# A Predictive Opportunistic Scheduler for 4G Wireless Systems

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*Abstract*— **Channel aware schedulers play a crucial role in the performance enhancement of wireless systems by taking advantage of channel fluctuations for leveraging the gain that results in user selections (multiuser diversity) to maximize the throughput performance. Opportunistic schedulers achieve a convenient trade-off between sum-rate and fairness by optimizing the resource allocation with respect to the channel performance over past assignments (e.g., channel quality measurements for the past users or the users' throughput).**

**This paper is focused on opportunistic schedulers based on cumulative density function with the introduction of the predicted channel quality values. The Predictive Score Based Scheduler (Predictive-SBS) proposed in this paper is computationally convenient and shows a better trade-off between sum-rate and fairness when compared to non-predictive schedulers based on cumulative density function.**

## I. Introduction

In broadcast wireless systems (e.g., downlink for B3G and 4G systems) the users experience independent fading on their links. Opportunistic schedulers take advantage of users' channel fluctuations in order to leverage multiuser throughput gain by adaptively allocating the contended resource to the users that experience the best channel quality. However, sum-rate maximization contrasts with other Quality of Service (QoS) requirements, such as the equal scheduling probability of different users (i.e., long-term fairness) and scheduling-jitter between the scheduling events of consecutive packets of the same flow. It is therefore of great interest to improve the trade-off among the contrasting requirements of sum-rate, fairness in the assignment among users and jitter achieved by scheduling algorithms.

The Proportional Fair Scheduler (PFS) [1] is based on past measurements of the channel quality for the scheduling optimization in the current time-slot. However, recent advances in channel power prediction (e.g., see [2] and references therein) proved meaningful advantages when the assignment in the current time-slot is based on both the past channel measurements and the predictions of the future channel conditions [3]. Bang et al. [4] recently proposed the Predictive Proportional Fair Scheduler (PPFS) that is aware of past and future channel values, thus leveraging a significant throughput gain without affecting fairness. By using the analytical framework in [5], the PPFS [4] maximizes a non-causal throughput utility function provided that error mismatch can be avoided in the analysis. Since the exhaustive numerical optimization required by the PPFS reduces the interests for any practical application, a simplified (but suboptimal in terms of sum-rate performance) iterative algorithm implementation is also provided in [4]. Moreover, the prediction uncertainty is never explicitly taken into account in the PPFS optimization objective [4]. In addition, when heterogeneous fading statistics are experienced by different users, the assignment of Proportional Fair algorithms is unfair [6-7]. Hence, even though the PPFS algorithm outperforms the PFS algorithm, there is the need to further exploit the prediction strategies in opportunistic schemes.

This paper addresses the problem of opportunistic predictive scheduling for wireless systems by considering a non-iterative scheme that optimally takes into account prediction uncertainty. We focus the attention on the Cumulative Density Function (CDF) based scheduler (e.g., see [7-9]) as it shows asymptotical (for a large number of users) throughput optimality and long-term fairness among users [9] that hold true even when different users experience heterogeneous fading statistics [7]. The idea of the CDF scheduler is to opportunistically select those users that experience the best channel according to their cumulative function of the channel distribution that can be empirically estimated e.g. from the past channel measurements [7].

Throughout this paper we develop a framework for the optimization of the CDF based scheduler by taking advantage of both *past* and *future* (i.e., predicted) channel values. The proposed algorithm is robust with regards to the unreliable channel predictions as (differently from [4]) the scheduler is made aware of the prediction error power. Furthermore, an extremely efficient implementation is allowed because the proposed non-iterative algorithm does not require the explicit estimation of the future samples, but just the less computationally demanding knowledge of their statistical properties.

In the last part of this paper it is shown by numerical analysis that the predictive CDF based scheduler meaningfully improves the trade-off between sum-rate and fairness with respect to other scheduling approaches.

