Diversity-Multiplexing tradeoff in multi-user scenario with selective feedback

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Abstract— **In this paper, we explore the existing tradeoff in using either OSTBC or opportunistic beamforming techniques with multiple beams (RB-MUX) in multi-user systems with selective feedback. We derive a closed-form expression of the ergodic system capacity for OSTBC and an approximation for the high-SNR regime for RB-MUX. By doing so, we *analytically* assess the impact of the number of terminals and bandwidth restrictions in the feedback channel on both the OSTBC and RB-MUX approaches. In particular, we show that RB-MUX schemes are more effective for exploiting multi-user diversity gain. However, OSTBC approaches give more benefits when either the feedback channel is restricted or the SNR of the system is increased.**

I. Introduction

In the downlink of a wireless multi-user system, it is well known that the average cell throughput can be increased when in each slot the user with better channel conditions is scheduled [1]. Such an effect is referred to as multi-user diversity (MUD) and relies on the assumption that different users experience independent fading processes [2]. On the other hand, communication schemes employing multiple antennas at the transmit and/or receive edges are known to provide remarkable improvements with respect to single-antenna configurations. Since multiple antennas systems have been proposed in novel wireless networks, much attention has been recently paid to the combined use of MUD and multi-antenna techniques.

Multiple antennas can be employed to increase the reliability of the transmission in presence of fading by using spatial diversity mechanisms. In particular, Orthogonal Space-Time Block Coding (OSTBC) is known to provide full diversity order schemes by using low complexity receivers [3][4]. Recently, several papers (e.g., [5], [6]) have analyzed the performance of OSTBC scheme in multiuser systems. It has been shown that OSTBC limits the MUD performance gain compared to the Single-Input-Single-Output (SISO) transmission scheme. The reason for that being that OSTBC schemes are designed to reduce the probability of deep fades but, by averaging over different transmit diversity branches, SNR peaks are suppressed too. However, in multiuser systems with imperfect feedback, the increased robustness of OSTBC schemes against fading provides significant gains with respect to those of SISO approaches. For instance, in [7] the author analyzed the impact of *delays* in the feedback channel. The consequences of *bandwidth restrictions* were explored in [8].

Nonetheless, multi-antenna capabilities can be exploited to serve several users simultaneously. In particular, the capacity region of the Gaussian multi-antenna broadcast channel can be achieved with dirty paper coding (DPC) [9]. However, DPC may not be considered an appropriate scheme for real applications, since it is not easy implementable due to the successive encoding and decodings. Furthermore, DPC requires *perfect* CSI, which is seldom available at the BS. For that reason, opportunistic beamforming schemes with multiple beams based on partial channel side information (CSI) at the base station (BS) has been recently attracted significant interest [10]. The main idea of RB-MUX[1] is to generate a random beamforming at the BS and to schedule the users that maximize the signal-to-noise ratio (SNR). In this scheme, users have to report only the SNR relative to the selected precoding, thus the amount of information to be sent in the feedback is considerably reduced. Performance has been shown to be very effective for systems with a large number of users, as the sum capacity for partial CSI has the same grow-rate as for DPC [10].

In this paper, we asses the tradeoff between OSTBC and RB-MUX schemes in a selective MUD scenario [11]. We focus on these schemes due to their applicability in wireless networks: low complexity receivers can be used and a low amount of information is required in the feedback channel. In particular, we conduct an analytical study of the impact of bandwidth restrictions in the feedback channel on both the OSTBC and RB-MUX approaches. To do that, we derive closed-form expressions of the system capacity for the former case, whereas an approximation for the high-SNR regime is obtained for RB-MUX. We analytically show that MUD is better exploited by RB-MUX schemes in scenarios without restrictions in the feedback channel. As soon as the feedback channel begins to be restricted (i.e., when the feedback load is reduced), the OSTBC strategy becomes to be more effective due to its inherent robustness against fading deeps. Furthermore, OSTBC scheme is more suitable for very high SNR, as RB-MUX technique is interference limited.

