uniformly distributed in $[0, T_f]$ that includes propagation delay and TH (time shift t_i is assumed to independent from frame to frame). For the sake of simplicity, let us consider the single frame case $M = 1$ (for the multi-frame case, see Remark 1 in Sec. III). The received signal, for $m = 0$, $k = 0$, $c_{0,0} = 0$, reduces to the equivalent model

$$y(t) = a_0 h_0\left(t - b_0\frac{\Delta}{2}\right) + \sum_{i=1}^{N_U} a_i h_i\left(t - \tau_i - b_i\frac{\Delta}{2}\right) + n(t+t_0) \qquad (3)$$

where $\tau_i = t_i - t_0$ is the random delay of the MAI contributions with respect to the reference user. Notice that, from the statistical independence of the delays t_i and from the assumption of their uniform distribution, the probability density function $p(\tau_i)$ of the differential delays τ_i is triangular over the support $\tau_i \in [-T_f, T_f]$: $p(\tau_i) = (T_f - |\tau_i|)/T_f^2$.

An optimal (with respect to AWGN only) single user receiver evaluates the decision variable according to the filter matched to the user of interest. In PAM ($a_i \in \{-1, +1\}$ and $b_i = 0$) the decision variable

$$\begin{aligned} z &= \int_0^{T_f} y(t)h_0(t)dt = a_0 \int_0^{T_f} h_0(t)^2 dt + \\ &\quad \sum_{i=1}^{N_U} a_i \int_0^{T_f} h_0(t)h_i(t-\tau_i)dt + \int_0^{T_f} n(t)h_0(t)dt \\ &= a_0 H + I(\tau) + N \end{aligned} \qquad (4)$$

includes the instantaneous channel energy H ($\mathbb{E}[H] = E_h$), the MAI $I(\tau)$ that depends on the set of delays $\tau = [\tau_1, ..., \tau_{N_U}]^T$ and the AWGN component N. The bit error probability conditioned to the fading and channel impairments is

$$P(E|\tau, \{h_i(t)\}, \{a_i\}) = P(z < 0 | a_0 = 1, \tau, \{h_i(t)\}, \{a_i\}). \qquad (5)$$

In this paper we assume that, for fading channels, the interference $I(\tau)$ conditioned to the delay pattern τ is Gaussian: $I(\tau) \sim \mathcal{N}(0, \sigma_\tau^2)$. The power σ_τ^2 depends on the degree of interference experienced by the user $i = 0$ according to the set of delays τ and it is given by

$$\begin{aligned} \sigma_\tau^2 &= \sum_{i=1}^{N_U} a_i^2 \mathbb{E}\left[\int h_0(t)h_0(\zeta)h_i(t-\tau_i)h_i(\zeta-\tau_i)d\zeta dt\right] \\ &= \sum_{i=1}^{N_U} \int \mathbb{E}[h_0(t)^2] \cdot \mathbb{E}[h_i(t-\tau_i)^2]dt = \sum_{i=1}^{N_U} \sigma^2(\tau_i) \end{aligned} \qquad (6)$$

where $\sigma^2(\tau_i)$ denotes the MAI power for a delay τ_i (independent from the modulation levels a_i). In order to evaluate the error probability conditioned to the delay pattern τ we use the Gaussian approximation obtaining

$$P(E|\tau, \{h_0(t)\}) = Q\left(\sqrt{\frac{H}{\sigma_\tau^2/H + \sigma^2}}\right). \qquad (7)$$

Hence the average error probability

$$P(E) = \int \mathbb{E}[P(E|\tau, \{h_0(t)\})]p(\tau)d\tau \qquad (8)$$

is evaluated by using the multivariate probability density function $p_\tau(\tau) = \prod_i p(\tau_i)$ of the delays and the statistical properties of H and σ_τ^2 according to the system definition.

In PPM ($a_i = 1$ and $b_i \in \{-1, +1\}$) the decision variable, for $b_0 = 1$, is defined accordingly as

$$\begin{aligned} z &= \int_0^{T_f} y(t)\bar{h}_0(t)dt = \int_0^{T_f} h_0(t - \frac{\Delta}{2})^2 dt - \\ &\quad \int_0^{T_f} h_0(t + \frac{\Delta}{2})h_0(t - \frac{\Delta}{2})dt + \int_0^{T_f} n(t)\bar{h}_0(t)dt + \\ &\quad \sum_{i=1}^{N_U} \int_0^{T_f} h_i(t - \tau_i - b_i\frac{\Delta}{2})\bar{h}_0(t)dt \\ &= H + I_0 + N + I(\tau) \end{aligned} \qquad (9)$$