## II. System Settings And Problem Definitions

Let us consider the downlink of a cellular system where the Base Station (BS) serves $K$ active Mobile Stations (MSs) each with an infinite buffer of stored data (to avoid specific assumptions on the incoming traffic model). Here the focusing is towards a single antenna single carrier TDMA system even if the extension to OFDMA and multiple antenna systems (e.g., SDMA schemes) is straightforward. Let $h_k(\mathrm{t}) \sim \mathrm{CN}(\mu_k, \nu_k^2)$ be the narrowband Rice distributed channel in time-slot $t$ between the BS and the $k$-th MS with Rician LoS factor $\phi_k = |\mu_k|^2 / \nu_k^2$. Channels are stationary random processes arbitrarily correlated in time and mutually uncorrelated among different users. Let $\gamma_k(t) = |h_k(t)|^2$ be the SINR value for the $k$-th user in time slot $t$ (we assume stationary noise and Gaussian interference with unitary power).

The scheduler chooses in each time slot the user $k^*(t)$ according to a score function $\psi_k(t)$:

$$k^*(t) = \arg\max_k \{\psi_k(t)\}. \tag{1}$$

Under the assumption of continuously distributed channel SINR values $\gamma_k(t)$, the score function in the Score Based Scheduler (SBS) [7] algorithm is

$$\psi_k(t) = \frac{1}{N_w} \sum_{\tau=1}^{N_w} 1(\gamma_k(t) - \gamma_k(t-\tau)), \tag{2}$$

where $N_w$ is the scheduling time window. Function $1(x) = 1$ if $x \geq 0$ and $1(x) = 0$ if $x < 0$. The score $\psi_k(t)$ is regarded as the relative quality of the current SINR $\gamma_k(t)$ for the user $k$ compared to its empirical distribution over the scheduling time window $N_w$. Therefore, the objective of the SBS (2) is to select those users that experience a good channel realization according to their own channel statistical distribution and compared with the percentile of the others.

According to the frequentist definition, the asymptotic value of the score (2) is

$$\lim_{N_w \to \infty} \psi_k(t) = F_k(\gamma_k(t)) \tag{3}$$

where $F_k(x) = p(\gamma_k(t) \leq x)$ is the CDF of the SINR at time slot $t$ and thus the SBS is asymptotically (for $N_w \to \infty$) equivalent to a CDF based scheduler.

In the following the score function for the scheduling will be based on the prediction of the SINR $\hat{\gamma}_k(t+\tau)$ based on the knowledge of the channel for τ<0.

## III. Predictive Score Based Scheduler

Let the scheduling time widow be split into $N_f$ future samples (with regards to the current time slot $t$) and $N_p$ past samples, so that $N_w = N_f + N_p$. The new predictive-based score function $\psi_k(t)$ becomes

$$\begin{aligned} \psi_k(t) &= \underbrace{\frac{1}{N_w} \sum_{\tau=1}^{N_p} 1(\gamma_k(t) - \gamma_k(t-\tau))}_{\widehat{\psi}_k(t)} \\ &+ \underbrace{\frac{1}{N_w} \sum_{\tau=1}^{N_f} 1(\gamma_k(t) - \gamma_k(t+\tau))}_{\breve{\psi}_k(t)} \\ &= \widehat{\psi}_k(t) + \breve{\psi}_k(t), \end{aligned} \tag{4}$$

here explicitly divided between the causal function $\widehat{\psi}_k(t)$ based only on the past SINR measurements, and the anti-causal function $\breve{\psi}_k(t)$ for the future SINR values. Of course, for $N_f = 0$ the score (4) reduces to the SBS (2). Score (4) cannot be evaluated in real time implementation as the future SINR samples $\gamma_k(t+\tau)$ for the evaluation of $\breve{\psi}_k(t)$ are still not available. Hence, we present in the following subsections two different approaches for the evaluation of the anti-causal term in (4) based on the prediction values $\hat{\gamma}_k(t+\tau)$ (Predictive-SBS Type 1) or the direct estimation of (4) (Predictive-SBS Type 2).