II. Signal model

Consider the downlink of a Multi-Input-Single-Output (MISO) cellular system with one base station (BS) equipped with M antennas and K single-antenna terminals. For an arbitrary time instant, the received signal at the terminal k is given by:

$$r_k = \mathbf{h}_k^T \mathbf{s} + n_k \tag{1}$$

where $\mathbf{h}_k \in \mathbb{C}^M$ is the channel vector gain between the BS and the terminal k, for which each component is assumed to be independent and identically distributed (i.i.d.), circularly symmetric Gaussian random variable (Rayleigh fading) with

[1]In the sequel, the random beamforming scheme with multiplexing capabilities is referred to as RB-MUX.

zero mean and variance σ_h^2 (i.e., $\mathbf{h}_k \sim \mathcal{CN}(\mathbf{0}, \sigma_h^2\mathbf{I}_M)$). Vector $\mathbf{s} \in \mathbb{C}^M$ contains the symbols sent from the BS and $n_k \sim \mathcal{CN}(0,1)$ denotes additive Gaussian noise (AWGN) with unitary power. Further, we consider quasi-static fading, i.e, the channel response remains constant during one time-slot and, then, it abruptly changes to a new independent realization. Concerning channel state information (CSI), we assume perfect knowledge for *each* user at the receive side, and the availability of a low-rate error-free feedback channel to convey partial CSI to the transmitter. Finally, the power transmitted by the BS is $P_t = E[\mathbf{s}^H\mathbf{s}]$, so that the average SNR (equal for all the users) is denoted as $\bar{\gamma} = \frac{P_t}{M}E[||\mathbf{h}_k||^2] = P_t\sigma_h^2$. At the BS, we will consider two transmission schemes: OSTBC and RB-MUX strategies.

- *OSTBC (Orthogonal Space-Time Block Coding):*

For the spatial diversity scheme the effective SNR experienced by user k in time-slot s, after coherent combining, conforms

$$\gamma_{k\_ST}(s) = \frac{P_t}{M}\sum_{i=1}^{M}|h_{i,k}|^2. \quad (2)$$

It is easy to notice that the SNR $\gamma_{k\_ST}(s)$ is a χ^2_{2M} random variable (RV) with average $E[\gamma_{k\_ST}(s)] = \bar{\gamma}$. In each time-slot the scheduler selects a single user to be served according to the instantaneous SNR $\gamma_{k\_ST}(s)$ as detailed in Sect. III. The probability density function (PDF) and the cumulative density function (CDF) are respectively

$$f_{\gamma_{k\_ST}}(\gamma) = \frac{1}{(M-1)!(\bar{\gamma}/M)^M}\gamma^{M-1}e^{-\gamma M/\bar{\gamma}}, \gamma \geq 0 \quad (3)$$

$$F_{\gamma_{k\_ST}}(\gamma) = 1 - e^{-\gamma M/\bar{\gamma}}\sum_{m=0}^{M-1}\frac{(\gamma M/\bar{\gamma})^m}{m!}, \gamma \geq 0 \quad (4)$$

- *RB-MUX (Random Beamforming-Multiplexing):*

At the beginning of any time slot s the BS constructs a set of M orthogonal beamforming vectors $\mathbf{u}_m$ so that the transmitted signal $\mathbf{s} = \sum_{m=1}^{M}\mathbf{u}_m x_m$ is the superposition of M beams ,where x_m stands for the unitary power information symbols stream. Power is evenly distributed to the beams, thus it is $||\mathbf{u}_m(t)||^2 = \frac{P_t}{M}$. The receiver k is assumed to perfectly know the value $\mathbf{h}_k^T\mathbf{u}_m$ (this can be readily arranged during training) and it computes the $SINRs$ by assuming that x_m is the desired signal and x_p is interference (for $p \neq m$)