with $\bar{h}_0(t) = h_0(t - \Delta/2) - h_0(t + \Delta/2)$. If the PPM offset Δ is greater than the channel support, the self interference I_0 is negligible and only one of the two terms that compose each interference contribution in $I(\tau)$ dominates. Rearranging the terms accordingly it is:

$$\begin{aligned} I(\tau) &= \sum_{i=1}^{N_U} \int_{-\Delta/2}^{T_f-\Delta/2} h_i\left(t - \tau_i - b_i\frac{\Delta}{2} + \frac{\Delta}{2}\right)h_0(t)dt - \\ &\quad \int_{\Delta/2}^{T_f+\Delta/2} h_i\left(t - \tau_i - b_i\frac{\Delta}{2} - \frac{\Delta}{2}\right)h_0(t)dt \\ &\simeq \sum_{i=1}^{N_U}\left(c_k \int_0^{T_f} h_i(t - \overline{\tau}_i)h_0(t)dt\right) \end{aligned} \qquad (10)$$

where $|\overline{\tau}_i| = min\{|\tau_i + b_i\Delta/2 - \Delta/2|, |\tau_i + b_i\Delta/2 + \Delta/2|$ and $c_k = \pm 1$ depending on the dominant term in (10). For each term in (10), the two events $b_i = \pm 1$ generate the following conditions for $\overline{\tau}_i$:

- if $b_i = 1$, $\overline{\tau}_i = \tau_i$ if $\tau_i \geq -\Delta/2$ and $\overline{\tau}_i = \tau_i + \Delta$ if $\tau_i < -\Delta/2$.
- if $b_i = -1$, $\overline{\tau}_i = \tau_i$ if $\tau_i \leq \Delta/2$ and $\overline{\tau}_i = \tau_i - \Delta$ if $\tau_i > \Delta/2$.

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Hence the mutual delay $\overline{\tau}_i$ turns out to have a probability density different from the triangular $p(\tau_i)$ but can be easily derived from the rules above. Similarly to (7), the conditioned error probability becomes

$$P(E|\tau,\{h_0(t)\}) \quad = \quad Q\left(\sqrt{\frac{H}{\sigma_\tau^2/H+2\sigma^2}}\right) \qquad (11)$$

Finally, when the same symbol is transmitted over $M > 1$ frames, it is easy to prove that error probability (7) depends on the extended set of delays $\tau = [\tau_1^{(0)},...,\tau_{N_U}^{(0)},\tau_1^{(1)},...,\tau_{N_U}^{(M-1)}]^T$ that accounts for the differential delays over each frame (i.e., $\tau_i^{(m)} = t_i^{(m)} - t_0^{(m)}$, $m=0,\cdots,M-1$). For PAM (and similarly for PPM) the conditional error probability is

$$P(E|\tau,\{h_0(t)\}) = Q\left(\sqrt{\frac{MH}{\sigma_\tau^2/MH+\sigma^2}}\right), \qquad (12)$$

and the average error probability is evaluated similarly to (8).

III. Collision model for Nakagami-m fading channels

A discrete time model reflects the way the RAKE receiver is employed as a bank of filters matched to the pulse waveform $w(t)$ followed by a maximum ratio combiner that collects all the available energy. The chip-spaced model can be employed in order to have the analytical model of the decision variable in case of multipath channel. Let the equivalent channel be described by the chip-spaced model

$$h_i(t) = \sum_{\ell=0}^{L-1} h_{i,\ell}\delta(t-\ell T) \qquad (13)$$

where the faded amplitudes $h_{i,\ell}$ are zero mean random variables distributed as Nakagami-m with fading figure m and exponential multipath intensity profile

$$\mathbb{E}[|h_{i,\ell}|^2] = \Omega_0 \exp(-\delta\cdot\ell). \qquad (14)$$

The average energy is

$$E_h = \sum_{\ell=0}^{L-1} \mathbb{E}\left[|h_{i,\ell}|^2\right] = \Omega_0 \frac{1-e^{-\delta L}}{1-e^{-\delta}} = \Omega_0 q(L,\delta). \qquad (15)$$