### A. *Predictive-SBS Type 1*

The straightforward approach is to trivially substitute the future SINR values $\gamma_k(t+\tau)$ in the score function (4) with their predicted values $\hat{\gamma}_k(t+\tau)$

$$\psi_k^{Type1}(t) = \widehat{\psi}_k + \frac{1}{N_w} \sum_{\tau=1}^{N_f} 1(\gamma_k(t) - \hat{\gamma}_k(t+\tau)). \tag{5}$$

Eq. (5) defines the Predictive-SBS Type 1. The way to optimally estimate the future SINR values $\gamma_k(t+\tau)$ is tackled in Section IV.A by considering an unbiased estimator.

### B. *Predictive-SBS Type 2*

The Predictive-SBS Type 1 can be shown (see Section V) not to be robust with respect to the prediction errors, as the prediction uncertainty of the SINR values $\hat{\gamma}_k(t+\tau)$ is not taken into account in the estimation of (4). Therefore, we define the score $\psi_k^{Type2}(t)$ of the Predictive-SBS Type 2 to be used in (1) as:

$$\psi_k^{Type2}(t) \equiv E[\psi_k(t)] = \widehat{\psi}_k(t) + E[\breve{\psi}_k(t)]. \tag{6}$$

The Predictive-SBS Type 2 is optimal with respect to the prediction error, as Eq. (6) is the Minimum Mean Squared Error [10] estimator of the score $\psi_k(t)$ in (4) conditioned to the knowledge of the faded channels over time. The closed form solution for (6) is based on specific assumptions on fading as it will be shown in Section IV.B for the optimal linear channel predictor.

## IV. Closed Form Solutions For The Predictive-SBS

### A. *Predictive-SBS Type 1*

Here we provide the optimal unbiased MMSE estimator of the predicted power $\hat{\gamma}_k(t+\tau)$ that has to be plugged in (5).

Following the main results in [4], let the BS be aware of the instantaneous channel values

$$\mathbf{h}_k(t)=[h_k(t),\quad h_k(t-1),\quad ...,\quad h_k(t-L+1)]^T \tag{7}$$

for all the users. This assumption is reasonable in Time Division Duplex (TDD) systems where the BS exploits channel reciprocity for the downlink channel estimation, and for Frequency Division Duplex (FDD) systems where the BS is aware of $\mathbf{h}_k(t)$ from a feedback channel. However, since the feedback for FDD systems is often based on the instantaneous SINR values $\gamma_k(t)$ for all the users, the results in Section IV have to be fully reconsidered based on SINR rather than $\mathbf{h}_k(t)$.

The MMSE estimation of the channel $h_k(t+\tau)$ based on the available channel observations $\mathbf{h}_k(t)$ is [10]:

$$\hat{h}_k(t+\tau)=E[h_k(t+\tau)\,|\,\mathbf{h}_k(t)]$$

$$=\mathbf{G}_k(\tau)(\mathbf{h}_k(t)-\mu_k\mathbf{1})+\mu_k \tag{8}$$

where

$$\mathbf{G}_k(\tau)=\mathbf{c}_k^{\mathbf{H}}(\tau)\mathbf{R}_k^{-1} \tag{9}$$

$$\underset{L\times L}{\mathbf{R}_k}=E\left[(\mathbf{h}_k(t)-\mu_k 1)(\mathbf{h}_k(t)-\mu_k 1)^H\right] \tag{9-a}$$

$$\underset{L\times 1}{\mathbf{c}_k^{\mathbf{H}}}(\tau)=E\left[(\mathbf{h}_k(t)-\mu_k\mathbf{1})(h_k(t+\tau)-\mu_k)^*\right]. \tag{9-b}$$

