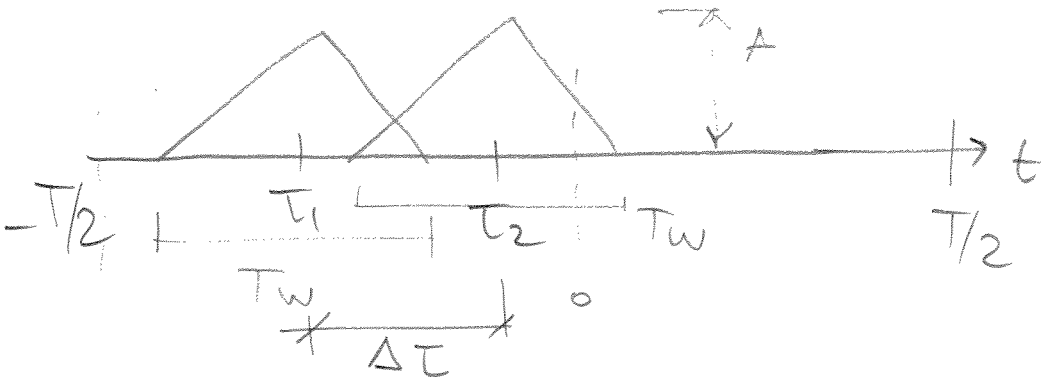
Stime TOA multiple

$$x(t) = \underbrace{w(t-\tau_1) + w(t-\tau_2)}_{s(t|\underline{\tau})} + n(t)$$

$n(t)$: rumore AWGN bilat. $N_0/2$.

(segnale somma di 2 echi ritardati (non attenuati) con forma d'onda nota (ex. radar/sonar attivo/sismica esplorazione/...))



$T \gg T_w$ (finestra di misura ampia)

$\Delta\tau > T_w$ echi non sovrapposti (non interferenti)

$\Delta\tau < T_w$ echi sovrapposti (interferenti)

$$\boxed{\eta \triangleq 1 - \Delta\tau/T_w}$$ sovrapposizione echi (frazione percentuale).

Stima ML

$$\min_{\underline{\tau}} \left\{ \int \left(x(t) - s(t|\underline{\tau})\right)^2 dt \right\} =$$

$\phi_{ww}(\tau_2-\tau_1)$ ← autocorrelaz.

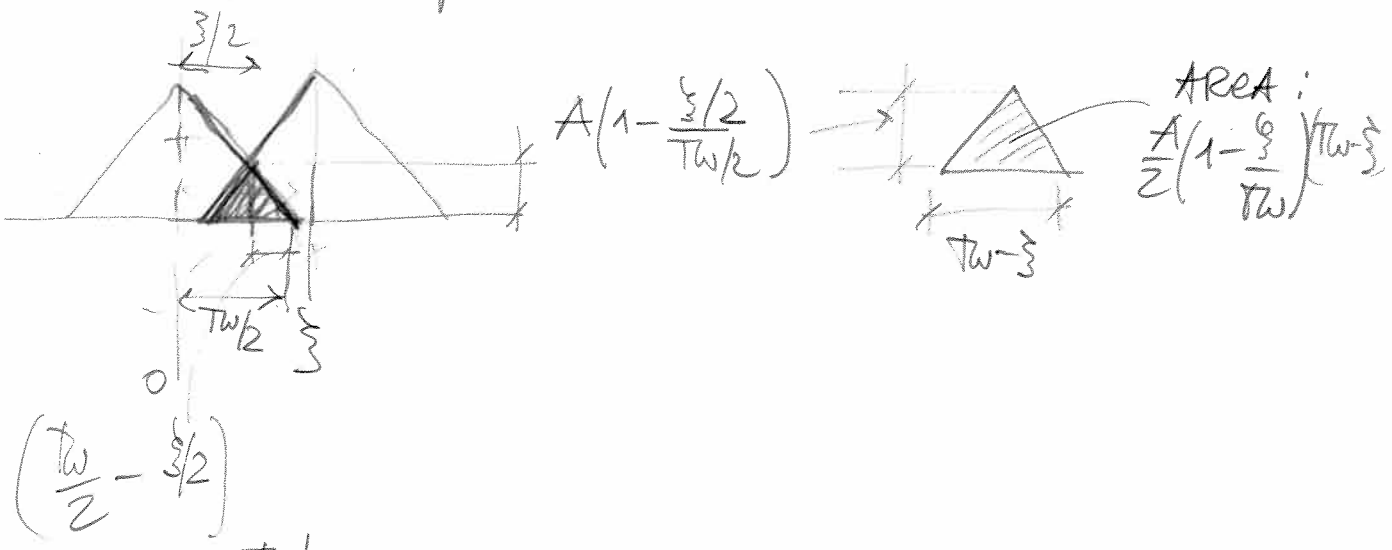
$$= \int x^2(t)\,dt + 2E_w + 2\int w(t-\tau_1)\,w(t-\tau_2)\,dt$$