According to (14), the power of MAI (6) can be evaluated in closed form for any arbitrary delay τ_i in the discretized set $\{0,\pm T,\cdots,\pm(N-1)T\}$. If $0 \le \tau_i \le (L-1)T$ (or similarly for $(1-L)T \le \tau_i \le 0$), the power of the ith interferer is

$$\sigma^2(\tau_i) \quad = \quad \Omega_0^2 \sum_{\ell=|\tau_i|}^{L-1} e^{-\delta\cdot\ell}\cdot e^{-\delta\cdot(\ell-|\tau_i/T|)} = E_h\cdot\Omega_0 \frac{q(L-|\tau_i/T|,2\delta)}{q(L,\delta)} e^{-\delta\cdot|\tau_i/T|}. \qquad (16)$$

Now we use the collision model for evaluating the average error probability in PAM with random TH. Let the i-th interfering user be assigned to one of two disjoint set as colliding or not colliding user (hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$, respectively) according to the following rule:

$$\begin{array}{l} i\in\mathcal{H}_0 \text{ if } |\tau_i| \le (L-1)T \\ i\in\mathcal{H}_1 \text{ if } |\tau_i| > (L-1)T \end{array} \qquad (17)$$

The collision probability for this discrete-time model is

$$p_c = P[i\in\mathcal{H}_0] = \sum_{n=-L+1}^{L-1} \frac{1}{N}\left(1-\frac{|n|}{N}\right) \qquad (18)$$

from the discretized triangular $p(\tau_i)$ while the average interfering power for each colliding interferer (regardless of its delay) is

$$\sigma_c^2 = \sum_{n=-L+1}^{L-1} \sigma^2(\tau_i) P(\tau_k = nT|\mathcal{H}_0) = E_h \Omega_0 \rho(L,N,\delta), \qquad (19)$$

where

$$\rho(L,N,\delta) = C_T^{-1} \sum_{n=-L+1}^{L-1} \frac{q(L-|n|,2\delta)}{q(L,\delta)} e^{-\delta\cdot|n|}\left(1-\frac{|n|}{N}\right) \qquad (20)$$

being $C_T = (1+(L-1)(2N-L)/N)$ a normalization term for the delay density conditioned to $\mathcal{H}_0$. The total interference power σ_τ^2 when k users are colliding (regardless of their delays) is the sum of k powers σ_c^2 and the corresponding density is binomial:

$$P[\sigma_\tau^2 = k\cdot\sigma_c^2] = \binom{N_U}{k} p_c^k (1-p_c)^{N_U-k}. \qquad (21)$$

The error probabilities (7) or (11) can be approximated as

$$P(E|\tau,\{h_0(t)\}) \simeq Q\left(\sqrt{S\cdot\gamma_k}\right) \qquad (22)$$

with

$$\begin{array}{l} S = \frac{1}{\Omega_0}\sum_{\ell=0}^{L-1} h_{0,\ell}^2 \\ \gamma_k = \frac{\Omega_0}{\sigma_\tau^2/E_h+\sigma^2} = \gamma\left(1+k\gamma\rho(L,N,\delta)\right)^{-1} \end{array} \qquad (23)$$

for k interfering users, and $\gamma = \Omega_0/\sigma^2$. The approximation in (22) is included because we are neglecting the power fluctuations induced by instantaneous energy into MAI (i.e., from (7) it is $\sigma_\tau^2/H \simeq \sigma_\tau^2/E_h$).

The PPM case with random TH can be similarly solved by substituting the triangular discrete probability density of the mutual delays τ_i with the modified one in Sec. II. The probability of collision p_c (18) and the definition of the function $\rho(L,N,\delta)$ (20) are modified accordingly. The conditioned error probability is still expressed by (22) but with

$$\gamma_k = \frac{\Omega_0}{\sigma_\tau^2/E_h+2\sigma^2} = \gamma\left(2+k\gamma\rho(L,N,\delta)\right)^{-1}. \qquad (24)$$

Now, in order to average (22) with respect to signal fading, we approximate the sum S of L squares of Nakagami random variables as the square of another Nakagami random variable [3] with the equivalent parameters

$$m_S = m\frac{q(L,\delta)^2}{q(L,2\delta)} \qquad \Omega_S = q(L,\delta) \quad (25)$$

evaluated from moments' matching. Following the reasoning in [2] we can easily express the conditional error probability $\mathbb{E}[P(E|\tau,\{h_0(t)\})]$ as

$$P(E \mid \sigma_\tau^2 = k\sigma_c^2) = \sqrt{\frac{\overline{\gamma}_k}{1+\overline{\gamma}_k}} \cdot \frac{(1+\overline{\gamma}_k)^{-m_S}\Gamma(m_S+1/2)}{2\sqrt{\pi}\Gamma(m_S+1)} \cdot {}_2F_1(1, m_S+1/2; m_S+1, (1+\overline{\gamma}_k)^{-1}) \quad (26)$$

where it is highlighted the dependence on k in

$$\overline{\gamma}_k = \gamma\left(1 + k\cdot\gamma\rho(L,N,\delta)\right)^{-1} \cdot \frac{q(L,2\delta)}{2m\cdot q(L,\delta)}. \quad (27)$$