From (8) the predicted power can be obtained in a straightforward way as

$$\hat{\gamma}_k^b(t+\tau)=\left|\hat{h}_k(t+\tau)\right|^2. \tag{10}$$

By applying simple analytical derivations, the expected value of the SINR which is estimated by (10) conditioned to a given observation $\mathbf{h}_k(t)$ is derived as:

$$E\left[\hat{\gamma}_k^b(t)\,|\,\mathbf{h}_k(t)\right]=E[\gamma_k(t)\,|\,\mathbf{h}_k(t)]-\sigma^2_{\varepsilon_h,k}(t+\tau) \tag{11}$$

where

$$\sigma^2_{\varepsilon_h,k}(t+\tau)=E\left[\left|\hat{h}_k(t+\tau)-h_k(t+\tau)\right|^2\right]$$

$$=\nu_k^2-\mathbf{c}_k^{\mathbf{H}}(\tau)\mathbf{R}_k^{-1}\mathbf{c}_k(\tau) \tag{12}$$

is the power of the complex channel prediction error.

Eq. (11) highlights that (10) is a biased estimator of SINR whose bias is given by (12). Therefore, the unbiased MMSE estimator of the SINR to be used in (5) is finally obtained by removing the bias (12) from (10):

$$\hat{\gamma}_k(t+\tau)=\left|\hat{h}_k(t+\tau)\right|^2+\sigma^2_{\varepsilon_h,k}(t+\tau). \tag{13}$$

Eq. (13) extends the results in [2] for the Rayleigh channel model to the Rice channel.

### B. *Predictive-SBS Type 2*

Here we derive the closed form expression for (6) which allows for the practical implementation of the Predictive-SBS Type 2.

The predictive-score term in (6) is reduced to:

$$E[\breve{\psi}_k(t)]=\frac{1}{N_w}E\left[\sum_{\tau=1}^{N_f}1(\gamma_k(t)-\gamma_k(t+\tau))\right]$$

$$=\frac{1}{N_w}\sum_{\tau=1}^{N_f}\Pr(\gamma_k(t+\tau)\le\gamma_k(t)), \tag{14}$$

where $\Pr(\gamma_k(t+\tau)\le\gamma_k(t))$ is the probability that the future (unknown) SINR $\gamma_k(t+\tau)$ does not exceed $\gamma_k(t)$.

In order to evaluate (14), the conditional density for the future channel values is [10]:

$$h_k(t+\tau)\sim CN\left(\hat{h}_k(t+\tau),\sigma^2_{\varepsilon_h,k}(t+\tau)\right), \tag{15}$$

where $\hat{h}_k(t+\tau)$ is given by (8) and $\sigma^2_{\varepsilon_h,k}(t+\tau)$ is given by (12). The future SINR $\gamma_k(t+\tau)=|h_k(t+\tau)|^2$ is a non central chi-squared random variable with two degrees of freedom with the following pdf [11]:

$$\Pr(\gamma_k(t+\tau)\,|\,\mathbf{h}_k(t)) \tag{16}$$

$$=\frac{\xi_k+1}{\Omega_k}e^{\left(-\xi_k-\frac{(\xi_k+1)\gamma_k(t+\tau)}{\Omega_k}\right)}I_0\left(2\sqrt{\frac{\xi_k(\xi_k+1)\gamma_k(t+\tau)}{\Omega_k}}\right)$$

where

$$\xi_k=\left|\hat{h}_k(t+\tau)\right|^2/\sigma^2_{\varepsilon_h,k}(t+\tau) \tag{17}$$

$$\Omega_k=\left|\hat{h}_k(t+\tau)\right|^2+\sigma^2_{\varepsilon_h,k}(t+\tau) \tag{18}$$

and $\Pr(\gamma_k(t+\tau)\,|\,\mathbf{h}_k(t)=0)$ for $\gamma_k(t+\tau)<0$. Eq. (14) is solved by evaluating

$$\Pr(\gamma_k(t+\tau)\le\gamma_k(t))=\int_0^{\gamma_k(t)}\Pr(\gamma_k(t+\tau)\,|\,\mathbf{h}_k(t))d\gamma_k(t+\tau) \tag{19}$$