$$- \underbrace{\int x(t)\,w(t-\tau_1)\,dt}_{\phi_{xw}(\tau_1)} - \underbrace{\int x(t)\,w(t-\tau_2)\,d\tau}_{\phi_{xw}(\tau_2)}$$

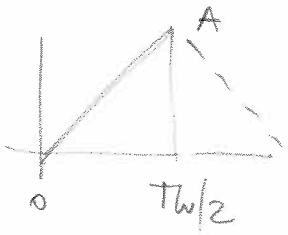
$$\phi_{xw}(\xi) = \int x(t)\,w(t-\xi)\,dt$$ cross-correlazione misure (calcolata 1 volta)

$E_w = \int w^2(t)\,dt = \phi_{ww}(0)$. energie f. fenêtre

Calculons $\phi_{ww}(\xi)$ par convolution :



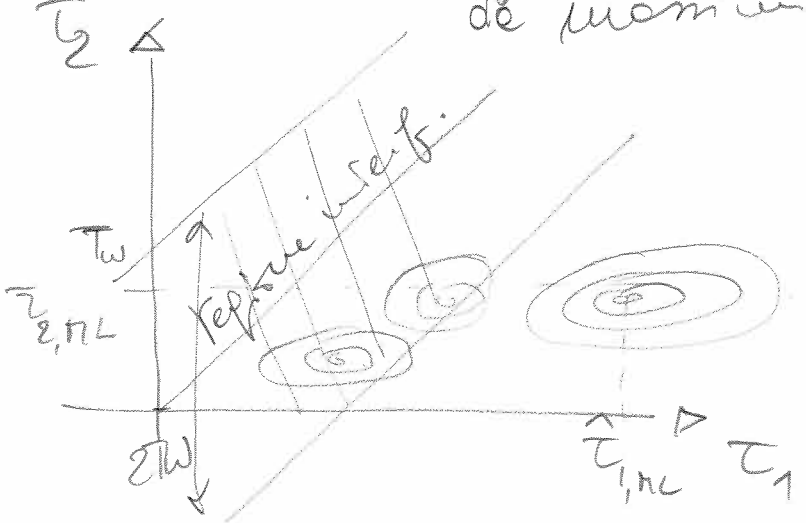
$$E_w = 2 \cdot \int_0^{Tw/2} \left(\frac{2A}{Tw}\,t\right)^2 dt = \frac{8A^2}{Tw^2}\,\frac{(Tw/2)^3}{3} = \frac{A^2 Tw}{3}$$



Sortie ML :

$$\max_{\tau_1,\tau_2} \left\{ \phi_{xw}(\tau_1) + \phi_{xw}(\tau_2) - \phi_{wN}(\tau_2 - \tau_1) \right\}$$