Finally the average error probability is obtained by conditioning the interference power to all the possible collision events

$$P(E) = \sum_{k=0}^{N_U} \binom{N_U}{k} p_c^k (1-p_c)^{N_U-k} \cdot P(E|\sigma_\tau^2 = k\sigma_c^2). \quad (28)$$

Few remarks are useful to define some additional model properties.

Remark 1: The extension to $M > 1$ frames transmission can rely on the assumption that M frames are independent colliding events. Therefore the collision probability is still binomial over the total $N_U M$ delays

$$P[\sigma_\tau^2 = k\cdot\sigma_c^2] = \binom{N_U M}{k} p_c^k (1-p_c)^{N_U M-k} \quad (29)$$

and the average error probability is similar to (28) with straightforward substitutions.

Remark 2: In the interference model the inter-frame collisions is neglected (i.e., the interference that arises when the support of the multipath channel has a leakage on the adjacent frames). However, if the channel support is larger than T_f (or $N \gg L$) the error degradation due to inter-frame collisions is negligible.

Remark 3: When frame length is comparable to channel support ($N \simeq L$) the TH becomes ineffective as $p_c \simeq 1$ and thus the system is heavy-loaded. In this case the collision model is not strictly necessary. This case could be of interest in some practical cases even if the benefits of TH would become questionable. The trade-off between frame length T_f and the number of frames M could be easily investigated once adopted the analytical relationships from the collision model.

IV. Numerical Results

For validating the analytical results presented in the previous sections, we have considered the following case studies:

- Channel model A with exponential profile (14), baseband transmission and path amplitudes subject to Gaussian density (Nakagami $m = 0.5$). This fading statistics relies on a commonly employed reflectivity model for stratified medium in remote sensing. Also in UWB channel models for low data rate applications, the Nakagami factor m is close to 0.5 in several environments [10].
- Channel model B with exponential profile (14), carrier-modulated transmission, path amplitude subject to a Rayleigh density (Nakagami $m = 1$) and phase uniformly distributed in $[0, 2\cdot\pi)$.
- UWB channel model C with time resolution 0.167 ns, high number of paths and baseband transmission of a Gaussian monocycle with $T = 0.167$ ns (-3 dB bandwidth equal to 4.75 GHz). The channel response model corresponds to typical indoor LOS realizations with range between 0 and 4 meters and log-normal shadowing [11].

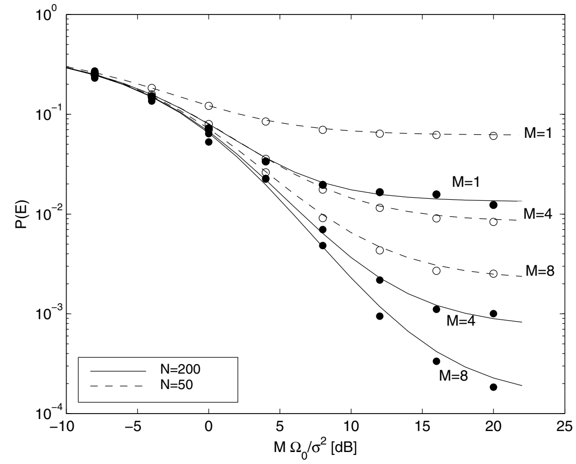

Fig. 1. $P(E)$ vs Ω_0/σ^2 in channel A for analytical (solid lines) and simulation (markers): $\delta = 0.2$, $L = 5$, $N_U = 15$.

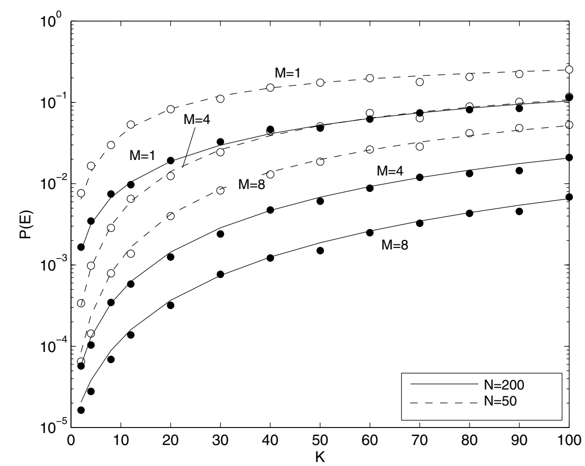

Fig. 2. $P(E)$ vs N_U in channel A for analytical (solid lines) and simulation (markers): $\delta = 0.2$, $L = 5$, $\Omega_0/\sigma^2 = 20$ dB.