$$\gamma_{k,m\_MUX}(s) = \frac{|\mathbf{h}_i^T\mathbf{u}_m|^2}{1+\sum_{p=1,p\neq m}^{M}|\mathbf{h}_i^T\mathbf{u}_p|^2} = \frac{z}{\frac{M}{\bar{\gamma}}+y}, \quad (5)$$

for $m = 1,..,M$. Each beam is assigned to a different user according to the SINR metric $\gamma_{k,m\_MUX}(s)$ (see Sect. III). In order to evaluate the performance we have to obtain the distribution of the SINR. Since the beamforming vectors are orthogonal and the users experience Rayleigh fading, $|\mathbf{h}_i^T\mathbf{u}_m|^2$ are i.i.d over m (and also over i) with χ^2_2 distribution and average value $\bar{\gamma}/M$. Thus, it holds that z and y are independent RVs with distributions $z \sim \chi^2_2$ and $y \sim \chi^2_{2M-2}$. Finally, the CDF of the SINR yields to

$$F_{\gamma_{k,m\_MUX}}(\gamma) = 1 - \frac{e^{-\gamma/\bar{\gamma}}}{(1+\gamma)^{M-1}} \quad (6)$$

Evaluation of the overall system performance from the CDF of the SINR (6) is far from being trivial and numerical integration turns out to be mandatory. In order to make an analytical derivation feasible, we assume that the system is interference limited and the additive noise as negligible. The assumption is reasonable for large average SNR $\bar{\gamma}$ and/or large number of antennas M. Simulation results validate the assumption in the analyzed scenario. The SINR $\gamma_{k,m\_MUX}(s)$ reduces to a SIR, that is recognized as the ratio z/y of independent chi-squares variables, thus it holds

$$f_{\gamma_{k,m\_MUX}}(\gamma) = \frac{M-1}{(1+\gamma)^M}, \quad (7)$$

$$F_{\gamma_{k,m\_MUX}}(\gamma) = 1 - \frac{1}{(1+\gamma)^{M-1}}. \quad (8)$$

III. Post-scheduling SNR statistics

The scheduling process is organized in a slot-by-slot basis following a *max SNR (for OSTBC)* or *SINR (for MUX-OB)* rule. More specifically, the OSTBC strategy selects in each time-slot the user with the maximum SNR $\gamma_{k\_ST}(s)$, whereas the RB-MUX scheduler allocates each beam to the active user with the highest SINR $\gamma_{k,m\_MUX}(s)$ under the constraint that different beams are assigned to different users. If one user achieves the highest metric over more than one beam, it is scheduled on the strongest beam and it does not compete to the allocation of the other beams. In this particular case, the scheduler does not select the best user over each beam. Anyway, the probability of this event is negligible when the number of users is large compared to the number of antennas ($K >> M$) [10]. In the sequel we will assume that the each user can not have the highest metric over more than one beam, thus the scheduler can always select the strongest metric over each beam. Under this assumption we can provide a unified framework by considering that in both schemes the scheduler maximizes the metric $\gamma_k(s)$ standing respectively for the SNR $\gamma_{k\_OSTBC}(s)$ in OSTBC and for the SINR $\gamma_{k\_MUX}(s)$ (for a given m) in RB-MUX.

In order to reduce bandwidth requirements in the feedback channel, a $Selective$ Multi-user Diversity (SMUD) approach is adopted [11]. In other words, only terminals experiencing metric above a pre-defined threshold (γ_{th}) in a specific time-slot are allowed to report their channel state information to the BS. Thus, the max-scheduler conducts the search over such a subset of the active users only, that is

$$\gamma^*(s) = \max{}_k\{\gamma_k(s)\} \text{ s.t. } \gamma_k(s) > \gamma_{th} \quad (9)$$