that unfortunately does not allow for a closed form solution that could be convenient for the practical implementation of the scheduler. However, the pdf (16) is tightly approximated by the Gamma density (according to the Rice-Nakagami duality [11]), and the approximated closed form for (19) is easily obtained as the Gamma distribution:

$$\Pr(\gamma_k(t+\tau)\le\gamma_k(t))\approx\frac{\Gamma(\eta_k,\lambda_k\gamma_k(t))}{\Gamma(\eta_k)}, \tag{20}$$

where $\Gamma(z)=\int_0^\infty u^{z-1}e^{-u}du$ is the Gamma function, $\Gamma(z,\alpha)=\int_0^\alpha u^{z-1}e^{-u}du$ is the incomplete Gamma function, $\eta_k=(\xi_k+1)^2/(2\xi_k+1)$ and $\lambda_k=\eta_k/\Omega_k$.

By plugging (20) into (14) we achieve the approximated closed form solution for (6) to be used in the Predictive-SBS Type 2.

The robustness of the Predictive-SBS Type II towards unreliably predicted samples is intuitively proved by observing that the value of Eq. (20) approaches $1/2$ for $\tau\to\infty$. Therefore, unreliable samples do not actively contribute to the assignment of the scheduler. This happens e.g., when the predicted samples fall beyond the channel coherence time.

Notice that to ease the implementation, the Gamma and the incomplete Gamma functions can be tabulated in a lookup table. The Predictive-SBS Type 2 has a simple implementation that does not need any suboptimal iterative optimization as it is required for other predictive schedulers [4].

## V. Performance Evaluation

In the following the performance of the Predictive-SBS proposed in this paper is numerically compared to those of the standard SBS and the PFS. The frame duration is set to $T_f = 5ms$ in order to perform the simulations with a granularity of the downlink resource assignment that is compliant to the 3GPP LTE (E-UTRA) [12] and the IEEE WiMAX 802.16e specifications [13]. All the $K$ active users move with speed $v = 3Km/h$ and are assumed to have infinite buffers of delay tolerant (e.g., best effort or video streaming) data to receive from the BS. The carrier frequency is set to $f_c = 2GHz$ . The scheduled users are assumed to receive data with an achievable rate (spectral efficiency) equal to $R_k(t) = \log_2(1+\gamma_k(t))\, b/s/Hz$ . The i.i.d. unit power channels are temporally correlated according to their speed, these are generated as autoregressive processes with 32 poles [13] obtained by solving the Yule-Walker equations for the band-limited channels with Jakes' correlation.

Unbiased SINR prediction is performed with 32 samples length MMSE power predictor (13), as for the channel stationarity the linear prediction shows to be optimal.

The scheduling window is chosen as $N_w = 50$ time-slots completely projected in the past for the SBS ( $N_{wf} = 0$ ) and is equally divided between $N_{wp} = 25$ past measurements and $N_{wf} = 25$ predicted (future) samples for the Predictive-SBS. The evaluation of Eq. (12) shows that extending the prediction window further than $N_{wf} = 25$ samples is useless, as the prediction MSE is already close to the channel power for $N_{wf} = 25$ . The effective window of the PFS is $N_w = 50$ time-slots.

For comparison we also consider the max SINR scheduler

$$k^*(t) = \arg\max_k \{\gamma_k(t)\} \tag{21}$$

as it maximizes the sum-rate but disregards the fairness constraints. Therefore, the max SINR scheduler (21) achieves an upper bound of the sum-rate. We recall that all the considered schedulers (including the Predictive-SBS) approach the performance of the max SINR scheduler when the channel coherence time is much shorter that the length of the scheduling window.

Figure 1 shows the sum-rate performance of the considered schedulers in a Non-LoS scenario $(\mu_k = 0\ \forall k)$. We highlight that the Predictive-SBS Type 2 (6) proposed in this paper achieves a significant sum-rate gain with respects to the SBS of [7] and the Predictive-SBS Type 1 (5). It should also be stressed that the relative sum-rate gain provided by the Predictive-SBS Type 2 towards the SBS increases with the number of users $K$.