We present the numerical results for the PAM average error probability, as a function of the signal-to-noise ratio (Ω_0/σ^2), for fixed numbers of interfering users N_U, or as a function of

N_U for a fixed Ω_0/σ^2 value. From Fig. 1 - 3 it is possible to notice how the collision model provides results (lines) that are close to the simulated performance (markers). Fig. 1 and 2 report the simulated and analytical results for a channel A with $L = 5$ and $\delta = 0.2$ at several values of the number of interfering users N_U and of the signal-to-noise ratio: the analysis reveals the robustness to interference in function of the system parameters N and M (number of frames). Similarly Fig. 3 shows an example of the channel model B, that is the radio frequency version of A. Fig. 4 shows an example regarding the UWB channel model C: the frame parameters generate a collision probability $p_c \simeq 1$ and thus it is not strictly necessary the use of a collision model even for a small number of users. This is due to the high number of paths that contributes, under the assumption of ideal receiver, to the Gaussian approximation in the total noise and interference power. In this channel model, the total energy of the channel impulse response is subject to a log-normal fluctuation and therefore the conditional error probability $\mathbb{E}[P(E|\tau, \{h_0(t)\})]$ cannot be evaluated in closed form as in (26) but it needs a numerical integration for any number of colliding users. It is also interesting to notice that the sophisticated characteristics of this channel model (as the double Poisson generation of clusters and paths) do not play a key role in this performance estimation as only the external exponential profile of the model is used for determining the average interference level.

V. Conclusions

In this work we have proposed an analytical approach for determining the average performance of transmission systems characterized by pulse position modulation and time hopping of radio pulses in presence of multi-user interference. The adopted collision model is used for including the effect of asynchronous interfering users and multipath faded channels. The analytical derivation applies to channel with exponential power intensity profile and Nakagami fading, and it has been shown to provide an effective and general way for easily predicting the average bit error rate in function of the fundamental system parameters.

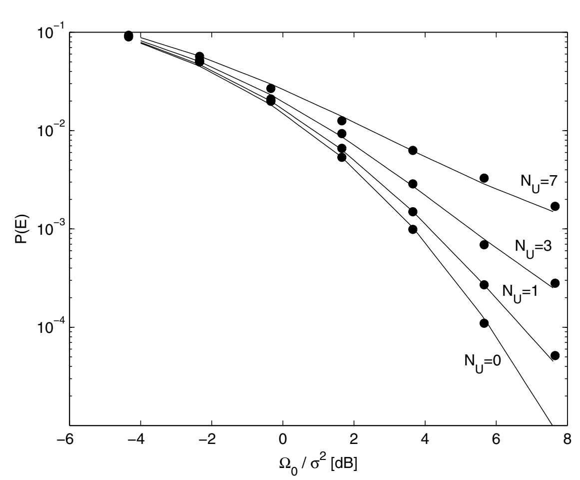

Fig. 3. Analytical (solid lines) and simulated (markers) results for channel B: $N = 64$, $M = 1$, $\delta = 0.2$, $L = 21$.

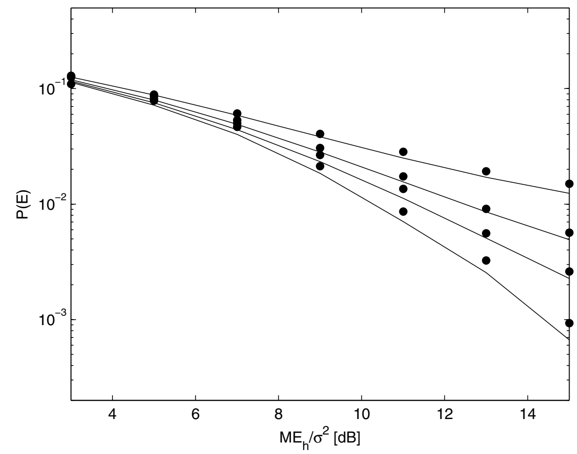

Fig. 4. Analytical (solid lines) and simulation (markers) results for channel C: $N = 32$, $M = 4$.

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