Conversely, when all the users remain silent (i.e. in the event of a *scheduling outage*) the scheduling rule amounts to

$$\gamma^*(s) = rand_k\{\gamma_k(s)\} \text{ for } k = 1,..,K \quad (10)$$

where rand is the random pick operator. In the sequel, subscript s will be dropped for the ease of notation. As for the *post-scheduling* metric γ^* the analysis must be conducted for two different SNR regions: $\gamma \leq \gamma_{th}$ (i.e., all users remain silent), and $\gamma > \gamma_{th}$ (at least one user reports its CSI to the BS). For the $\gamma \leq \gamma_{th}$ case and by recalling that all users experience i.i.d fading, we have:

$$F_{\gamma^*}(\gamma) = \text{Prob}(\gamma^* \leq \gamma, \gamma_k \leq \gamma_{th} \text{ for all } k = 1...K) = (F_\gamma(\gamma_{th}))^{K-1} F_\gamma(\gamma). \quad (11)$$

On the other hand, for $\gamma > \gamma_{th}$, the CDF function can be expressed as:

$$F_{\gamma^*}(\gamma) = \text{Prob}(\gamma_1 \leq \gamma, \ldots, \gamma_K \leq \gamma) = (F_\gamma(\gamma))^K. \quad (12)$$

IV. Feedback load

In [11], the authors define *normalized average feedback load* $\bar{F}$ as the usage ratio per time slot averaged over the total number of active users. This measure can also be interpreted as the probability for a given user to effectively send its CSI over the feedback channel. Thus, the metric threshold γ_{th} must be designed to meet a desired feedback load $\bar{F}$. Since all the users experience i.i.d. fading channels, it yields

$$\bar{F} = \text{Prob}(\gamma_k(s) > \gamma_{th}) = 1 - F_\gamma(\gamma_{th}) \quad (13)$$

In OSTBC strategy each user sends to the BS the instantaneous SNR γ_{k_ST}, thus the SNR threshold γ_{th} for a given feedback load $\bar{F}$ can be directly obtained by plugging Eq. (4) into (13).

RB-MUX scheduler is based on the SINR metric γ_{k,m_MUX}. Since each user can not be scheduled over more than one beam and we assume that each user can be the strongest over one beam at most, feedback the SINRs γ_{k,m_MUX} for all the M beams would be a waste of resource. The amount of information sent to the scheduler can be equivalently restricted to the transmission of the maximum SINR γ_{k,m^*_MUX} along with the index m^* of the beam where the SINR is maximized. Since the transmission of the index m^* requires a negligible rate, we remark that the amount of feedback in OSTBC and RB-MUX is similar (one real coefficient for each user), thus making possible a fair performance comparison. Said that, we further restrict the feedback channel by designing a threshold γ_{th} on the feedback of the maximum SINR γ_{k,m^*_MUX}. The threshold, γ_{th} for a given feedback load $\bar{F}$ can then be obtained by plugging in Eq. (13) the CDF of the maximum SINR, that can be derived from (6) as

$$F_{\gamma_{k,m^*\_MUX}}(\gamma) = \left(1 - \frac{e^{-\gamma/\bar{\gamma}}}{(1+\gamma)^{M-1}}\right)^M. \quad (14)$$

V. System capacity

Since the users channels are i.i.d., the scheduler guarantees a fair long-term resource allocation, thus we focus the attention on the average system performance. In a multi-user system, the instantaneous channel capacity achievable by the scheduled user over the *equivalent* SISO channel is given by $C^* = \log_2(1+\gamma^*)$,where the scheduled metric γ^* stands for the SNR in OSTBC and for the SINR in RB-MUX transmission scheme, respectively. Thus, the ergodic system capacity under the proposed scheduling policy can be expressed as

$$\overline{C}^* = E_{\gamma^*}[C^*] = \int_0^\infty r_{TX}(M) \log_2(1+\gamma) f_{\gamma^*}(\gamma) d\gamma \quad (15)$$

where $r_{TX}(M)$ stands for the rate of the transmission scheme.