$\psi(\tau_1, \tau_2)$ fonction 2D de maximisation

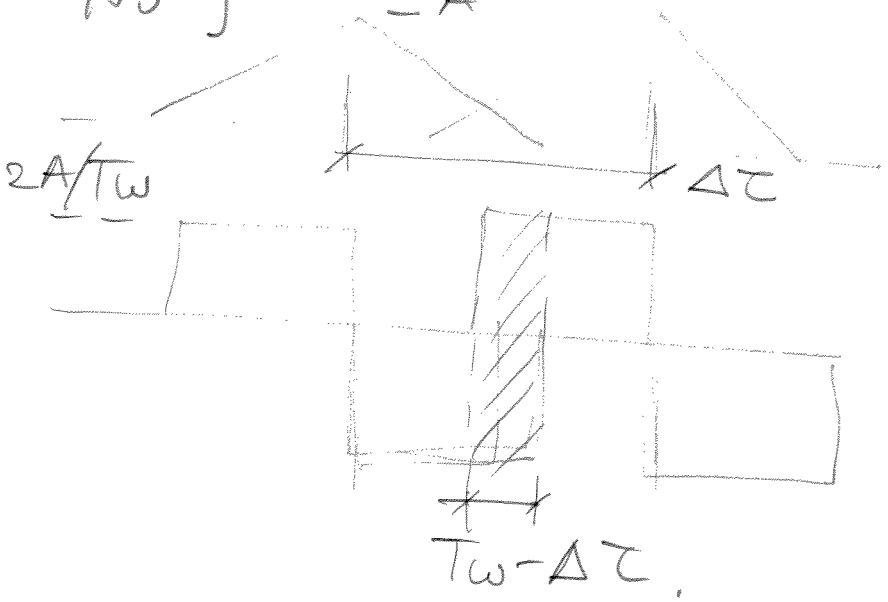


CRB

$$I_{\tau_1\tau_2} = \frac{2}{N_0}\int \frac{\partial s(t|\tau)}{\partial \tau_1}\,\frac{\partial s(t|\tau)}{\partial \tau_2}\,dt =$$

$$= \frac{2}{N_0}\int \frac{\partial w(t-\tau_1)}{\partial \tau_1}\cdot\frac{\partial w(t-\tau_2)}{\partial \tau_2}\,dt =$$

$$= \frac{2}{N_0}\int \dot{w}(t-\tau_1)\,\dot{w}(t-\tau_2)\,dt.$$



$$I_{\tau_1\tau_2} = \frac{2}{N_0}\cdot(T_w-\Delta\tau)\cdot\left(\frac{2A}{T_w}\right)^2 = \frac{8A^2}{N_0 T_w}\eta =$$

$$= \underbrace{\frac{A^2 T_w}{3}}_{E_w}\cdot\frac{3\times 8}{T_w^2}\cdot\frac{1}{N_0}\cdot\eta = \frac{2E_w}{N_0}\cdot\frac{12}{T_w^2}\cdot\eta$$

$$I_{\tau_1\tau_2} = \frac{2E_w}{N_0}\cdot\frac{12}{T_w^2}$$

$(\eta = 1)$

quindi la FIM è:

$$\underline{\underline{I}} = \frac{2\mathcal{E}_w}{N_0} \cdot \frac{12}{T_0^2} \begin{pmatrix} 1 & \eta \\ \eta & 1 \end{pmatrix}$$