Figure 2 compares the performance of the proposed schedulers for $K = 15$ users in terms of the CCDF of the scheduling jitter (i.e., the time that intercourses between consecutive scheduling events of a given user). Jitter is often regarded as the most stringent QoS requirement for B3G/4G systems, and we believe that the jitter CCDF provides a useful insight on the algorithm fairness. The Predictive-SBS Type 2 (6) and the Predictive-SBS Type 1 (5) provide enhanced fairness than the original SBS [7]. Therefore, we conclude that the Predictive-SBS Type 2 leverages a better trade-off between sum-rate and fairness than the SBS [7], in the considered scenario. It is also clear from Figure 1 that the Predictive-SBS Type 2 achieves a higher sum-rate than the Predictive-SBS Type 1, as the proof that a meaningful performance boost can be achieved by optimally taking into account the prediction uncertainty.

We now investigate the case of heterogeneously distributed user channels, where each user experiences Rice fading statistics with unbalanced Rice factors $|\mu_k|^2 / \nu_k^2$ . We address a $K = 4$ users scenario where user 1 experiences Rayleigh fading while the other users experience full LoS propagation ( $\nu_k = 0$ ). A similar scenario has already been evaluated in [7], in order to compare the SBS and the PFS.

Figure 3 shows the performance of the considered schedulers in terms of scheduling probability of each user. As it was already pointed out in [6-7] the PFS becomes unfair in this scenario, while the SBS and the Predictive-SBS proposed in this paper still achieve fairness in the scheduling probability of different users.

Simulations show that the scheduling is optimal, provided that the channel coherence time is smaller than the scheduling window $N_w$ . However, when the channel coherence time is larger than the scheduling window, it is necessary to provide the scheduler with additional fairness enforcement controls in order to prevent users to stack on favourable (and slowly varying) channel conditions.

## VI. Conclusions

Recent results on the topic of channel prediction have been employed for the optimization of CDF based schedulers such as the score-based scheduler. Numerical results prove that the predictive scheduler proposed in this paper improves the trade-off between sum-rate and fairness with respects to non predictive CDF schedulers with a definite advantage in term of practical implementation for next generation wireless systems.

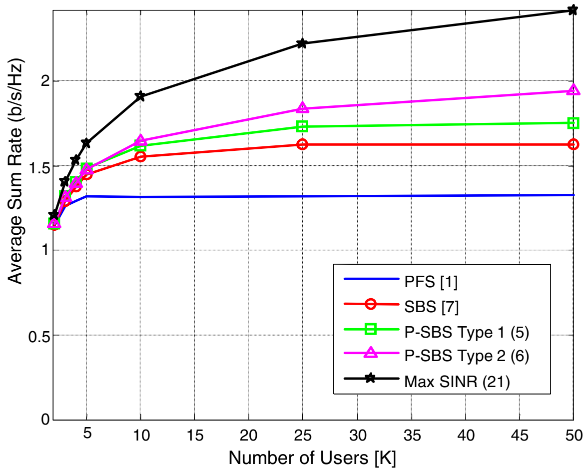


Figure 1. Sum-rate performance as a function of the number of users *K*.

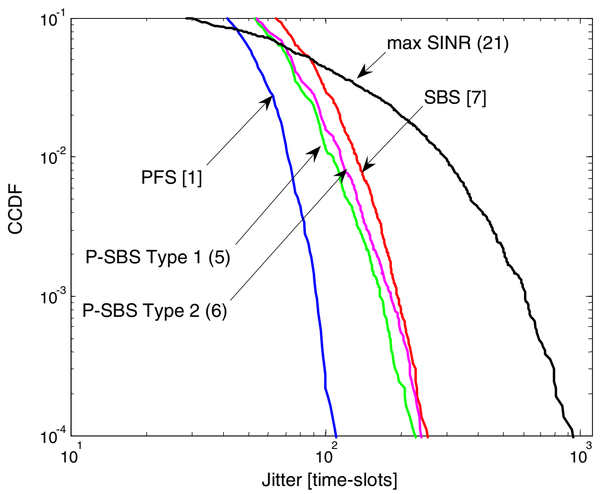


Figure 2. Complementary CDF of the scheduling jitter for *K*=15 users.