- *OSTBC (Orthogonal Space-Time Block Coding):*

Deriving a closed-form expression of the average capacity for the OSTBC scheme is somewhat involving. By using (4), (11) and (12) into (15) and after some algebra, the average capacity reduces to the following intermediate expression (see Appendix I)

$$\frac{\overline{C}^*}{\log_2(e)} = \frac{r_{ST}(M)M^M}{\bar{\gamma}^M(M-1)!}[F_\gamma(\gamma_{th})^{K-1} \cdot \Psi_M(\gamma_{th}, M/\bar{\gamma}) + K \sum_{k=0}^{K-1} \binom{K-1}{k} (-1)^k \sum_{n_1=0}^{k} \binom{k}{n_1} \sum_{n_2=0}^{n_1} \binom{n_1}{n_2} \left(\frac{1}{2!}\right)^{n_2} \cdots \sum_{n_{M-1}=0}^{n_{M-2}} \binom{n_{M-2}}{n_{M-1}} \left(\frac{2!3!..(M-2)!}{(M-1)!}\right)^{n_{M-1}} \left(\frac{M}{\bar{\gamma}_i}\right)^{m'} \cdot \overline{\Psi}_{M+m'}(\gamma_{th}, (k+1)M/\bar{\gamma}_i)], \quad (16)$$

where $m' = \sum_{j=1}^{M-1} n_j$ and we define the integrals $\Psi_m(a,\mu)$ and $\overline{\Psi}_m(a,\mu)$ as

$$\Psi_m(a,\mu) = \int_0^a \ln(1+\gamma)\gamma^{m-1}e^{-\mu\gamma}d\gamma, \quad (17)$$

$$\overline{\Psi}_m(a,\mu) = \int_a^\infty \ln(1+\gamma)\gamma^{m-1}e^{-\mu\gamma}d\gamma, \quad (18)$$

$$\text{for } \mu > 0 \text{ and } m = 1, 2, ..$$

The summation accounts for two different contributions: the capacity coming from the random scheduling (i.e., in the case of *scheduling outage*) and the contribution due to the opportunistic scheduling. The integrals in expressions (17) and (18) are solved in closed-form in [8], where the interested reader can find the details. Here we discuss the limiting case of no-feedback ($F = 0$) and perfect feedback ($F = 1$). In the former case it is $\gamma_{th} \to \infty$, the second term vanishes and the integral reduces to

$$\Psi_m(a \to \infty, \mu) = (m-1)! e^\mu \sum_{k=1}^{m} \frac{\Gamma(k-m,\mu)}{\mu^k} \quad (19)$$

where $\Gamma(\alpha, x) = \int_x^\infty t^{\alpha-1}e^{-t}dt$ is the complementary incomplete Gamma function defined in [[13], Eq. 8.350.2]. The capacity yields

$$\frac{\overline{C}^*}{\log_2(e)} = r_{ST}(M) e^{M/\bar{\gamma}_i} \sum_{k=0}^{M-1} \frac{\Gamma(-k, M/\bar{\gamma})}{(M/\bar{\gamma})^k}, \quad (20)$$

thus reducing (up to term $r_{ST}(M)$ and the power loss associated to OSTBC scheme [4]) to the performance of the maximum ratio combiner (MRC) receiver [12]. In the dual case of complete feedback ($F = 1$), it is $\gamma_{th} \to 0$ and the integral can be solved by plugging $\overline{\Psi}_m(0,\mu) = \Psi_m(\infty,\mu)$ into Eq. (16).

- *RB-MUX (Random Beamforming-Multiplexing):*

The average capacity of the RB-MUX is obtained here in closed-form under the assumption of interference dominated system. To simplify the analytical derivation, we insert the user capacity $C_m = \log_2(1+\gamma)$ (scheduled over the beam m) into integral (15) as

$$\overline{C}^* = r_{_{MUX}}(M)\int_0^\infty C \cdot f_{C_m}(C)\, dC. \qquad (21)$$