$\frac{2\mathcal{E}_w}{N_0} \triangleq SNR.$

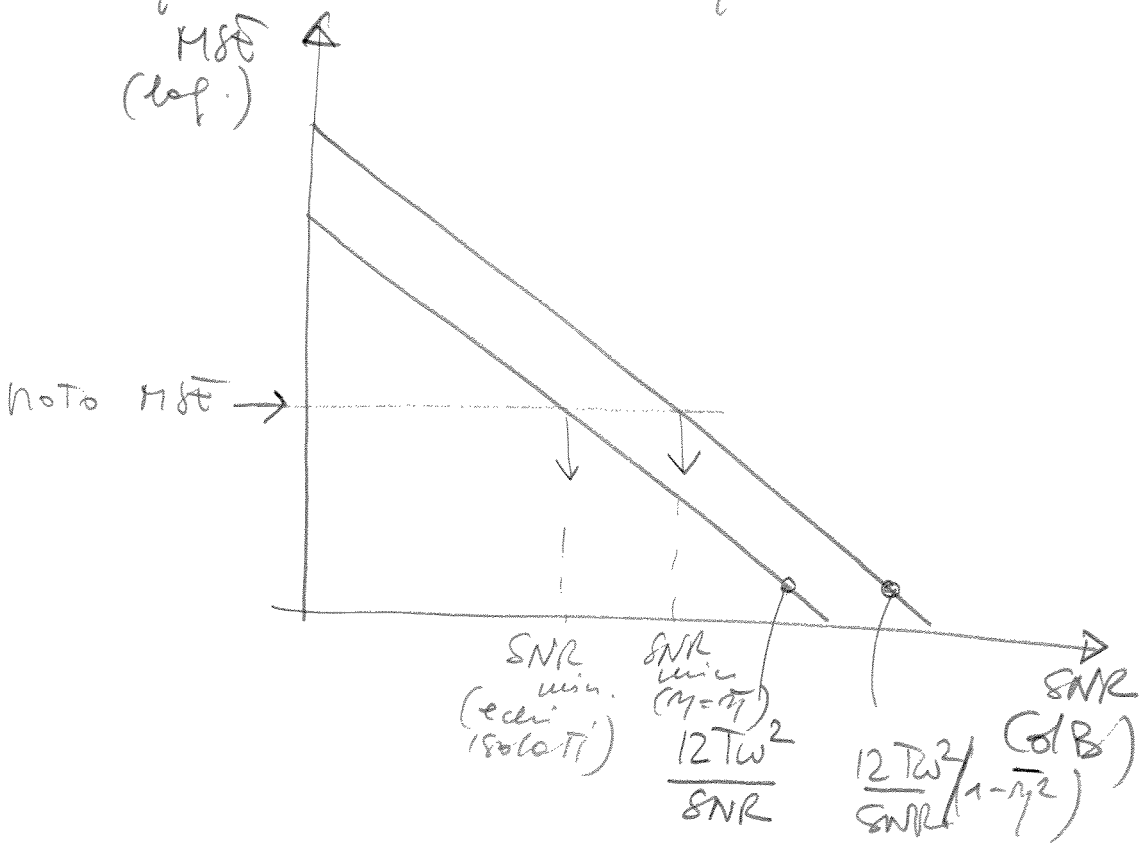
$$CRB = \frac{12T_w^2}{SNR} \cdot \frac{1}{1-\eta^2} \begin{pmatrix} 1 & -\eta \\ -\eta & 1 \end{pmatrix}$$

$$Var(\tau_{1,2}) \geq \frac{12T_w^2}{SNR} \cdot \frac{1}{1-\eta^2}.$$

se $\eta = 0$ (no interf.) allora la stima è disaccoppiata. $Var(\tau_{1/2}) = \frac{12T_w^2}{SNR}$.

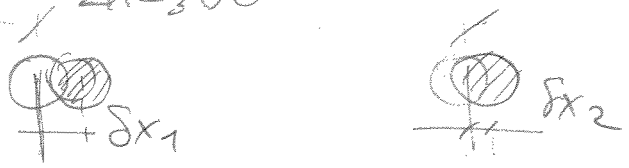
nota che:

$$Var(\tau_{1/2} \text{ per } \eta=0) \leq Var(\tau_{1,2} \text{ per } \eta \neq 0)$$



Esempio di progetto.

Si assume di avere 2 transponder con posizione nominale di $\Delta x = 10\,cm$. le vengono collocati ciascuno con una accuratezza di $\pm 1\,cm$. (uniforme).

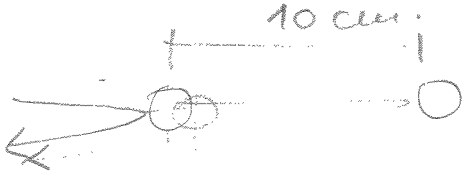


$$x_1 = x_{1nom} + \delta x_1 \rightarrow \delta x_i \sim \mathcal{U}(-1cm, +1cm)$$
$$x_2 = x_{2nom} + \delta x_2 \rightarrow$$

Si vuole misurare il loro posizionamento reale con una accuratezza di 0.3 cm., dimensionare il sistema sapendo che:

1) il sistema radar UWB ha una forma d'onda con risoluzione $T_W = 2\,ns$.
2) la velocità di propag. $c = 30\,cm/ns$.