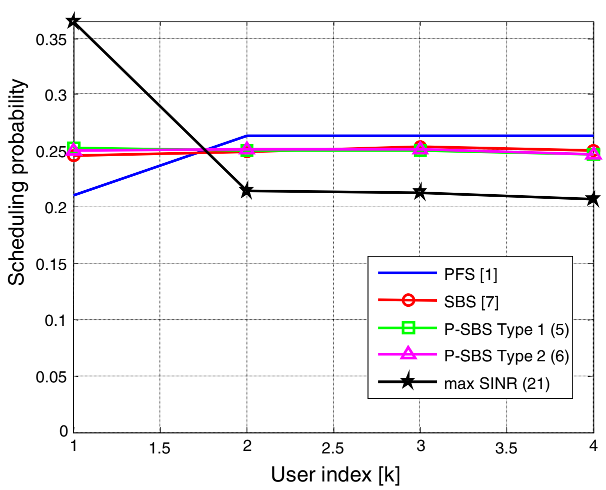


Figure 3. Scheduling probability of users with heterogeneous channel fading distributions (user 1 experiences N-LoS propagation, users 2-4 experience LoS channels).

## REFERENCES


[1] Viswanath, D. N. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," IEEE Trans. Inform., vol. 48, no. 6, pp. 1277–1294, June 2002

[2] T. Ekman, "Prediction of Mobile Radio Channels. Modeling and Design", PhD Thesis, Signals and Systems,Uppsala University, Sweden, 2002. Online: www.signal.uu.se/Publications/abstracts/a023.html

[3] S. Falahati, A. Svensson, T. Ekman and M. Sternad, "Adaptive modulation systems for predicted wireless channels," IEEE Trans. on Communications, vol. 52, Feb. 2004, pp. 307-316.

[4] H. Bang, T. EKman, D. Gesbert, "A channel predictive proportional fair scheduling algorithm ", Proceedings of IEEE Workshop on Signal Processing Advances in Wireless Communications, 2005, New York.

[5] H. Kushner, P. Whiting, "Convergence of Proportional Fair Sharing Algorithms under General Conditions", Technical report, Bell Laboratories, 2002.

[6] J.M. Holtzman, "Asymptotic analysis of Proportional Fair algorithm", Proc. of 12th IEEE PIMRC, 2001.

[7] T. Bonald, "A score-based opportunistic scheduler for fading radio Channels", Proc. of European Wireless, 2004

[8] D. Park, H. Seo, H. Kwon, B. Gi Lee, "Wireless Packet Scheduling Based on the Cumulative Distribution Function of User Transmission Rates", IEEE Trans. On Communications, vol. 53, Nov. 2005

[9] S. Patil and G. de Veciana, "Measurement-Based Opportunistic Scheduling for Heterogenous Wireless Systems", submitted to IEEE Trans. On Communications, 2006

[10] S. M. Kay, "Fundamentals of Statistical Signal Processing. Estimation Theory", Prentice Hall, 1993

[11] G. L. Stuber, "Principles of Mobile Communication. Second Edition.", Kluwer Academic Publishers, 2002

[12] 3GPP TR 25.814, "Physical layer aspects for evolved Universal Terrestrial Radio Access (UTRA)", Release 7, 2006

[13] WiMAX Forum, "Mobile WiMAX – Part I: A Technical Overview and Performance Evaluation", August 2006. Online: www.wimaxforum.org

[14] K. E. Baddour , N. C. Beaulieu, "Autoregressive Models for Fading Channel Simulation", IEEE Global Telecommunications Conference, Globecom 2001, pp. 1187-1192.