The transmission rate is $r_{_{MUX}}(M) = M$ as M independent beams are transmitted. From Eqs. (8), (11) and (12) the capacity CDF results

$$\begin{aligned} \text{Prob}\left(C_m < C\right) &= F_\gamma\left(\gamma_{th}\right)^{K-1}(1-e^{-C/\overline{C}}), \text{ for } C \le C_{th} \\ &= (1-e^{-C/\overline{C}})^K, \text{ for } C > C_{th} \end{aligned} \qquad (22)$$

where $C_{th} = \log_2(1+\gamma_{th})$ is the equivalent capacity threshold and $\overline{C} = \frac{\log_2 e}{M-1}$ is the average beam capacity. It is easy to notice that the single user scheduled capacity C_m is distributed as a χ_2^2 RV in random scheduling region ($C \le C_{th}$), whereas it is the maximum of K i.i.d RVs in the max-scheduling region ($C > C_{th}$). By plugging (22) into (21) the ergodic capacity can be expressed as

$$\begin{aligned} \overline{C}^* &= \frac{M}{\overline{C}}[F_\gamma\left(\gamma_{th}\right)^{K-1}\int_0^{C_{th}} C \cdot e^{-C/\overline{C}} dC \\ &\quad + K\int_{C_{th}}^\infty C(1-e^{-C/\overline{C}})^{K-1}e^{-C/\overline{C}} dC]. \end{aligned} \qquad (23)$$

Similarly to OSTBC (16), the first term accounts for random scheduling, whereas the second term reflects for max SINR scheduling. By using the binomial expansion and identities [[13], Eq. 3.351.7] and [[13], Eq. 3.351.2] the following expression results

$$\begin{aligned} \frac{\overline{C}^*}{M} &= \overline{C}\cdot[F_\gamma\left(\gamma_{th}\right)^{K-1}(1-\frac{1+C'_{th}/\overline{C}}{(1+\gamma_{th})^{M-1}}) + K \cdot \\ &\sum_{k=0}^{K-1}(-1)^k\binom{K-1}{k}\frac{e^{-(k+1)C'_{th}/\overline{C}}}{(1+k)^2}(1+(k+1)C'_{th}/\overline{C})]. \end{aligned} \qquad (24)$$

where $C'_{th} = C_{th}/\log_2(e)$. Expression (24) further simplifies when considering no-feedback channel ($F = 0$) or full feedback load ($F = 1$). In the former case, it is $\{\gamma_{th}, C_{th}\} \to \infty$ and the capacity specializes to

$$\overline{C}^* = M \cdot \overline{C}. \qquad (25)$$

In the full feedback case ($\overline{F} = 1$), it is $\{\gamma_{th}, C_{th}\} = 0$, thus leading to

$$\overline{C}^* = M \cdot \overline{C} \cdot \sum_{k=1}^{K}\frac{1}{k}. \qquad (26)$$

The term $\sum_{k=1}^{K}\frac{1}{k}$ stands for the multiuser diversity gain and for large K it can be approximated as $\sum_{k=1}^{K}\frac{1}{k} \simeq \log(K)+\varepsilon$, where ε stands for the Euler constant. Nevertheless, it would be incorrect to draw conclusions on the capacity scaling low by letting $K \to \infty$ as the approximation introduced by the assumption of interference dominated system ($\gamma_{k,m\_MUX} \simeq z/y$) increases with the number of users K. As a motivation, the opportunistic scheduler selects the users with the largest SINR (5), thus jointly maximizing the power of the desired signal $|\mathbf{h}_i^T\mathbf{u}_m|^2$ and minimizing that of the interference $\sum_{p\neq m}|\mathbf{h}_i^T\mathbf{u}_p|^2$. As a consequence, the contribution of the mutual interference vanishes by letting $K \to \infty$ and the additive noise can not be neglected. By taking the noise into account, the asymptotic rate for the capacity becomes $log(log(K))$ as shown in [10].