Calcolo overlapping (nominale) η:



$$\Delta\tau = \frac{2 \times 10\,cm}{30\,cm/ns} = \frac{2}{3}\,ns$$

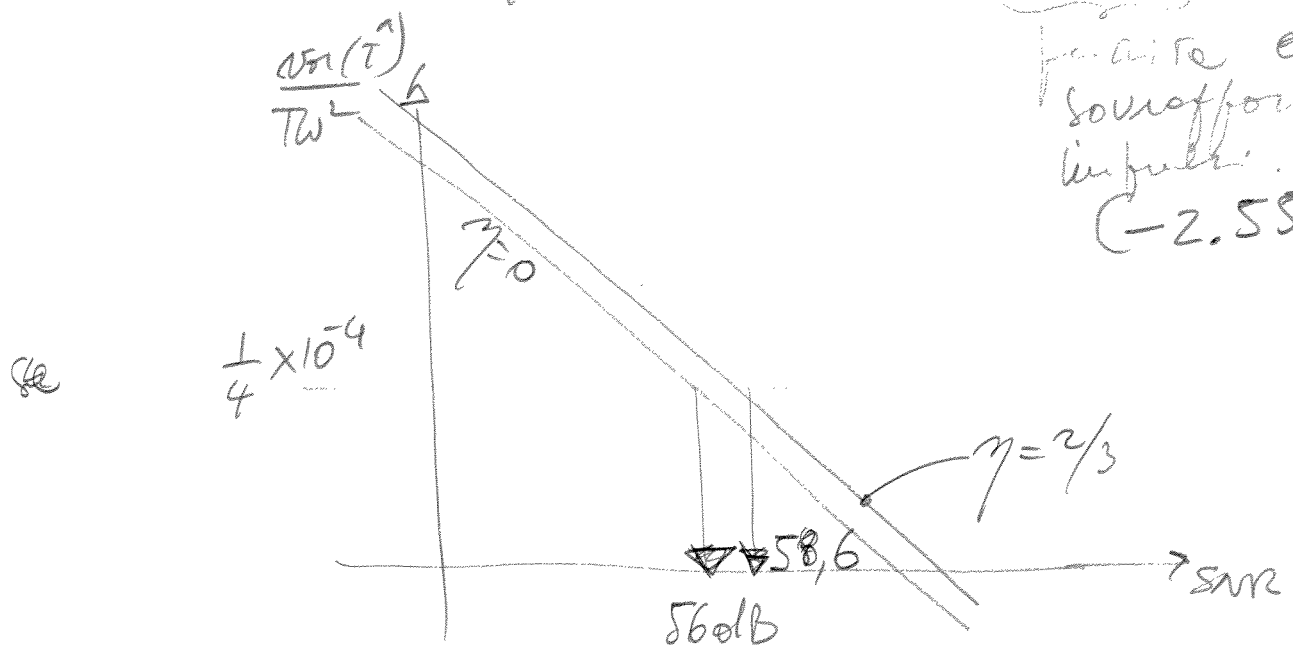
$$\eta = 1 - \frac{2/3\,ns}{2\,ns} = 1 - \frac{1}{3} \simeq 67\%$$

Requisiti: $\underset{max.}{var}(\hat{\tau}) = \frac{var(\hat{x})}{c^2} = \frac{(0.3\,cm)^2}{(30\,cm/ns)^2} = \left(\frac{1}{100}\,ns\right)^2$.

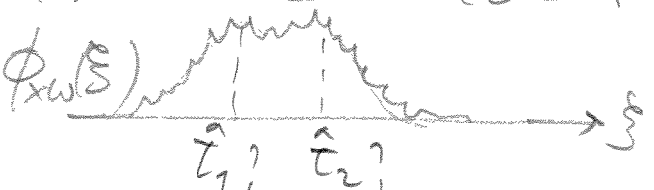
$$\frac{\mathrm{var}(\hat{\tau})}{T_W^2} = \frac{(1/100)^2}{4} = \frac{1}{4}\cdot 10^{-4} \geq \frac{12}{SNR}\cdot\frac{1}{1-\eta^2}$$

$$SNR \geq \frac{48\times 10^4}{1-\eta^2} = 56\,dB - (1-\eta^2)_{dB}$$

(fattore che sovrastima l'impulso: $-2.55\,dB$)



NOTA: adottando uno stimatore ML per 1 solo eco (equivale a ricercare il 1° max della cross-correlazione $\phi_{xw}(\xi)$) non è più possibile raggiungere l'accuratezza desiderata in questo caso (per $\eta=2/3$ il risultato sarebbe circa):



tuttavia non è prevedibile il SNR necessario in questo caso per raggiungere l'accuratezza desiderata — risulta $> 58,6\,dB$.
Risultato solo per simulazione.