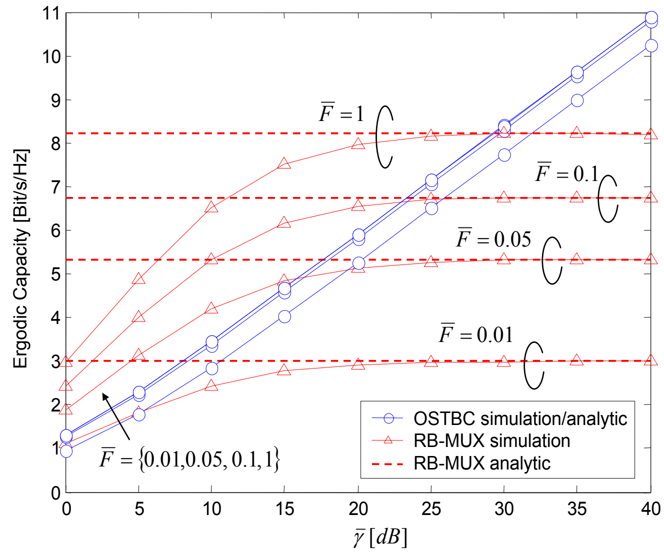

Fig. 1. Ergodic capacity of OSTBC and RB-MUX (MUX) vs. SNR ($\bar{\gamma}$) for $M = 4$ antennas, $K = 40$ users and feedback load $\overline{F} = 1, 0.1, 0.05, 0.01$

VI. Numerical results

Throughout this section, we will consider a system with a BS equipped with $M = 4$ antennas transmitting to K single antenna users. The OSTBC scheme over $M = 4$ antennas achieves a maximum transmission rate $r_{_{OSTBC}}(4) = 3/4$ [4].

Fig. 1 shows the *ergodic* capacity of OSTBC and RB-MUX strategies versus the average system SNR for $K = 40$ users and different average feedback load values ($\overline{F} = 1, 0.1, 0.05, 0.01$). First at all, it worth analyzing the match between the curves associated with the analytical expressions (analytical) and the corresponding computer simulation results (simulation). Perfect match over the whole range is obtained for OSTBC, while the analytical analysis of RB-MUX performance is close to the simulation results for high average SNR ($\bar{\gamma} > 15-20dB$) due to the assumption of interference dominated system. Anyway, this range is of crucial interest since it permits to estimate the threshold SNR where the methods switch in the performance order. In fact, the RB-MUX strategy is more effective at low SNR due to the spatial multiplexing capabilities, while OSTBC achieves larger ergodic capacity at very large SNR as RB-MUX is interference limited. The ergodic capacity as a function of the number of users K is shown in Fig. 2 for $\bar{\gamma} = 20dB$. Analytical expression of the RB-MUX capacity is close to the simulation results for different average feedback load values $\overline{F}$. An accurate analysis shows that the gap between the analytical and simulated curves increase with the number of users K, thus confirming that the approximation does not hold for infinite users as stated in Sect. V. In the full feedback case ($\bar{F} = 1$)

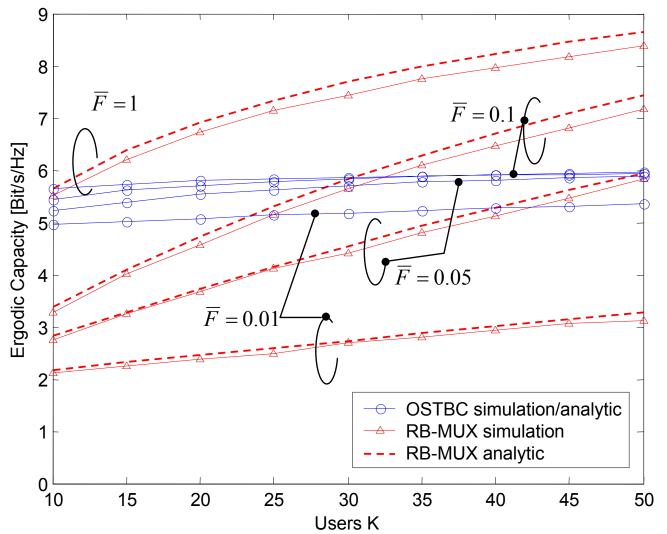

Fig. 2. Ergodic capacity of OSTBC and RB-MUX (MUX) vs. number of users K for $M = 4$ antennas, $\bar{\gamma} = 20dB$ and feedback load $\overline{F} = 1, 0.1, 0.05, 0.01$.

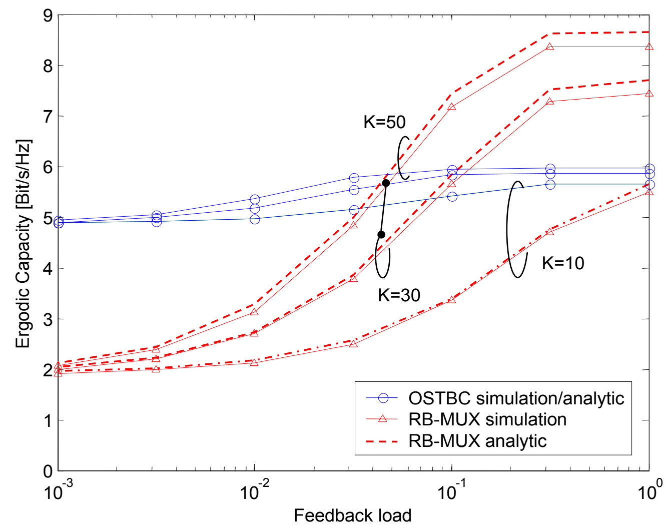

Fig. 3. Ergodic capacity of OSTBC and RB-MUX (MUX) vs. feedback load $\overline{F}$ for $K = 10, 30, 50$ users, $M = 4$ antennas and $\bar{\gamma} = 20dB$.

one can observe that the RB approach is far more effective than its OSTBC counterpart in exploiting multi-user diversity. In other words, the suppression of SNR peaks due to the SNR-stabilizing effect associated to OSTBC penalizes system performance. Conversely, when the average feedback load per user is reduced, the degradation experienced by the RB-based schemes is larger than that exhibited by the OSTBC ones. This follows from the fact that OSTBC approach provides additional robustness against unfavorable fading conditions resulting from random user selection. Similar conclusions can be drawn from Fig. 3, that shows the ergodic capacity as a function of the feedback load for $K = 10, 30, 50$ users.

VII. Conclusions

In this paper we have investigated the performance of spatial diversity and spatial multiplexing techniques in a multi-user scenario with selective feedback. Performance assessment is conducted both analytically and by means of simulation results in terms of ergodic capacity. By our evaluation, the spatial multiplexing approach is very effective in exploiting the MUD when opportunistic scheduling is employed at the BS, whereas the performance degrades considerably when bandwidth restrictions on feedback channel reduce the MUD. On the other hand, the spatial diversity is more suitable for limited feedback as it enhances the transmission robustness against channels fading deeps.

VIII. Acknowledgment

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Appendix I
Proof of Eq. (16)

The expression (16) is obtained by plugging Eqs. (4), (11), (12) into (15), by using the binomial expansion formula on the CDF of the maximum SNR and by exploiting the equality

$$\left(\sum_{m=0}^{M-1}\frac{1}{m!}\left(\frac{\gamma M}{\bar{\gamma}_i}\right)^m\right)^k=\sum_{n_1=0}^{k}\binom{k}{n_1}\sum_{n_2=0}^{n_1}\binom{n_1}{n_2}\left(\frac{1}{2!}\right)^{n_2} \cdot\cdot\sum_{n_j=0}^{n_{j-1}}\binom{n_{j-1}}{n_j}\left(\frac{1}{j!}\right)^{n_j}\cdots\cdot\sum_{n_{M-1}=0}^{n_{M-2}}\binom{n_{M-2}}{n_{M-1}}\left(\frac{2!3!\ldots(M-2)!}{(M-1)!}\right)^{n_{M-1}}\left(\frac{\gamma M}{\bar{\gamma}_i}\right)^{n_1+n_2+\ldots+n_{M-1}} \qquad (27)$$

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